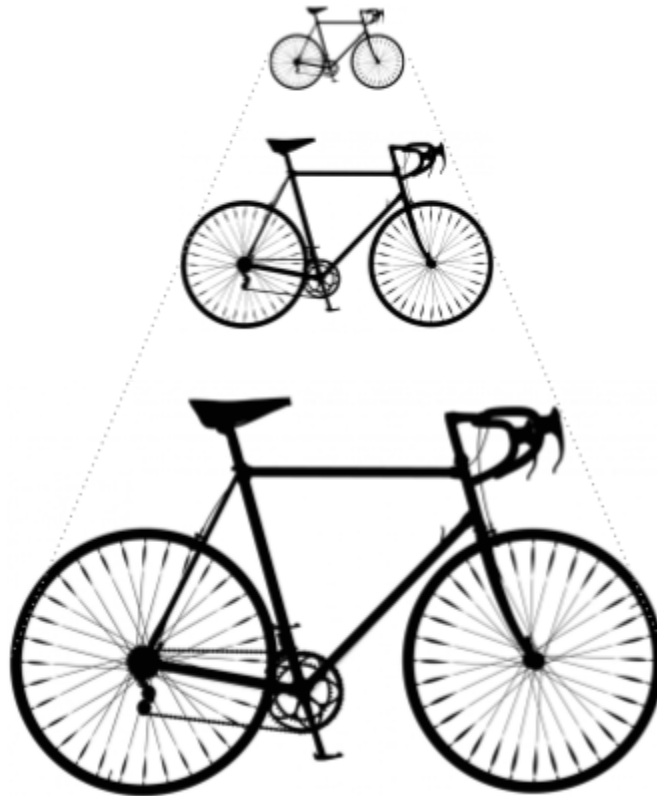


Proportional Reasoning

Fast Track GRASP Math Packet

Part 2



Version 1.0

3/1/2024



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<http://www.collectedny.org/ftgmp>

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Making Comparisons With Ratios

Which is a Better Deal?



5 oranges for \$3.00



7 oranges for \$5.00

- 1) Assuming the oranges are the same size and quality, which would you rather buy? Explain how you know.



4 cans of cat food for \$3.00



6 cans of cat food for \$4.00

- 2) Which cat food would you rather buy? Explain why.

Proportional Reasoning (Part 2)

3) Find an advertisement that shows one or more of the following:

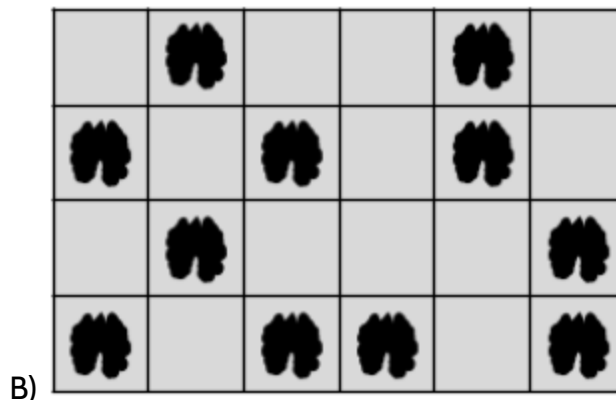
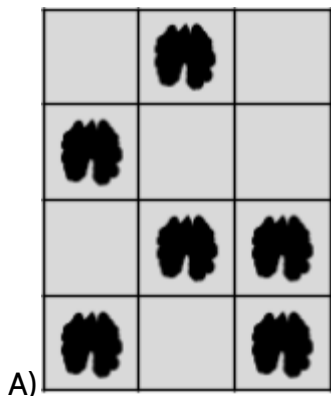
- dollar price per pound
- dollar price per number of items
- dollar price per liter, gallon, etc.
- sales price and normal price
- etc.

You can find the advertisement in a newspaper, in a store, or on the Internet. Take a photograph if possible, then write the item for sale and details about the price below.

4) Write at least three different ratios that are equivalent to the price you found above.

Comparing Ratios Using Bar Models

The two drawings below show two chocolate bars with nuts. Each square of chocolate is the same size, but the chocolate bars are different sizes.



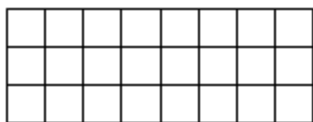
- 5) Which chocolate bar is nuttier? In other words, which bar has a greater number of nuts for the amount of chocolate? How do you know?

- 6) In chocolate bar A, the ratio of nuts to chocolate squares is 6:12. This ratio is equivalent to _____:24 and _____:48.

In chocolate bar B, the ratio of nuts to chocolate squares is _____:_____. This ratio is equivalent to _____:12 and _____:48.

The size of the chocolate bars is related to the mathematical idea of *area*, which is the size of a flat surface. Count the number of squares to find the area of each chocolate bar. Chocolate bar A has an area of 12 square units. Chocolate bar B has an area of 24 square units.¹

- 7) What is the area of the rectangle below?

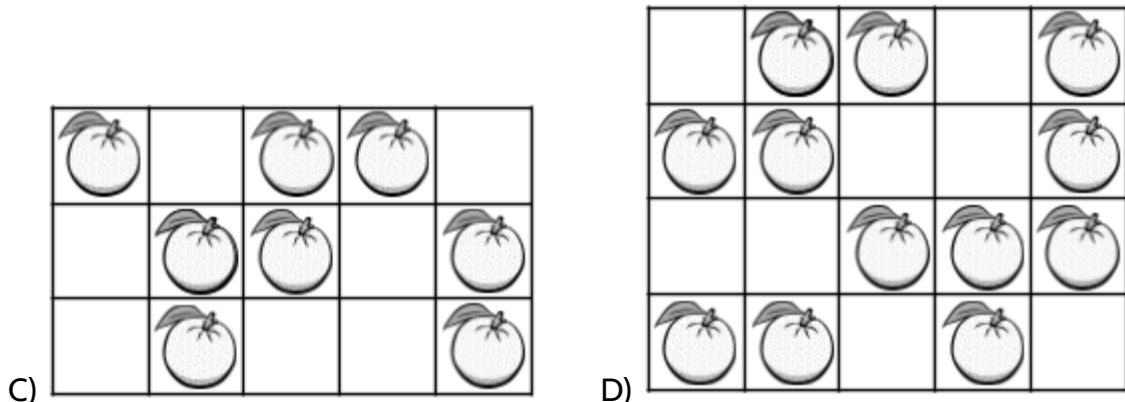


_____ square units

¹See **Two-Dimensional Geometry** for practice with area.

Proportional Reasoning (Part 2)

8) Which image below has a greater number of oranges per square unit?



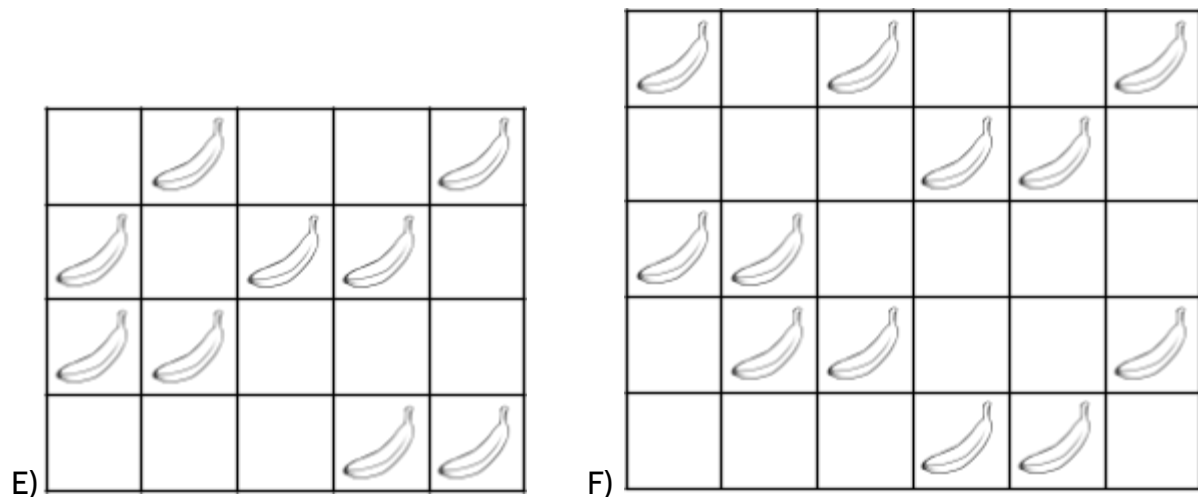
9) In image C, the ratio of $\frac{8 \text{ oranges}}{\text{squares}}$ is equivalent to $\frac{\quad}{30}$, $\frac{\quad}{45}$, and $\frac{\quad}{60}$.

In image D, the ratio of $\frac{\text{oranges}}{\text{squares}}$ is equivalent to $\frac{\quad}{40}$, $\frac{\quad}{60}$, and $\frac{\quad}{80}$.

10) Complete this sentence: I know that there is a larger ratio of oranges to squares in image

_____ because _____

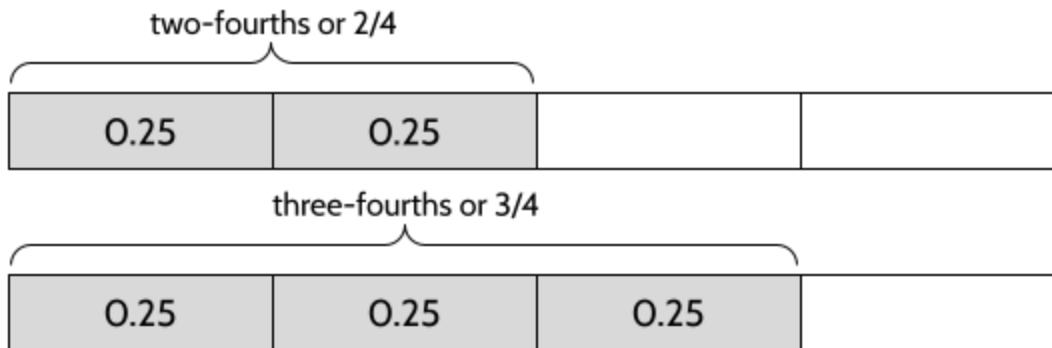
11) Which image below has a greater number of bananas per square unit? How do you know?



Proportional Reasoning (Part 2)

What if you want to compare ratios written as fractions? How can you decide which is larger?

For example, let's look $\frac{2}{4}$ and $\frac{3}{4}$ to see which is larger. We will use rectangles to see the fractions. This way of drawing ratios or fractions is sometimes called a *bar model*.



12) We can use money to think about fourths. Fill in the blanks below.

$\frac{1}{4}$ of a dollar is 1 quarter or \$0.25.

$\frac{5}{4}$ of a dollar is _____ quarters or \$_____.

$\frac{2}{4}$ of a dollar is 2 quarters or \$_____.

$\frac{6}{4}$ of a dollar is _____ quarters or \$_____.

$\frac{3}{4}$ of a dollar is _____ quarters or \$_____.

$\frac{7}{4}$ of a dollar is _____ quarters or \$_____.

$\frac{4}{4}$ of a dollar is _____ quarters or \$_____.

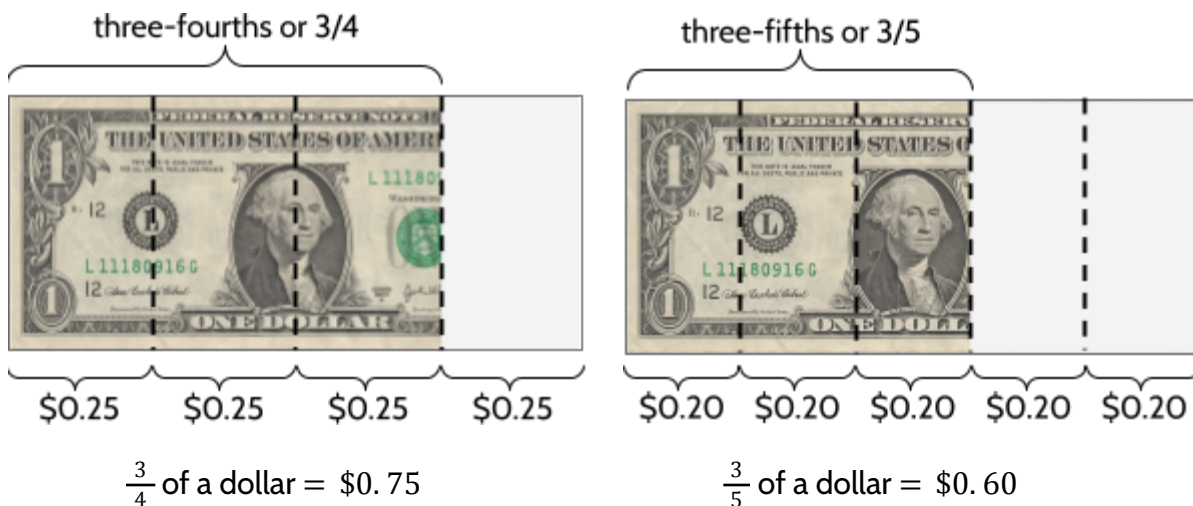
$\frac{8}{4}$ of a dollar is _____ quarters or \$_____.

13) Which ratio is greater, $\frac{3}{4}$ or $\frac{3}{5}$? Use the images of dollar bills below to explain.

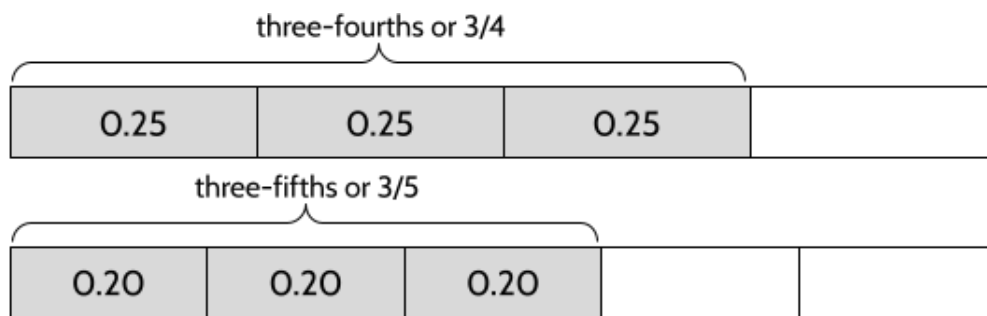


Proportional Reasoning (Part 2)

$\frac{3}{4}$ is greater than $\frac{3}{5}$. Here is one way to see this:



Bar models help us see the ratios. Do you see how $\frac{3}{4}$ is longer than $\frac{3}{5}$?

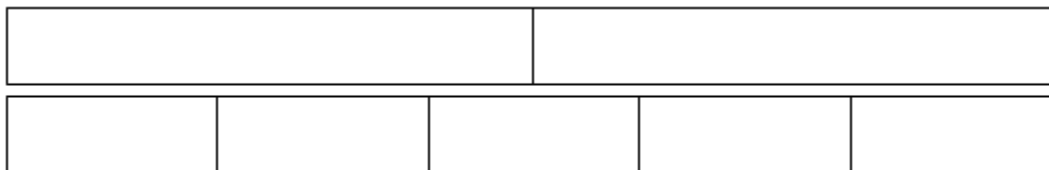


Shade in the bar models below to compare the two fractions. Then add > or < to show which fraction is bigger.

14) $\frac{3}{4} > \frac{3}{8}$



15) $\frac{1}{2} \quad \frac{3}{5}$



Proportional Reasoning (Part 2)

16) $\frac{3}{4}$ $\frac{5}{8}$

17) $\frac{2}{4}$ $\frac{1}{3}$

We can also use bar models to compare fractions that are larger than 1.

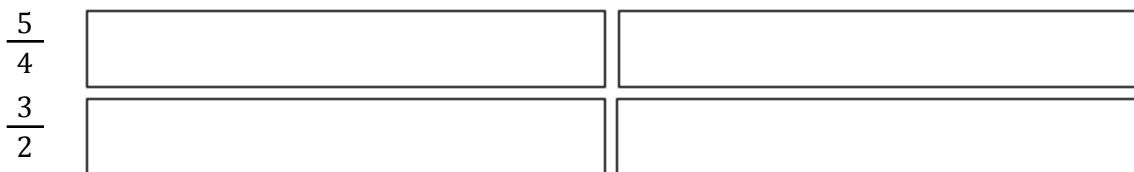
18) $\frac{6}{4}$ $\frac{8}{5}$



19) $\frac{7}{4}$ $\frac{13}{8}$



20) $\frac{5}{4}$ $\frac{3}{2}$

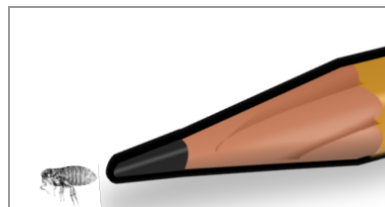


Who is the Best Jumper?

A kangaroo can jump 30 feet.

A bullfrog can jump 7 feet.

A flea can jump 1 foot.



Animal	Average Size
Kangaroo	About 6 feet tall
Bullfrog	About 6 inches long
Flea	About $\frac{1}{8}$ inch long

Obviously, a kangaroo can jump the farthest. 30 feet is a lot further than 7 feet or 1 foot. But what if we consider the length of each jump compared to the size of each animal?

- 21) Which animal is the best jumper? Use math to explain your thinking.
- 22) Martin says kangaroos are the best jumpers because they can jump 30 feet. Paola says if bullfrogs were the same size as kangaroos, they could jump more than 60 feet. Who do you agree with? Explain your thinking.

Proportional Reasoning (Part 2)

28) If the bullfrog was 12 inches long with the same jumping ability, it could jump 14 feet.
What if the bullfrog was a different size?

Size (inches)	6	12	24		
Size (feet)	0.5	1	2	4	6
Jump (feet)	7	14			

29) Fill in the blank: A bullfrog can jump _____ times its size.

A. 6

C. 12

B. 7

D. 14

30) What if the flea was a different size?

Size (inches)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	12
Jump (feet)	1	2			
Jump (inches)	12	24			

Size (inches)	$\frac{1}{8}$	1	12		
Size (feet)	$\frac{1}{96}$		1	2	6
Jump (feet)	1	8			

31) Fill in the blank: A flea can jump _____ times its size.

A. 12

C. 48

B. 24

D. 96

Proportional Reasoning (Part 2)

32) About how far could you jump if you...

... could jump like a kangaroo?

... could jump like a bullfrog?

... could jump like a flea?

33) Complete the table.

Animal	Size	Length of Jump	Ratio of Length of Jump to Size
Kangaroo	6 feet tall	30 feet	$\frac{\text{size}}{\text{length of jump}} = \frac{6 \text{ feet}}{30 \text{ feet}} = \frac{1 \text{ foot}}{\text{feet}}$
Bullfrog	6 inches long	7 feet	$\frac{\text{size}}{\text{length of jump}} = \frac{6 \text{ inches}}{7 \text{ feet}} = \frac{1 \text{ foot}}{\text{feet}}$
Flea	$\frac{1}{8}$ inch long	1 foot	$\frac{\text{size}}{\text{length of jump}} = \frac{\frac{1}{8} \text{ inch}}{1 \text{ foot}} = \frac{1 \text{ foot}}{\text{feet}}$

34) Which of these animals jumps the farthest *for its size*? How do you know?

Comparing Ratios Using Ratio Tables

Ratio tables can help us decide which ratio or fraction is larger.

35) Complete the following tables.

	× 5	× 5	
	↗ ↘		
3	15		
4			
	↖ ↗		
	× 5	× 5	

	× 2	× 5	
	↗ ↘		
7			
10			
	↖ ↗		
	× 2	× 5	

36) Explain how you know that $\frac{3}{4}$ is larger than $\frac{7}{10}$.

Make equivalent ratios in each table so that you have more information to make comparisons. Then add > or < to show which fraction is bigger.

37) $\frac{2}{5} > \frac{1}{4}$

	× 2	× 2	× 5	
	↗ ↘ ↘			
2				
5				
	↖ ↗ ↗			
	× 2	× 2	× 5	

	× 5	× 5	
	↗ ↘		
1			
4			
	↖ ↗		
	× 5	× 5	

38) $\frac{9}{15} \frac{6}{8}$

	÷ 3	× 2	× 10	
	↖ ↗ ↗			
9				
15				
	↖ ↗ ↗			
	÷ 3	× 2	× 10	

	÷ 2	× 5	× 5	
	↖ ↗ ↗			
6				
8				
	↖ ↗ ↗			
	÷ 2	× 5	× 5	

Proportional Reasoning (Part 2)

In these ratio tables, decide what numbers to multiply or divide by to make the ratios “out of” the same number. You don’t have to use all the boxes in the table.

39) $\frac{3}{5}$ $\frac{3}{4}$

3				
5				

3				
4				

How do you know?

40) $\frac{6}{4}$ $\frac{8}{5}$

6				
4				

8				
5				

How do you know?

41) $\frac{14}{20}$ $\frac{2}{3}$

How do you know?

42) $\frac{10}{4}$ $\frac{8}{3}$

How do you know?

Proportional Reasoning (Part 2)

Compare the following rates using ratio tables.

- 43) How would you rather be paid, \$45.00 for every 3 hours of work or \$80.00 for every 5 hours of work?

Hours worked	3				
Pay	\$45.00				

Hours worked	5				
Pay	\$80.00				

- 44) Which is a better deal, 4 oranges for \$3.00 or 5 oranges for \$4.00?

Oranges	4				
Cost	\$3.00				

Oranges	5				
Cost	\$4.00				

- 45) Which is faster, 200 miles in 4 hours or 90 miles in 1.5 hours?

Miles					
Distance					

Miles					
Distance					

Proportional Reasoning (Part 2)

- 46) Big Horn Ranch has 150 acres of pasture and raises 100 horses. Jefferson Ranch has 125 acres of pasture and raises 75 horses. Which ranch has more acres of pasture per horse?

Explain your answer using words, pictures, or diagrams.

- 47) Hemwanti makes \$25 per hour. Shahida makes \$3680 every 4 weeks. Both of them work 40 hours per week. Who makes more money?

Explain your answer using words, pictures, or diagrams.

- 48) Consider the following soccer (football) statistics:

- The Futbol Club Barcelona has scored 43 goals in 21 matches this season.
- The Arsenal Football Club has scored 44 goals in 22 matches.
- Borussia Dortmund has scored 40 goals in 19 matches.

Which team scores the most goals per football match? Which team scores the least?

Explain your answer using words, pictures, or diagrams.

Nurse-to-Patient Ratios

At a hospital in New York City, the nurses' union and the hospital management agreed in a contract that the medical-surgical unit must have at least six nurses on duty for every 30 patients.²



- 49) What is the maximum number of patients one nurse can take care of, according to the contract?

Since the contract was signed, nurses at the hospital have made complaints that the number of patients was often higher than what is allowed with a nurse-to-patient ratio of 6:30.

- 50) Let's imagine we are reviewing patient and staffing data for a week in September. There should never be less than six nurses for every 30 patients, but it is okay if there are more nurses than are needed. For each shift, decide whether the ratio violates the contract.

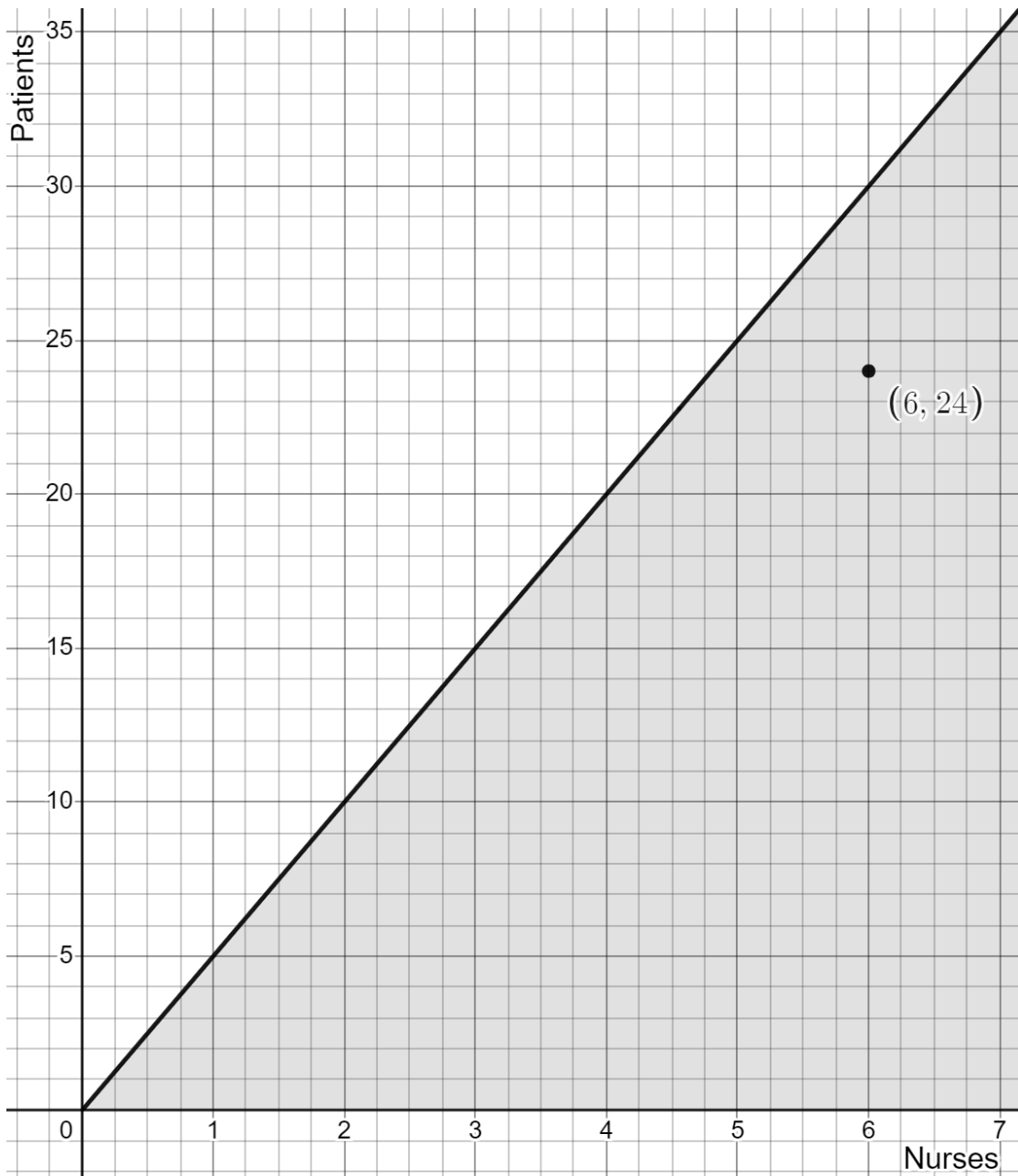
Day	Number of Nurses	Number of Patients	Does the ratio violate the agreement of no fewer than six nurses per 30 patients?
9/1	6	24	No
9/2	5	30	
9/3	4	20	
9/4	7	34	
9/5	3	16	
9/6	5	22	
9/7	2	10	

- 51) In New York State, hospitals that violate the nurse-to-patient ratio can be fined up to \$2,000 per violation. How much money could the hospital be fined in total based on the data above?

² NYU Brooklyn Hospital Understaffed Nurses 47 Times Last Year, Arbitrator Rules - The City (<https://www.thecity.nyc/2023/12/19/nyu-langone-brooklyn-understaffed-arbitrator>)

Proportional Reasoning (Part 2)

- 52) Now graph all the points on the previous page and see whether your predictions were correct. Data from the first day has been placed on the chart.



- 53) Which points are on the black line or are in the shaded area below the line?
- 54) Which points are in the white area above the line?

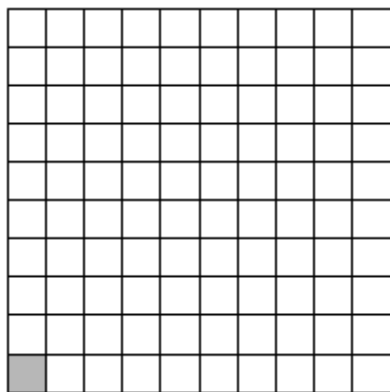
Visual Ways to Understand Percents and Ratios

Let's break down the word *percent*. *Per-* means "for every" and *-cent* means "100." *Percent* literally means "for every 100." In Spanish, *percent* is "por ciento," which means "for one hundred." A percent is a part-whole ratio which is "out of" 100. For example, "5 percent" means "5 for every 100." We use the symbol % to show percent.

By the way, you can find "cent" in many words related to 100. There are 100 *cents* (pennies) in \$1 dollar. A *century* is 100 years. A *centimeter* is a hundredth of a meter. A *centipede* (supposedly) has 100 legs. A *centenarian* is a person who has lived to be 100 years old!

55) The gray square in the grid below is equivalent to $\frac{1}{100}$ or 1% of the total squares.

Can you explain why?

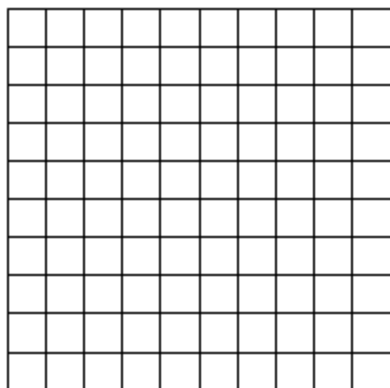


56) If we shade in two squares, what percent of the total squares would be shaded?

57) How many squares would you shade in to show $3\frac{1}{2}\%$?

58) To show 10% on the grid to the left, how many squares would you shade in?

Shade in 50 of the squares in the grid below.

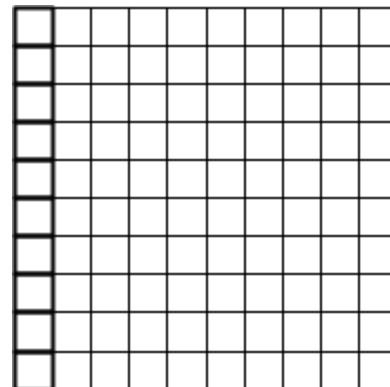


59) What percentage of the squares did you shade in?

60) What fraction of the squares did you shade in?

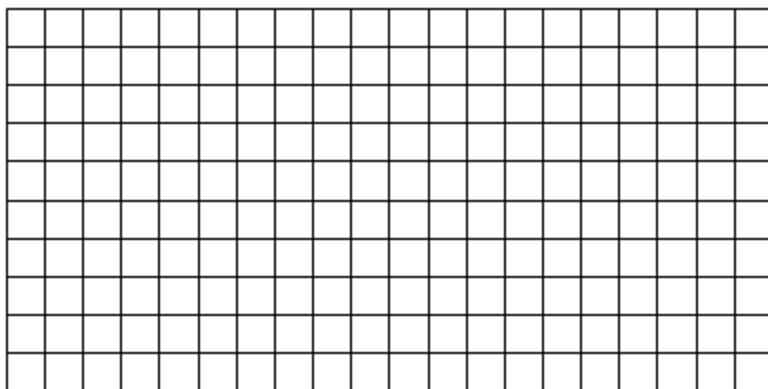
Proportional Reasoning (Part 2)

- 61) Shade in one column in the grid to the right. What percent of the total squares did you shade in?



The grids below have different numbers of squares. Some have more than 100 squares. Some have less than 100 squares. Percents allow us to compare ratios to 100 even when we don't have exactly 100 things.

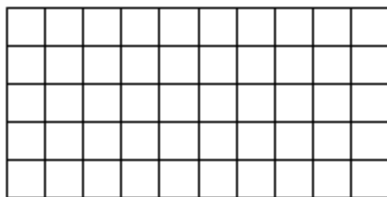
- 62) There are 200 total squares in the grid below. With a pencil or pen, shade in one of the squares.



What percent of the total squares are shaded?

$$\frac{1 \text{ shaded square}}{200 \text{ total squares}} = \frac{\text{shaded squares}}{100 \text{ total squares}} = \underline{\hspace{2cm}} \%$$

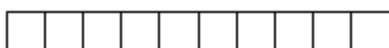
- 63) Count the number of squares in the grid below. Shade in one of the squares, then fill in the blanks.



What percent of the total squares are shaded?

$$\frac{1 \text{ shaded square}}{\text{total squares}} = \frac{\text{shaded squares}}{100 \text{ total squares}} = \underline{\hspace{2cm}} \%$$

- 64) Shade in 40% of the squares in the grid below, then fill in the blanks.



$$40\% = \frac{40 \text{ shaded squares}}{100 \text{ total squares}} = \frac{\text{shaded squares}}{10 \text{ total squares}}$$

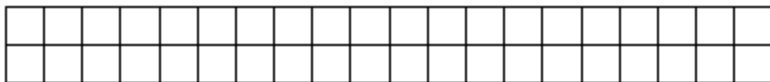
Proportional Reasoning (Part 2)

65) What percent of the squares are shaded in the grid below?

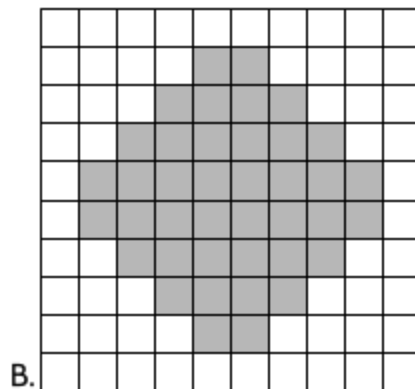
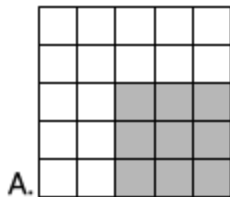


$$\frac{\text{shaded squares}}{\text{total squares}} = \frac{\text{shaded squares}}{100 \text{ total squares}} = \underline{\hspace{2cm}} \%$$

66) Shade in 20% of the squares below.



67) Which grid below has a larger percentage of its area (total number of squares) shaded?
How do you know?



68) Which of these shows a larger ratio of shaded to total squares? How do you know?

$$\frac{9 \text{ shaded squares}}{25 \text{ total squares}}$$

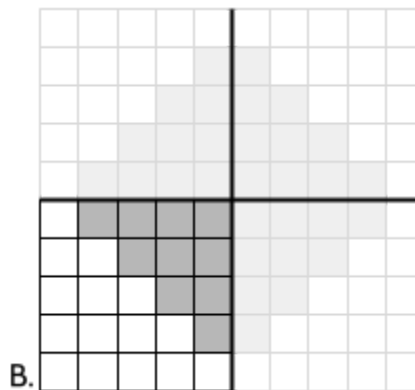
$$\frac{40 \text{ shaded squares}}{100 \text{ total squares}}$$

Proportional Reasoning (Part 2)

To compare a ratio of $\frac{9 \text{ shaded squares}}{25 \text{ total squares}}$ with a ratio of $\frac{40 \text{ shaded squares}}{100 \text{ total squares}}$, we could split Diagram B into four equal pieces, so that we have a total of 25 squares in each piece:



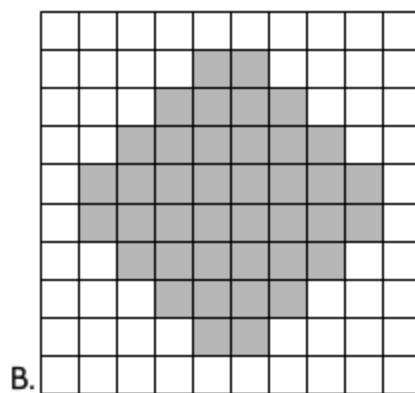
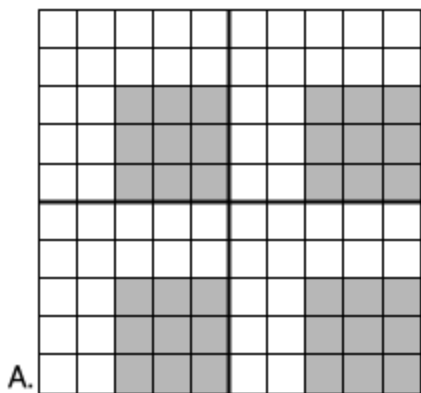
$$\frac{9 \text{ shaded squares}}{25 \text{ total squares}}$$



$$\frac{40 \text{ shaded squares} \div 4}{100 \text{ total squares} \div 4} = \frac{10 \text{ shaded squares}}{25 \text{ total squares}}$$

When both ratios are “out of 25,” we can see that Diagram B has more shaded squares.

69) We also could make four copies of Diagram A so that we have a total of 100 squares in each diagram. Fill in the blanks below.



$$\frac{9 \text{ shaded squares} \times 4}{25 \text{ total squares} \times 4} = \frac{\text{shaded squares}}{100 \text{ total squares}} = \underline{\hspace{2cm}} \%$$

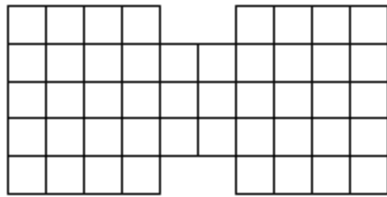
$$\frac{40 \text{ shaded squares}}{100 \text{ total squares}} = \underline{\hspace{2cm}} \%$$

Explain how you know Diagram B has a greater portion of its area shaded.

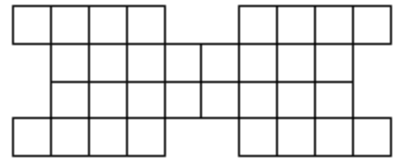
Proportional Reasoning (Part 2)

70) For each of the following, shade in the given percentage of its area.

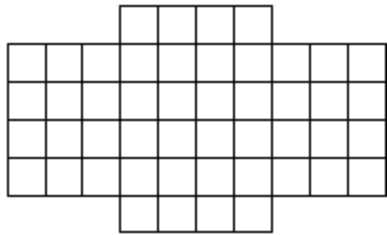
I. 50%



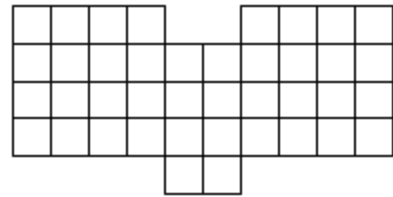
III. 75%



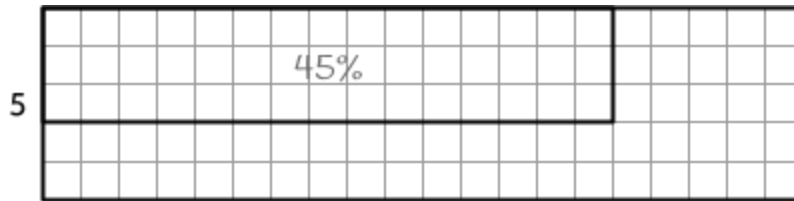
II. 25%



IV. 20%

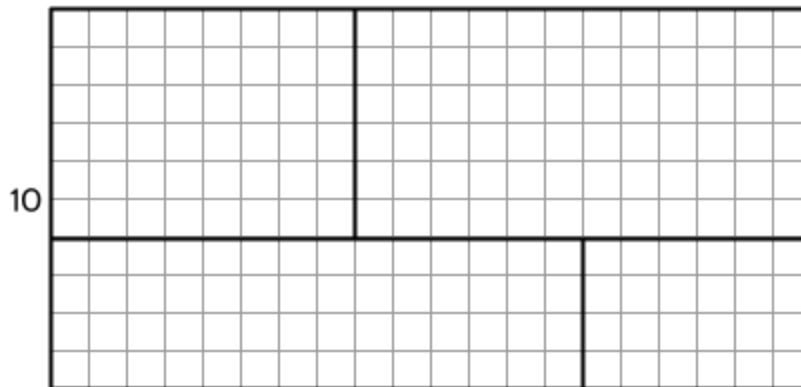


71) For each of the following, find what percent the area of each region is of the total shape. Part of the first one has been done for you.



I.

20



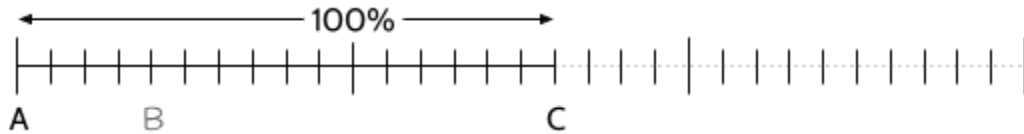
II.

20

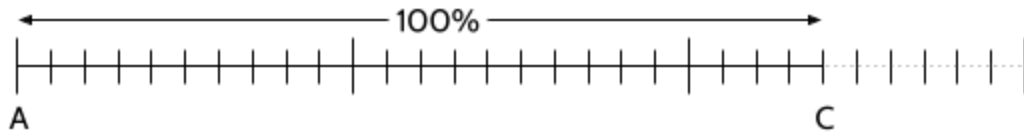
Proportional Reasoning (Part 2)

72) In each of the following, \overline{AC} is a line segment. Place B on the number line so that \overline{AB} shows the given percent. The first one has been done for you.

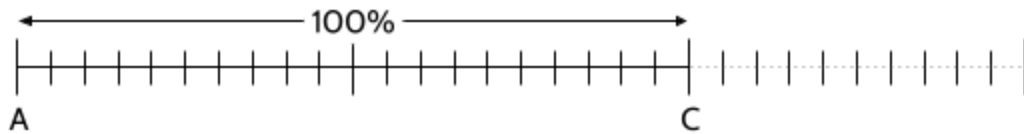
I. Place B at 25%.



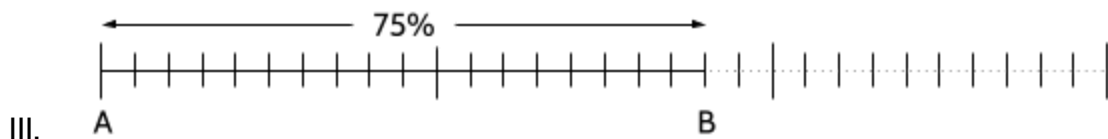
II. Place B at 50%.



III. Place B at 20%.



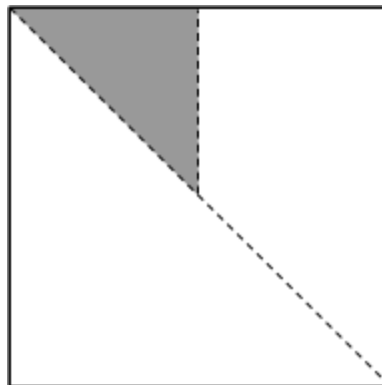
73) In each of the following, \overline{AB} is part of a line segment. Complete the line segment by placing C on the number line so that \overline{AC} is 100%. The first one has been done for you.



Proportional Reasoning (Part 2)

Consider the square on the right.

- 74) If the whole square is worth \$100.00, how much is the dark triangle worth? How do you know?³



- 75) Complete the table below.

Number of shaded triangles	Fraction	Percent	Portion of \$100
1	$\frac{1}{8}$		
2			
3			
4			\$50.00
5			
6		75%	
7			
8	$\frac{8}{8}$		\$100.00

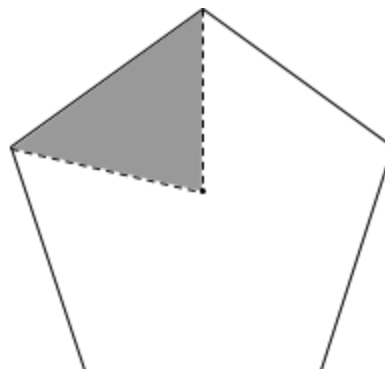
- 76) If the whole square is worth \$200.00, how much is the dark triangle worth?
- 77) If the whole square is worth \$200.00, how much are three dark triangles ($\frac{3}{8}$) worth? How do you know?

³ Hint: Can you draw more dashed lines to split the square into triangles of the same size?

Proportional Reasoning (Part 2)

Consider the pentagon on the right.

78) If the whole pentagon is worth \$100.00, how much is the dark triangle worth? How do you know?



79) Complete the table below.

Number of shaded triangles	Fraction	Percent	Portion of \$100
1			
2			
3			
4			\$80.00
5		100%	

80) If the whole pentagon is worth \$200.00, how much is the dark triangle worth?

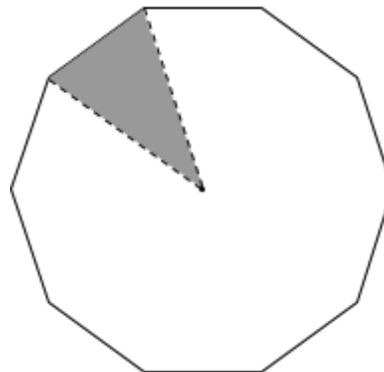
81) If the whole pentagon is worth \$400.00, how much are three dark triangles ($\frac{3}{5}$) worth?
How do you know?

82) Use the pentagon to explain $\frac{2}{5}$ of \$1,000.00.

Proportional Reasoning (Part 2)

Consider the decagon on the right.

83) If the whole decagon is worth \$100.00, how much is the dark triangle worth? How do you know?



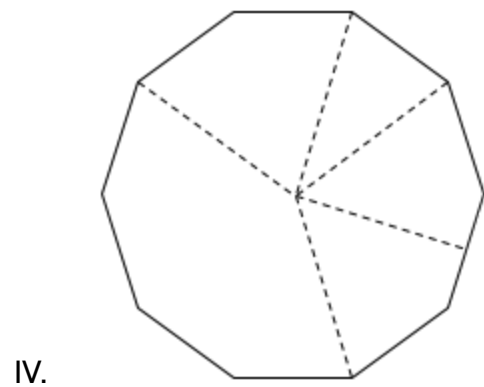
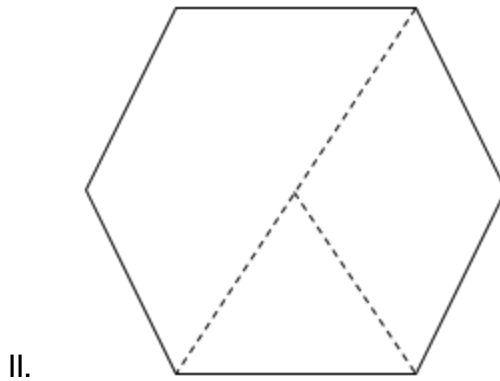
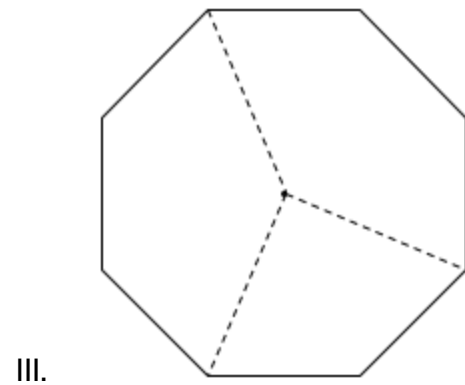
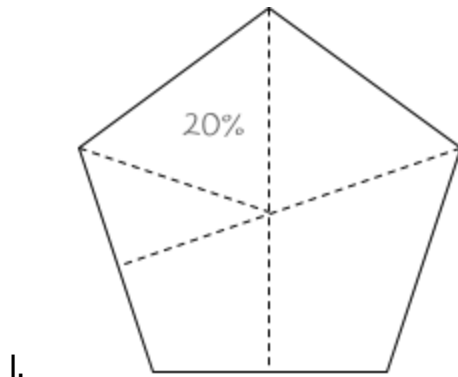
84) Complete the table below.

Number of shaded triangles	Fraction	Percent	Portion of \$100
1			
2			
3			
4			
5		50%	\$50.00
6			
7			
8			
9			
10			

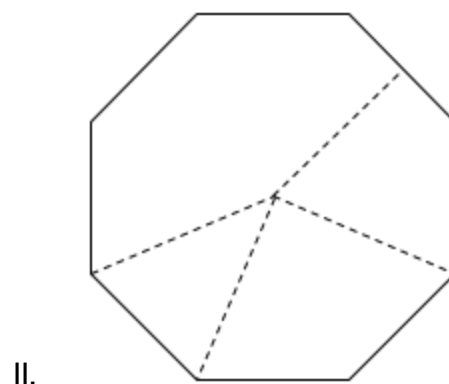
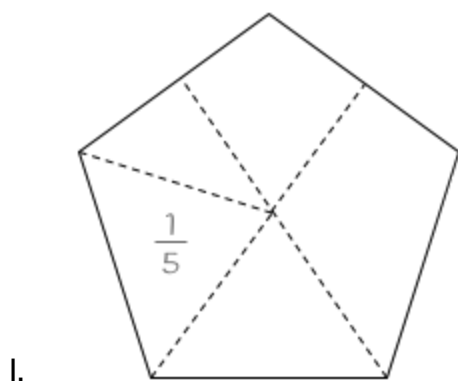
85) Use the decagon to explain 40% of \$300.00.

Proportional Reasoning (Part 2)

- 86) For each of the following, find what percent the area of each region is of the total shape. Part of the first one has been done for you.



- 87) For these shapes, find what fraction the area of each region is of the total shape. Part of the first one has been done for you.



Comparing Ratios Using Decimals and Percents

Ratios can be expressed as percents and decimals. A percent is a part-whole ratio which is “out of 100.” When a ratio is shown as a decimal, it compares one quantity “for every 1” of another quantity.

Let’s consider $\frac{1}{2}$ (one half) of a dollar, which is 50 cents. The decimal equivalent can be written as \$0.50, which is equivalent to two quarters, five dimes, or 50 pennies.

$$50\% \text{ of 1 dollar} = \frac{2 \text{ quarters}}{4 \text{ quarters}} = \frac{5 \text{ dimes}}{10 \text{ dimes}} = \frac{50 \text{ pennies}}{100 \text{ pennies}} = 0.50$$

88) Complete the table. The ratios in each row are equivalent.

Number of pennies	“out of 100”	Percent	Written as money (decimal)
50	$\frac{50}{100}$	50%	\$0.50
25	$\frac{\quad}{100}$	%	
	$\frac{80}{100}$	%	
	$\frac{\quad}{100}$	8%	
	$\frac{\quad}{100}$	%	\$0.05

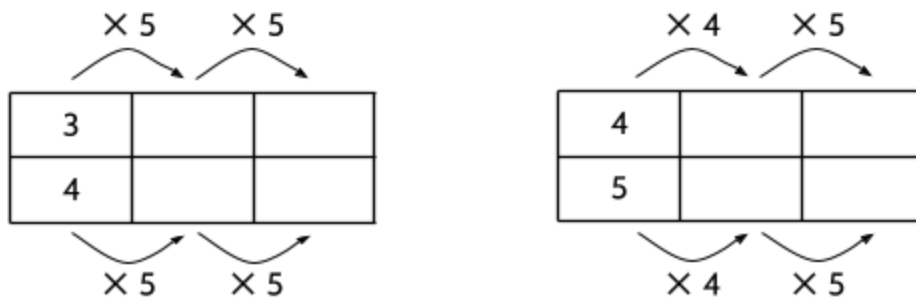
89) Fill in the missing information. The ratios in each row are equivalent.

“out of 5”	“out of 10”	“out of 50”	“out of 100”	Percent	Decimal
$\frac{3}{5}$	$\frac{\quad}{10}$	$\frac{\quad}{50}$	$\frac{\quad}{100}$	60%	0.6
$\frac{\quad}{5}$	$\frac{2}{10}$	$\frac{\quad}{50}$	$\frac{20}{100}$	%	
$\frac{\quad}{5}$	$\frac{\quad}{10}$	$\frac{40}{50}$	$\frac{\quad}{100}$	%	
$\frac{\quad}{5}$	$\frac{\quad}{10}$	$\frac{\quad}{50}$	$\frac{\quad}{100}$	%	0.4
$\frac{\quad}{5}$	$\frac{\quad}{10}$	$\frac{\quad}{50}$	$\frac{\quad}{100}$	100%	

Proportional Reasoning (Part 2)

Let's review how we can compare ratios and fractions with a ratio table:

90) $\frac{3}{4} < \frac{4}{5}$

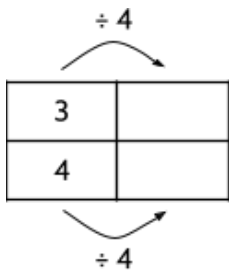


How do decimals help us compare ratios and fractions?

When a ratio is written as a decimal, an amount is being compared to 1. For example, we can see which is larger if both ratios are written as fractions with 1 as the denominator.

But how can we change a ratio like $\frac{3}{4}$ so that the bottom number is 1?

91) Fill in the blanks. You may want to use a calculator.



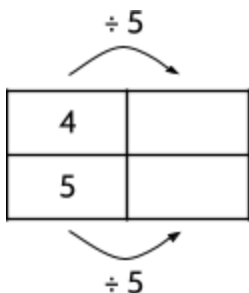
Since $3 \div 4$ is 0.75, the new numerator is 0.75.

Since $4 \div 4$ is 1, the new denominator is 1.

$\frac{3}{4}$ is equal to the decimal _____.

Note: Any number divided by 1 is itself. For example, $3 \div 1 = 3$. The ratio $\frac{0.75}{1}$ is the same as $0.75 \div 1$, which equals 0.75.

92) Fill in the blanks. You may want to use a calculator.



$\frac{4}{5}$ is equal to the decimal _____.

$\frac{4}{5}$ is larger than $\frac{3}{4}$. Explain how you know.

Proportional Reasoning (Part 2)

Fill in the blanks in the following ratio tables. Add > and < to compare the size of the fractions. You may want to use a calculator.

93)

7	
5	1

decimal: 1.4

5	
4	1

decimal: _____

$$\frac{7}{5} > \frac{5}{4}$$

How do you know?

94)

3	
8	1

decimal: _____

4	
10	1

decimal: _____

$$\frac{3}{8} \quad \frac{4}{10}$$

How do you know?

95)

7	
16	

decimal: _____

2	
5	

decimal: _____

$$\frac{7}{16} \quad \frac{2}{5}$$

How do you know?

Proportional Reasoning (Part 2)

96)

$$\frac{7}{8} = \frac{\quad}{1}$$

$\div 8$

 $\div 8$

decimal: _____

$$\frac{17}{20} = \frac{\quad}{1}$$

$\div 20$

 $\div 20$

decimal: _____

$$\frac{7}{8} \quad \frac{17}{20}$$

How do you know?

97)

$$\frac{7}{16} = \frac{\quad}{1}$$

decimal: _____

$$\frac{11}{25} = \frac{\quad}{1}$$

decimal: _____

$$\frac{7}{16} \quad \frac{11}{25}$$

How do you know?

98)

$$\frac{\quad}{\quad} = \frac{\quad}{1}$$

decimal: _____

$$\frac{\quad}{\quad} = \frac{\quad}{1}$$

decimal: _____

$$\frac{6}{5} \quad \frac{23}{20}$$

How do you know?

99) Which is greater, $\frac{4}{5}$ or $\frac{6}{8}$? Use decimals to explain.

How do percentages help us compare ratios?

Since we often need to compare ratios that are “out of” different amounts, we can use percentages to make any ratio into a percent, which is an “for every 100” ratio.

Percent (%) and “for every 100” mean the same thing:

$$\frac{25}{100} = 25\%$$

$$\frac{40}{100} = 40\%$$

$$\frac{50}{100} = 50\%$$

$$\frac{62.5}{100} = 62.5\%$$

To use percents to compare ratios, the first step is to convert the ratio into a percent. In other words, you want to change the ratio so that it is “out of 100.” Here is one way to do that.

100) Fill in the blanks. You may want to use a calculator.

original	decimal	percent
4		
5	1	100

$\div 5$ $\times 100$
 $\div 5$ $\times 100$

Divide numerator and denominator by 5 to get an equivalent decimal.

Decimal: 0.8

Multiply the new numerator and denominator by 100 to get an equivalent percent.

Percent: 80%

Use ratio tables to convert the following ratios into decimals and percentages.

101)

original	decimal	percent
3		
8	1	100

$\div 8$ $\times 100$
 $\div 8$ $\times 100$

Decimal: _____

Percent: _____

Proportional Reasoning (Part 2)

102)

Decimal:

7		
5	1	100

$\div 5$ $\times 100$
 $\div 5$ $\times 100$

Percent:

103)

10		
25	1	100

Decimal:

Percent:

104)

5		
4		

Decimal:

Percent:

105)

13		
4		

Decimal:

Percent:

106)

$$\begin{array}{c}
 \div 20 \qquad \qquad \times 100 \\
 \curvearrowright \qquad \qquad \curvearrowright \\
 \frac{19}{20} = \frac{\quad}{1} = \frac{\quad}{100} = \text{---} \% \\
 \div 20 \qquad \qquad \times 100 \\
 \curvearrowleft \qquad \qquad \curvearrowleft
 \end{array}$$

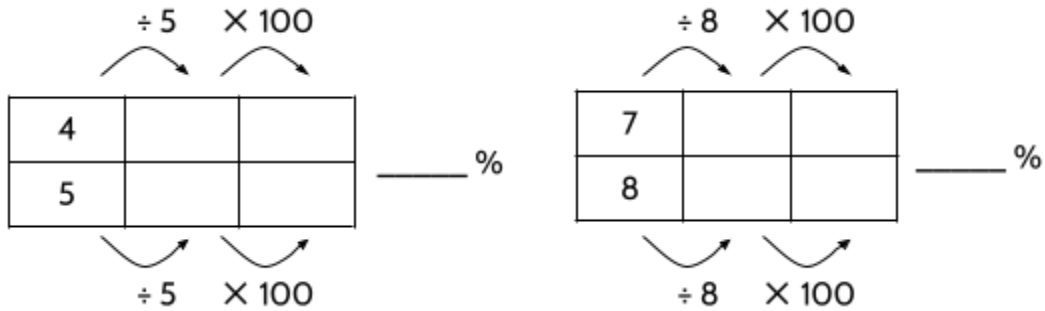
107)

$$\frac{7}{16} = \frac{\quad}{1} = \frac{\quad}{100} = \quad \%$$

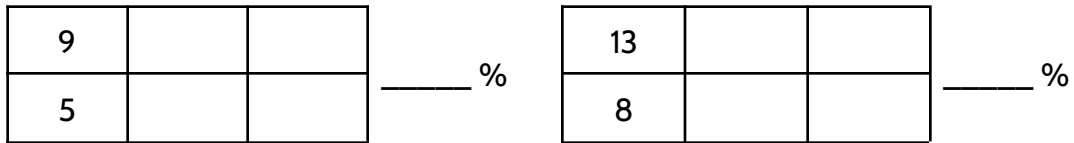
Proportional Reasoning (Part 2)

Once you convert two fractions into decimals or percents, you can see which is greater.

108) Which is greater, $\frac{4}{5}$ or $\frac{7}{8}$?



109) Which is greater, $\frac{9}{5}$ or $\frac{13}{8}$?



110) Which is greater, $\frac{9}{16}$ or $\frac{11}{20}$?

$$\frac{9}{16} = \frac{\quad}{1} = \frac{\quad}{100} = \quad\%$$

$$\frac{11}{20} = \frac{\quad}{1} = \frac{\quad}{100} = \quad\%$$

111) Which is greater, $\frac{7}{25}$ or $\frac{5}{16}$?

Proportional Reasoning (Part 2)

112) Which is greater, 30% or $\frac{2}{5}$? (You don't have to use all of the boxes.)

30					
100					
2					
5					

113) Which is greater, 0.48 or $\frac{9}{20}$?

48					
100					
9					
20					

114) Which is greater, 63 out of 300 or 22%?

115) Which is greater, 145% or $\frac{8}{5}$?

Proportional Reasoning (Part 2)

- 116) Using the digits 0 to 9 at most one time each, place a digit in each box to create a true statement.

The diagram shows a fraction with two two-digit boxes in the numerator and denominator, followed by an equals sign and a two-digit box followed by a percent sign. The boxes are represented by dashed lines.

Here is one solution: $\frac{13}{26} = 50\%$

How many solutions can you find?

This is based on an activity from openmiddle.com, where similar challenges can be found.

Making Comparisons Between Ratios - Answer Key

Which is a Better Deal?

- 1) 5 oranges for \$3 is cheaper.
- 2) 6 cans of cat food for \$4.00 is cheaper.
- 3) Answers will vary. Please share the ratios you found with us. Email: Eric (eric.appleton@cuny.edu) and Mark (mark.trushkowsky@cuny.edu)
- 4) Answers will vary.

Comparing Ratios Using Bar Models

- 5) Chocolate bar A is nuttier. 6 nuts:12 squares is equal to 12:24. Chocolate bar B has 11 nuts:24 squares.
 Chocolate bar A) $\frac{6 \text{ nuts}}{12 \text{ squares}} = \frac{12}{24}$ Chocolate bar B) $\frac{11 \text{ nuts}}{24 \text{ squares}}$
- 6) In chocolate bar A, the ratio of nuts to chocolate squares is 6:12. This ratio is equivalent to 12:24 and 24:48.
 In chocolate bar B, the ratio of nuts to chocolate squares is 11:24. This ratio is equivalent to 5.5:12 and 22:48.
- 7) 8 square units
- 8) Image D has more oranges per square unit.
- 9) In image C, the ratio of $\frac{8 \text{ oranges}}{15 \text{ squares}}$ is equivalent to $\frac{16}{30}$, $\frac{24}{45}$, and $\frac{32}{60}$.
 In image D, the ratio of $\frac{12 \text{ oranges}}{20 \text{ squares}}$ is equivalent to $\frac{24}{40}$, $\frac{36}{60}$, and $\frac{48}{80}$.
- 10) Answers will vary.
- 11) Image E has more oranges per square unit.
 In image E, the ratio of $\frac{9 \text{ bananas}}{20 \text{ squares}}$ is equivalent to $\frac{18}{40}$ and $\frac{27}{60}$.
 In image F, the ratio of $\frac{12 \text{ bananas}}{30 \text{ squares}}$ is equivalent to $\frac{24}{60}$.
- 12) $\frac{1}{4}$ of a dollar is 1 quarter or \$0.25. $\frac{2}{4}$ of a dollar is 2 quarters or \$0.50.

Proportional Reasoning (Part 2)

$\frac{3}{4}$ of a dollar is 3 quarters or \$0.75.

$\frac{6}{4}$ of a dollar is 6 quarters or \$1.50.

$\frac{4}{4}$ of a dollar is 4 quarters or \$1.00.

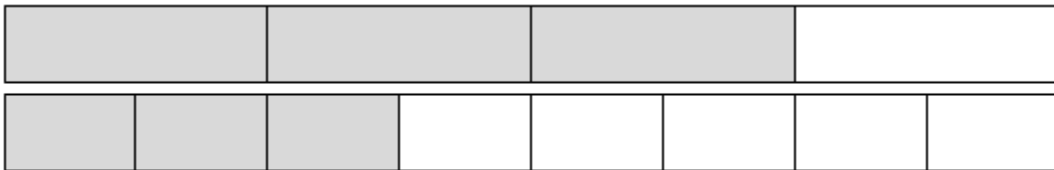
$\frac{7}{4}$ of a dollar is 7 quarters or \$1.75.

$\frac{5}{4}$ of a dollar is 5 quarters or \$1.25.

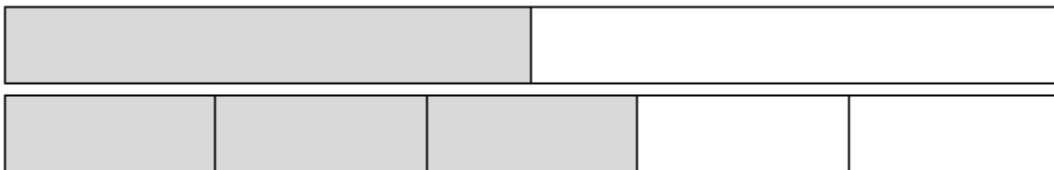
$\frac{8}{4}$ of a dollar is 8 quarters or \$2.00.

13) $\frac{3}{4} > \frac{3}{5}$. See the explanation on the following page.

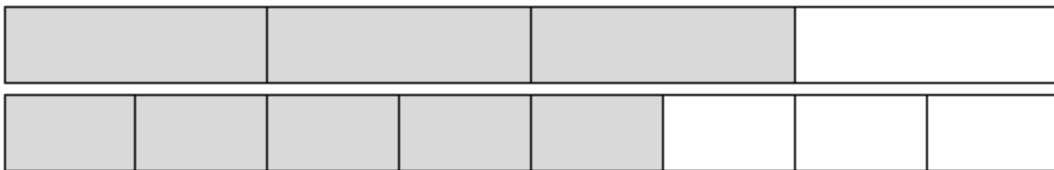
14) $\frac{3}{4} > \frac{3}{8}$



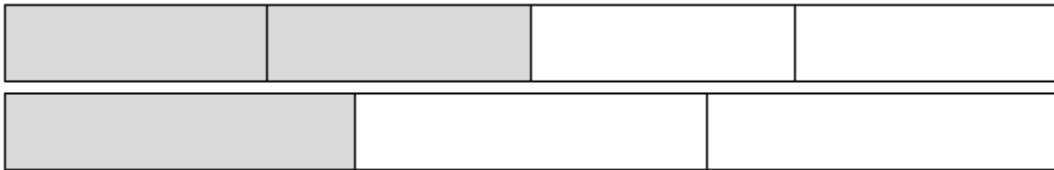
15) $\frac{1}{2} < \frac{3}{5}$



16) $\frac{3}{4} > \frac{5}{8}$



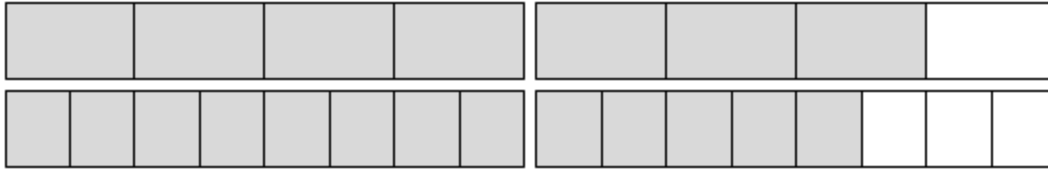
17) $\frac{2}{4} > \frac{1}{3}$



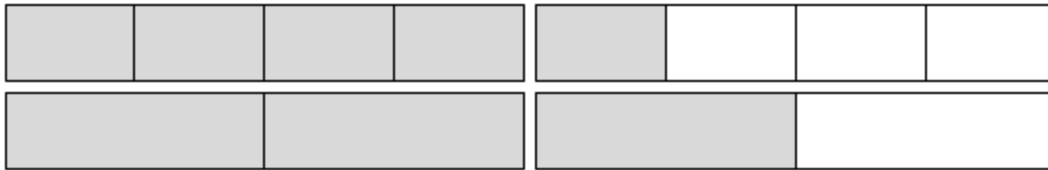
18) $\frac{6}{4} < \frac{8}{5}$

Proportional Reasoning (Part 2)

19) $\frac{7}{4} > \frac{13}{8}$



20) $\frac{5}{4} < \frac{3}{2}$



Who is the Best Jumper?

21) Answers will vary.

22) Answers will vary.

23)

Feet	1	2	6	7	14
Inches	12	24	72	84	168

Feet	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{12}$
Inches	12	6	3	1.5	1

Feet	1	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{48}$	$\frac{1}{96}$
Inches	12	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

24) 72 inches

25) 1 inch

26)

Proportional Reasoning (Part 2)

Size (feet)	6	3	1	4	$\frac{1}{2}$
Jump (feet)	30	15	5	20	2.5

27) A

28)

Size (inches)	6	12	24	48	72
Size (feet)	0.5	1	2	4	6
Jump (feet)	7	14	28	56	84

29) D

30)

Size (inches)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	12
Jump (feet)	1	2	4	8	96
Jump (inches)	12	24	48	96	1152

Size (inches)	$\frac{1}{8}$	1	12	24	72
Size (feet)	$\frac{1}{96}$	$\frac{1}{12}$	1	2	6
Jump (feet)	1	8	96	192	576

31) D

32) Answers will vary because people have different heights, but here are some estimates based on someone who is 6 feet tall:

If a 6 foot tall person could jump like a kangaroo, they could jump 30 feet.

$$\frac{\text{kangaroo jump of 30 feet}}{\text{kangaroo size of 6 feet}} = \frac{\text{jump of 30 feet}}{\text{human size of 6 feet}}$$

Proportional Reasoning (Part 2)

If a 6 foot tall person could jump like a bullfrog, they could jump 84 feet.

$$\frac{\text{bullfrog jump of 7 feet}}{\text{bullfrog size of 0.5 feet}} = \frac{14 \text{ feet}}{1 \text{ foot}} = \frac{\text{jump of 84 feet}}{\text{human size of 6 feet}}$$

If a 6 foot tall person could jump like a flea, they could jump 576 feet!

$$\frac{\text{flea jump of 1 foot}}{\text{flea size of } \frac{1}{8} \text{ inch}} = \frac{8 \text{ feet}}{1 \text{ inch}} = \frac{96 \text{ feet}}{1 \text{ foot}} = \frac{\text{jump of 576 feet}}{\text{human size of 6 feet}}$$

33)

Animal	Size	Length of Jump	Ratio of Length of Jump to Size
Kangaroo	6 feet tall	30 feet	$\frac{\text{size}}{\text{length of jump}} = \frac{6 \text{ feet}}{30 \text{ feet}} = \frac{1 \text{ foot}}{5 \text{ feet}}$
Bullfrog	6 inches long	7 feet	$\frac{\text{size}}{\text{length of jump}} = \frac{6 \text{ inches}}{7 \text{ feet}} = \frac{1 \text{ foot}}{14 \text{ feet}}$
Flea	$\frac{1}{8}$ inch long	1 foot	$\frac{\text{size}}{\text{length of jump}} = \frac{\frac{1}{8} \text{ inch}}{1 \text{ foot}} = \frac{1 \text{ foot}}{96 \text{ feet}}$

34) The flea jumps farthest for its size because it jumps 96 times farther than its size. The bullfrog jumps 14 times farther than its size. The kangaroo jumps 6 times farther than its size.

Comparing Ratios Using Ratio Tables

	× 5	× 5	
	↘	↘	
3	15	75	
4	20	100	
	↗	↗	
	× 5	× 5	

	× 2	× 5	
	↘	↘	
7	14	90	
10	20	100	
	↗	↗	
	× 2	× 5	

35)

36) Answers will vary.

37) $\frac{2}{5} > \frac{1}{4}$

Proportional Reasoning (Part 2)

3	15	75
4	20	100

$\times 5$ $\times 5$
 $\times 5$ $\times 5$

7	14	90
10	20	100

$\times 2$ $\times 5$
 $\times 2$ $\times 5$

38) $\frac{9}{15} < \frac{6}{8}$

9	3	6	60
15	5	10	100

$\div 3$ $\times 2$ $\times 10$
 $\div 3$ $\times 2$ $\times 10$

6	3	15	75
8	4	20	100

$\div 2$ $\times 5$ $\times 5$
 $\div 2$ $\times 5$ $\times 5$

39) $\frac{3}{5} < \frac{3}{4}$. Ratio tables and explanations will vary.

40) $\frac{6}{4} < \frac{8}{5}$. Ratio tables and explanations will vary.

41) $\frac{14}{20} > \frac{2}{3}$. Ratio tables and explanations will vary.

42) $\frac{10}{4} < \frac{8}{3}$. Ratio tables and explanations will vary.

43) $\$45.00/3 \text{ hours} = \$15.00/\text{hour}$ and $\$80.00/5 \text{ hours} = \$16.00/\text{hour}$

44) 4 oranges for \$3.00 is cheaper.

45) 90 miles in 1.5 hours is faster.

46) Jefferson Ranch has more acres of pasture per horse.

47) Hemwanti makes more money.

48) Borussia Dortmund has the most goals per football match. Arsenal has the least goals per match.

Nurse-to-Patient Ratios

49) 5

Proportional Reasoning (Part 2)

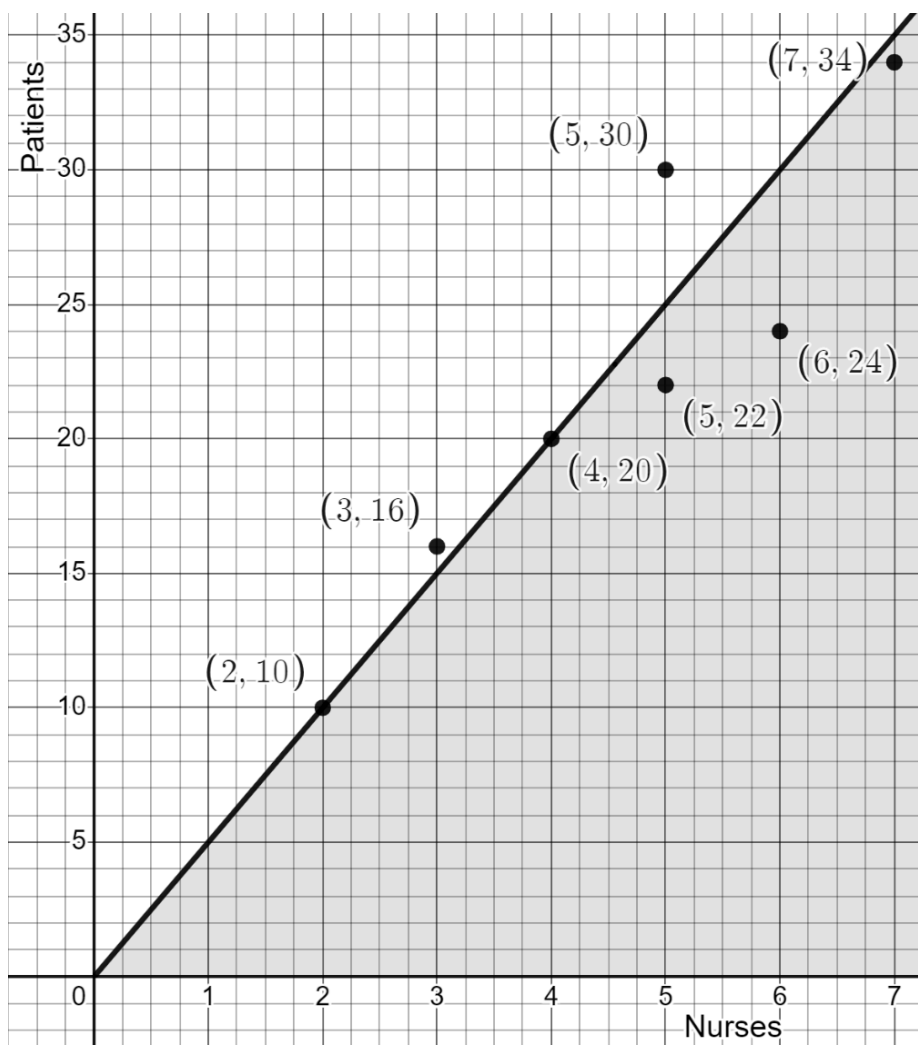
50)

Day	Number of Nurses	Number of Patients	Does the ratio violate the agreement of no fewer than six nurses per 30 patients?
9/1	6	24	No
9/2	5	30	Yes
9/3	4	20	No
9/4	7	34	No
9/5	3	16	Yes
9/6	5	22	No
9/7	2	10	No

51) \$4,000 (2 violations)

Proportional Reasoning (Part 2)

52)



- 53) The points on the black line or in the shaded area have a nurse-to-patient ratio equal to or greater than 6:30. There were enough nurses on these shifts.
- 54) The points in the white area above the line have a nurse-to-patient ratio equal to or less than 6:30. There were not enough nurses on these shifts.

Visual Ways to Understand Percents and Ratios

- 55) 2%
- 56) $3\frac{1}{2}$ squares
- 57) All of them. 100.
- 58) To show 10% on a grid of 100 squares, you should shade in 10 squares.

Proportional Reasoning (Part 2)

59) 50%

60) $\frac{50}{100}$ or $\frac{1}{2}$

61) 10%

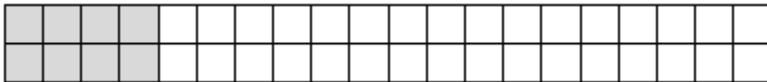
62) $\frac{1 \text{ shaded square}}{200 \text{ total squares}} = \frac{0.5 \text{ shaded squares}}{100 \text{ total squares}} = 0.5\%$

63) $\frac{1 \text{ shaded square}}{50 \text{ total squares}} = \frac{2 \text{ shaded squares}}{100 \text{ total squares}} = 2\%$

64) 

$$40\% = \frac{40 \text{ shaded squares}}{100 \text{ total squares}} = \frac{4 \text{ shaded squares}}{10 \text{ total squares}}$$

65) $\frac{7 \text{ shaded squares}}{20 \text{ total squares}} = \frac{35 \text{ shaded squares}}{100 \text{ total squares}} = 35\%$

66) 

There are different ways to find the number of squares. Here is one way:

$$20\% = \frac{20 \text{ shaded squares}}{100 \text{ total squares}} = \frac{4 \text{ shaded squares}}{20 \text{ total squares}} = \frac{8 \text{ shaded squares}}{40 \text{ total squares}}$$

67) See explanation below on the following page.

68) See explanation on the following page.

69) A. $\frac{9 \text{ shaded squares} \times 4}{25 \text{ total squares} \times 4} = \frac{36 \text{ shaded squares}}{100 \text{ total squares}} = 36\%$

B. $\frac{40 \text{ shaded squares}}{100 \text{ total squares}} = 40\%$

70) You might shade the different squares. These are the number of squares that should be shaded in:

I. 23 squares

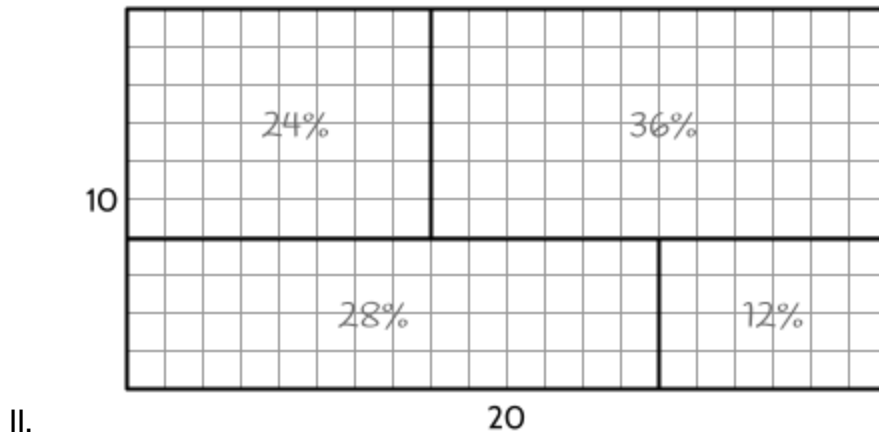
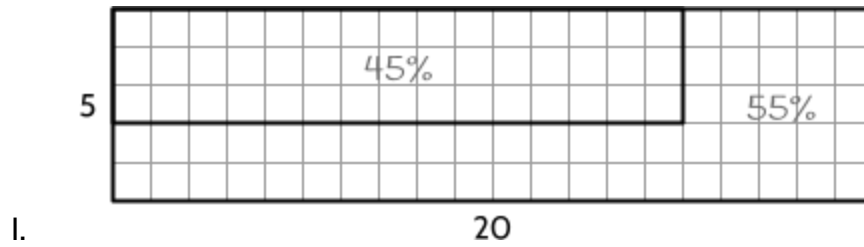
II. 12 squares

III. 24 squares

IV. 8 squares

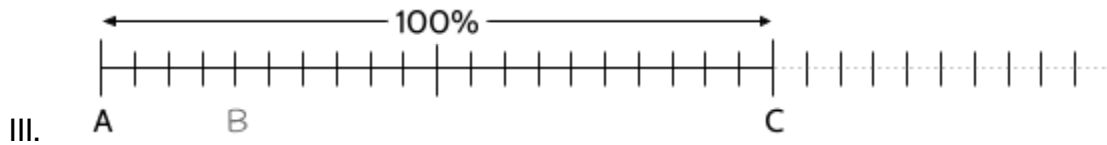
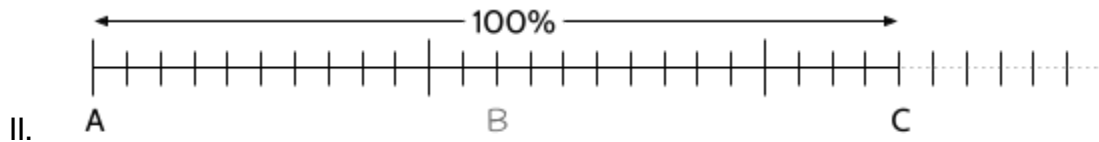
Proportional Reasoning (Part 2)

71)



72)

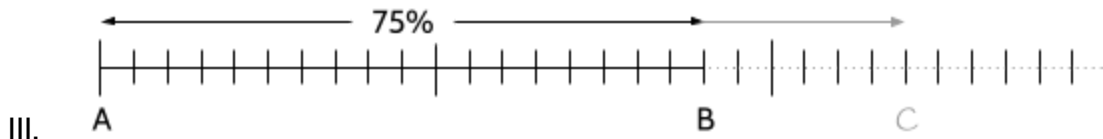
I. This one was done for you.



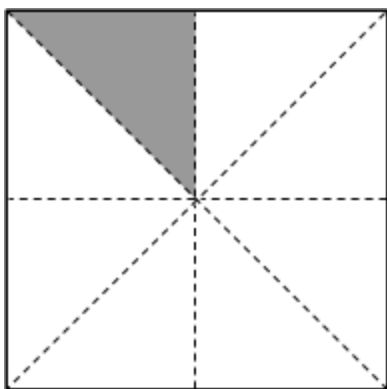
73)



Proportional Reasoning (Part 2)



- 74) The dark triangle is worth \$12.50. One explanation is because the square can be cut into 8 pieces of the same size. The dark triangle is one of these pieces.
 $8 \times \$12.50 = \100.00 .



75)

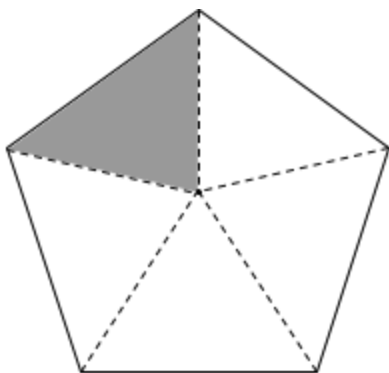
Number of shaded triangles	Fraction	Percent	Portion of \$100
1	$\frac{1}{8}$	12.5%	\$12.50
2	$\frac{2}{8}$	25%	\$25.00
3	$\frac{3}{8}$	37.5%	\$37.50
4	$\frac{4}{8}$	50%	\$50.00
5	$\frac{5}{8}$	62.5%	\$62.50
6	$\frac{6}{8}$	75%	\$75.00
7	$\frac{7}{8}$	87.5%	\$87.50
8	$\frac{8}{8}$	100%	\$100.00

76) \$25.00

Proportional Reasoning (Part 2)

77) \$75.00

78) The dark triangle is worth \$20.00. One explanation is because the pentagon can be cut into 5 pieces of the same size. The dark triangle is one of these pieces.
 $5 \times \$20.00 = \100.00 .



79)

Number of shaded triangles	Fraction	Percent	Portion of \$100
1	$\frac{1}{5}$	20%	\$20.00
2	$\frac{2}{5}$	40%	\$40.00
3	$\frac{3}{5}$	60%	\$60.00
4	$\frac{4}{5}$	80%	\$80.00
5	$\frac{5}{5}$	100%	\$100.00

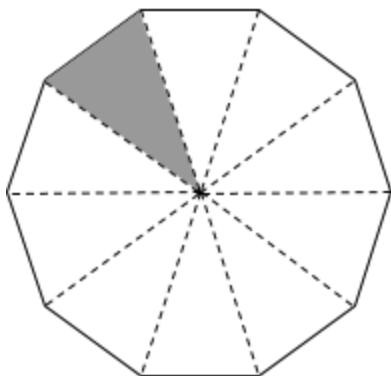
80) \$40.00

81) \$240.00. If the whole pentagon is worth \$400.00, each dark triangle would be worth \$80.00. $3 \times \$80.00 = \240.00 .

82) Explanations will vary, but $\frac{2}{5}$ of \$1,000.00 is \$400.00.

Proportional Reasoning (Part 2)

- 83) The dark triangle is worth \$10.00. One explanation is because the decagon can be cut into 10 pieces of the same size. The dark triangle is one of these pieces.
 $10 \times \$10.00 = \100.00 .



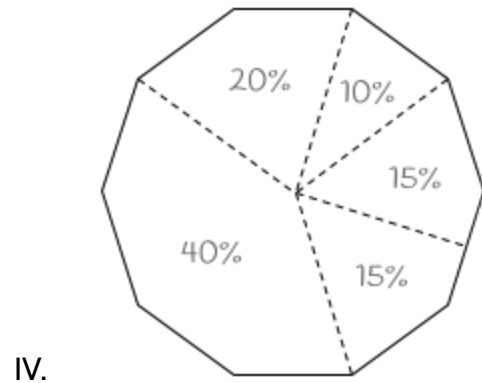
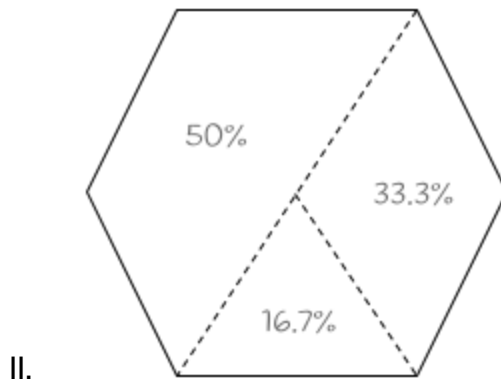
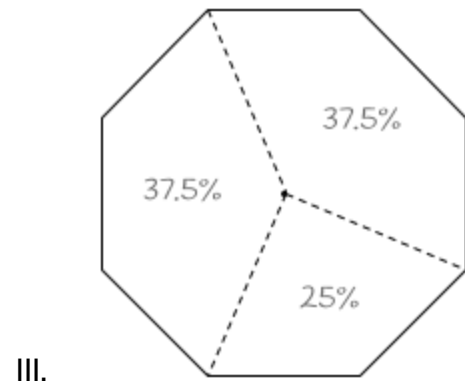
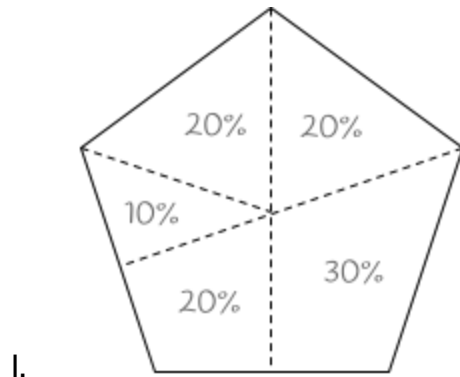
84)

Number of shaded triangles	Fraction	Percent	Portion of \$100
1	$\frac{1}{10}$	10%	\$10.00
2	$\frac{2}{10}$	20%	\$20.00
3	$\frac{3}{10}$	30%	\$30.00
4	$\frac{4}{10}$	40%	\$40.00
5	$\frac{5}{10}$	50%	\$50.00
6	$\frac{6}{10}$	60%	\$60.00
7	$\frac{7}{10}$	70%	\$70.00
8	$\frac{8}{10}$	80%	\$80.00
9	$\frac{9}{10}$	90%	\$90.00
10	$\frac{10}{10}$	100%	\$100.00

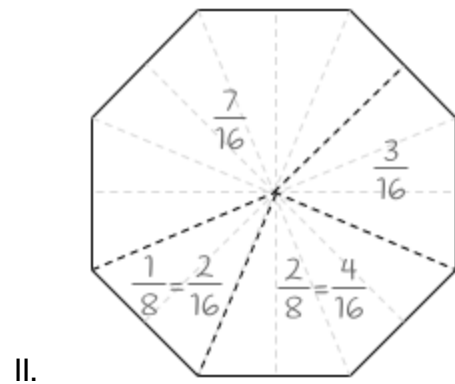
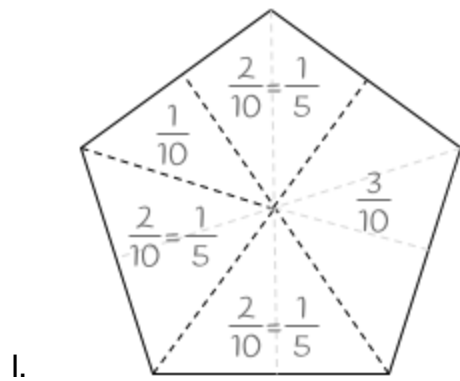
Proportional Reasoning (Part 2)

85) Explanations will vary, but 40% of \$300.00 is \$120.00.

86)



87)



Proportional Reasoning (Part 2)

Comparing Ratios Using Decimals and Percents

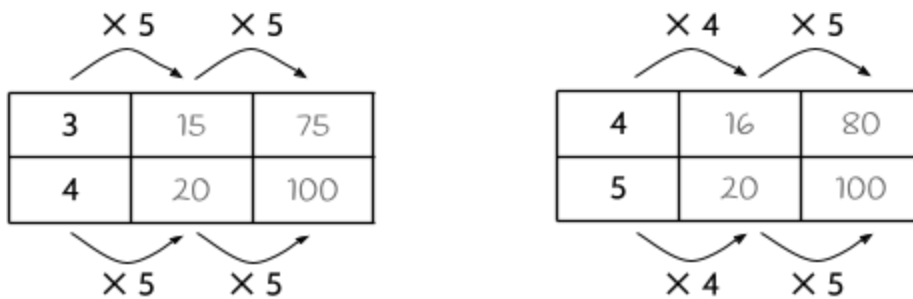
88)

Number of pennies	"out of 100"	Percent	Written as money (decimal)
50	$\frac{50}{100}$	50%	\$0.50
25	$\frac{25}{100}$	25%	\$0.25
80	$\frac{80}{100}$	80%	\$0.80
8	$\frac{8}{100}$	8%	\$0.08
5	$\frac{5}{100}$	5%	\$0.05

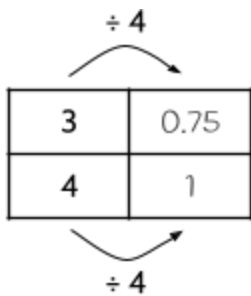
89)

"out of 5"	"out of 10"	"out of 50"	"out of 100"	Percent	Decimal
$\frac{3}{5}$	$\frac{6}{10}$	$\frac{30}{50}$	$\frac{60}{100}$	60%	0.6
$\frac{1}{5}$	$\frac{2}{10}$	$\frac{10}{50}$	$\frac{20}{100}$	20%	0.2
$\frac{4}{5}$	$\frac{8}{10}$	$\frac{40}{50}$	$\frac{80}{100}$	80%	0.8
$\frac{2}{5}$	$\frac{4}{10}$	$\frac{20}{50}$	$\frac{40}{100}$	40%	0.4
$\frac{5}{5}$	$\frac{10}{10}$	$\frac{50}{50}$	$\frac{100}{100}$	100%	1

90)



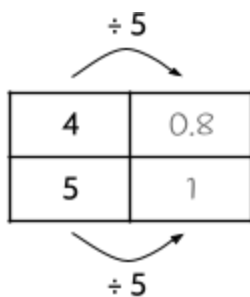
Proportional Reasoning (Part 2)



91)

$\frac{3}{4}$ is equal to the decimal 0.75.

92)

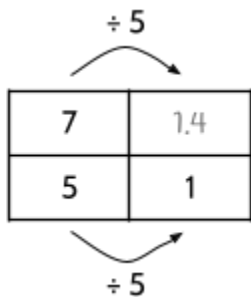


$\frac{4}{5}$ is equal to 0.80.

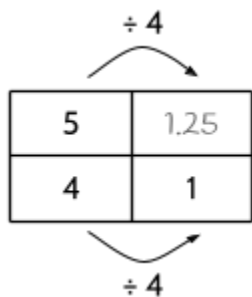
$\frac{3}{4}$ is equal to 0.75.

$\frac{4}{5}$ is greater because the decimal 0.80 is greater than the decimal 0.75.

93)



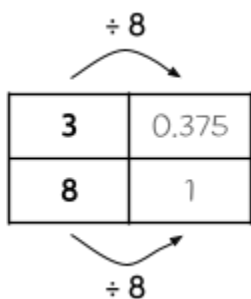
decimal: 1.4



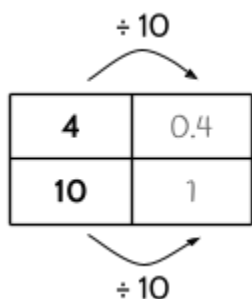
decimal: 1.25

$$\frac{7}{5} > \frac{5}{4}$$

94)



decimal: 0.375



decimal: 0.4

$$\frac{3}{8} < \frac{4}{10}$$

Proportional Reasoning (Part 2)

95)

$\div 16$	
7	0.4375
16	1
$\div 16$	

$\div 5$	
2	0.4
5	1
$\div 5$	

 $\frac{7}{16} > \frac{2}{5}$

decimal: 0.4375 decimal: 0.4

96)

$\div 8$	
$\frac{7}{8}$	$= \frac{0.875}{1}$
$\div 8$	

$\div 20$	
$\frac{17}{20}$	$= \frac{0.85}{1}$
$\div 20$	

 $\frac{7}{8} > \frac{17}{20}$

decimal: 0.875 decimal: 0.85

97)

 $\frac{7}{16} = \frac{\quad}{1} \quad \frac{11}{25} = \frac{\quad}{1} \quad \frac{7}{16} < \frac{11}{25}$

decimal: 0.4375 decimal: 0.44

98)

 $\frac{6}{5} = \frac{\quad}{1} \quad \frac{23}{20} = \frac{\quad}{1} \quad \frac{6}{5} > \frac{23}{20}$

decimal: 1.2 decimal: 1.15

99) $\frac{4}{5} = 0.8$ and $\frac{6}{8} = 0.75$. $\frac{4}{5}$ is greater.

Proportional Reasoning (Part 2)

original decimal percent

$\div 5$ $\times 100$

4	0.8	80
5	1	100

$\div 5$ $\times 100$

100)

original decimal percent

$\div 8$ $\times 100$

3	0.375	37.5
8	1	100

$\div 8$ $\times 100$

101)

Decimal: 0.375

Percent: 37.5%

$\div 5$ $\times 100$

7	1.4	140
5	1	100

$\div 5$ $\times 100$

102)

Decimal: 1.4

Percent: 140%

103)

10	0.4	40
25	1	100

Decimal: 0.4

Percent: 40

104)

5	1.25	125
4	1	100

Decimal: 1.25

Percent: 125%

Proportional Reasoning (Part 2)

105)

13	3.25	325	Decimal: 3.25 Percent: 325%
4	1	100	

106)

$$\frac{19}{20} \xrightarrow{\div 20} \frac{0.95}{1} \xrightarrow{\times 100} \frac{95}{100} = 95\%$$

107) $\frac{7}{16} = \frac{0.4375}{1} = \frac{43.75}{100} = 43.75\%$

108) $\frac{7}{8}$ is greater

4	0.8	80	80%
5	1	100	

7	0.875	87.5	87.5%
8	1	100	

109) $\frac{13}{8}$ is greater

9	1.8	180	180%
5	1	100	

13	1.625	162.5	162.5%
8	1	100	

110) $\frac{9}{16}$ is greater

$$\frac{9}{16} = \frac{0.5625}{1} = \frac{56.25}{100} = 56.25\%$$

$$\frac{11}{20} = \frac{0.55}{1} = \frac{55}{100} = 55\%$$

Proportional Reasoning (Part 2)

111) $\frac{5}{16}$ is greater

114) 22% is greater

112) $\frac{2}{5}$ is greater

115) $\frac{8}{5}$ is greater

113) 0.48 is greater

116) There are many correct answers. Here are a few more correct answers:

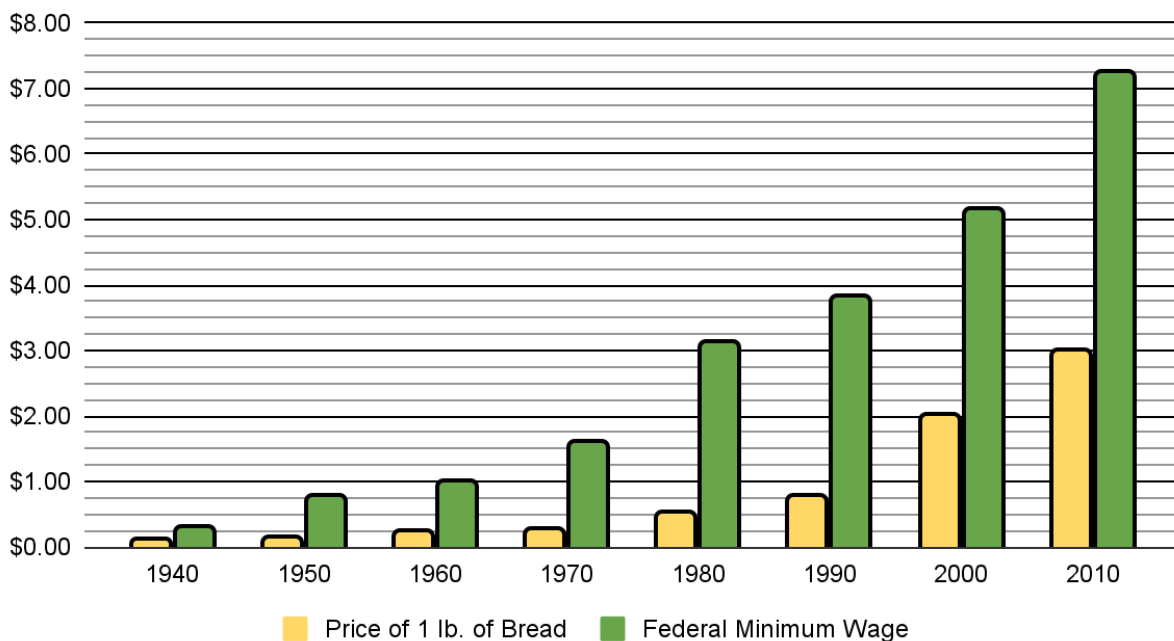
$\frac{14}{28} = 50\%$, $\frac{23}{46} = 50\%$, $\frac{36}{48} = 75\%$, and more....

Proportions in the Real World

Bread and Wages

Look at the graph and table of data below.

The Price of Bread & The Federal Minimum Wage (1930 to 2010)



Year	Price of 1 lb. of Bread	Federal Minimum Wage
1940	\$0.10	\$0.30
1950	\$0.12	\$0.75
1960	\$0.23	\$1.00
1970	\$0.25	\$1.60
1980	\$0.50	\$3.10
1990	\$0.75	\$3.80
2000	\$1.99	\$5.15
2010	\$2.99	\$7.25

- 1) Write four different ratios based on the data above.

Proportional Reasoning (Part 2)

- 2) In which year could people making the federal minimum wage buy the most bread with an hour of work? Explain your answer.

- 3) In which year could people making the federal minimum wage buy the least bread with an hour of work? Explain your answer.

- 4) What would you guess is the ratio of bread to minimum wage this year?

- 5) Choose one of the following sentence frames and continue writing:

When I first looked at this graph...

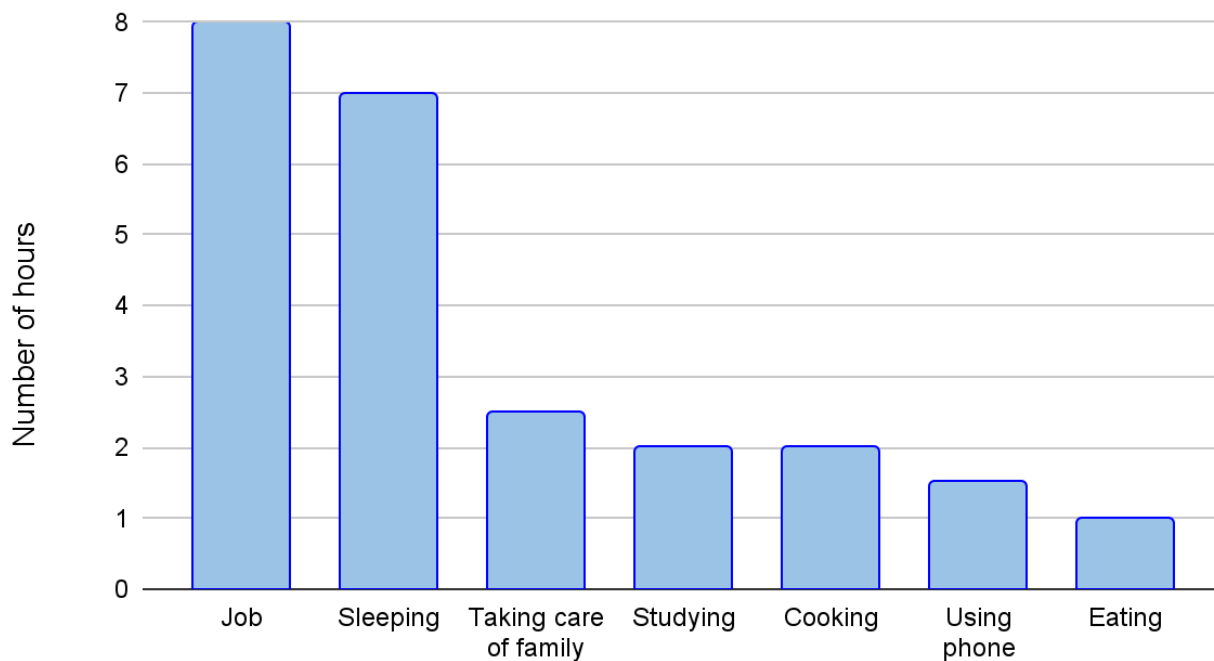
One thing about the graph that surprises me is...

I would like to show this graph to _____ because...

A Typical Day

Look at the graph below.

How Jakima spends a typical day



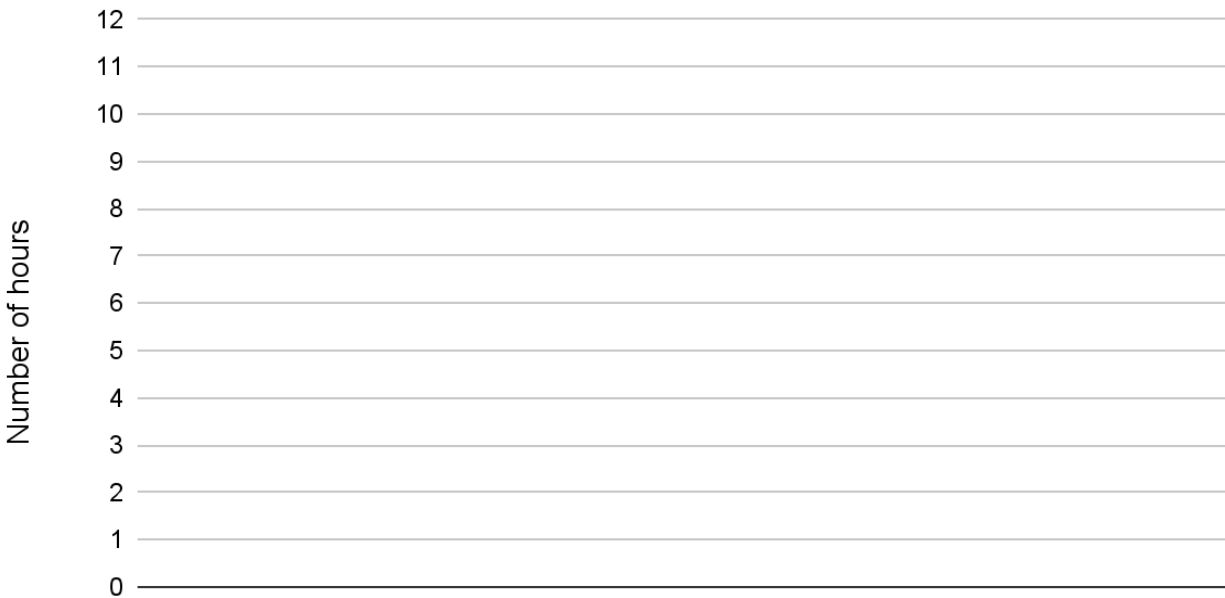
6) Mark the following statements true or false.

Statements	T or F
Jakima typically works at her job 8 hours per day.	T
All of Jakima's activities on the graph add up to 20 hours.	
Jakima works more hours than she sleeps.	
$\frac{1}{3}$ of Jakima's day is spent at her job.	
Jakima studies $\frac{1}{8}$ of a typical day.	
For Jakima, the ratio of working at her job to cooking is 4:1.	
Jakima uses approximately 10% of a typical day taking care of her family.	

Proportional Reasoning (Part 2)

7) Complete the following diagram to show 24 hours in one of your typical days.

How I spend a typical day



Write mathematical statements about how you spend a typical day. Use what you know about ratios, fractions, and percents.

Essential Workers: Pay Them, Protect Them, Empower Them

by Adriana Herrera López (from The Change Agent, <http://changeagent.nelrc.org>)

BEFORE YOU READ:

- 1) What does undervalued mean? Do you think certain workers are undervalued? If so, who?
- 2) What does *fair share* and *proportionate* mean? (Add the prefix *dis* to *proportionate*. Now what does it mean?)
- 3) What does *overrepresented* mean? Can you name an occupation in which men or women or people of color are overrepresented?

Essential Workers Have Been Undervalued

The virus affected my family. My cousin, who worked in a hospital in Colombia, got Covid. He gave it to his parents. They all almost died. My sister, who works as a cleaner in New York City, did not get Covid, but her workload increased a lot. Now she has severe back pain. These family members did essential work during the pandemic, and they paid a price.

Without a doubt, 2020 taught us many things, including: the people who do the vital jobs for society are often undervalued. These workers are our essential workers. They are more exposed to infection. They work longer hours. Many of them do not get paid more for the extra risk they take. I am talking about women and people of color—particularly Blacks and Latinos. Society has long *undervalued* these workers. Now the pandemic has reminded us how important they are!

Essential and Frontline Workers Are Mostly People of Color

In New York City where I live, people of color do more than their fair share of the essential work. They are *disproportionately* represented. According to one report from the city, 75% of

Proportional Reasoning (Part 2)

all frontline workers are people of color. Black people are *overrepresented* in public transit, trucking and delivery services, healthcare, and childcare, food and family services. Latinos are *overrepresented* in cleaning services and as grocery store workers. In the U.S., 76% of the healthcare workers are women. Many of these women also take care of their families, so they have a double workload, which is especially difficult during a pandemic!

Now They Are More Visible

The pandemic has made frontline workers more visible. Now, some people call them heroes. We should not hit the back button and go back to how it was before when these workers were invisible. They are the ones who take care of our health and our children. They make sure we have food and transportation. They drive trucks and deliver our mail and our packages. We should have laws and policies that make sure these workers earn a living wage and have safe working conditions. We should make their work visible. Life would not be possible without them. We should pay them, protect them, and empower them.

Sources:

<https://Comptroller.nyc.gov/reports/new-york-citys-frontline-workers>

<https://www.census.gov/quickfacts/newyorkcitynewyork>

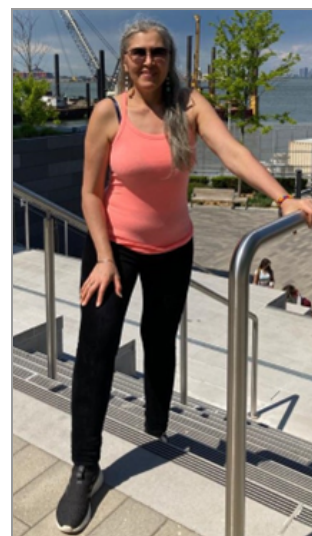
<https://www.census.gov/library/stories/2019/08/your-health-care-in-womens-hands.html>

<https://www.brookings.edu/research/a-policy-manifesto-for-paying-protecting-and-empowering-essential-workers>

Adriana Herrera López is a student of the Queensborough Community College/CUNY Adult Literacy Program in New York City. She was born in Cali, Colombia, and studied at the Santiago de Cali University. Her hobbies include swimming, dancing, traveling, and reading.

The Change Agent is a biannual magazine for adult educators and learners published by the **New England Literacy Resource Center (NELRC)** at **World Education**.

You can submit your own writing for publication with The Change Agent. Visit <http://changeagent.nelrc.org> for more information.



Is It in Proportion?

by Eric Appleton (from The Change Agent, <http://changeagent.nelrc.org>)

When we read, write, and talk about statistics (the study of numbers in the world), the idea of proportion is often an important part of the conversation.

Based on the information we have, a proportion tells us what size or share we might expect in a different situation. For example, if oranges cost \$1.50 for one pound, we would expect two pounds of oranges to cost \$3.00. Paying \$3.00 dollars for two pounds of oranges is proportionate to paying \$1.50 for one pound of oranges. If we had to pay \$4.00 for 2 pounds of oranges, that would be disproportionate; we would think it was unfair.

- 1) Here's another example. Ndeye works 10 hours and earns \$150. Her co-worker, Moise, has the same job. He works 15 hours and earns \$300. Is the pay Ndeye and Moise received *in proportion*? You could also ask, "Is this fair?" What do you think?

Learn More about Proportion

Proportion (noun): The size of something compared to something else

Example: A large **proportion** of essential workers are employed in health care.

When you look at the word proportion, you might notice that it includes the word "portion." A *portion* is part or a share of a whole. Example: *At the end of the shift, each worker took her **portion** of tips.*

Related words: *share, percentage, ratio*

The size, share, or cost we expect	Larger or smaller size, share, or cost than we expect
proportionate	disproportionate
in proportion	out of proportion
representative	overrepresented or underrepresented

Proportional Reasoning (Part 2)

Let's practice thinking about proportions and populations of people. In 2020, the U.S. Census counted all people living in the United States. The population data in the Census is often used in proportions.

- 2) Approximately what proportion of people in the U.S. do you think identify as “white alone”?
 - A. 20%
 - B. 40%
 - C. 60%
 - D. 80%

- 3) Approximately what proportion of people in the United States do you think are Black?
 - A. 15%
 - B. 25%
 - C. 35%
 - D. 45%

- 4) Approximately what proportion of people in the United States do you think are female?
 - A. 49%
 - B. 50%
 - C. 51%
 - D. 52%

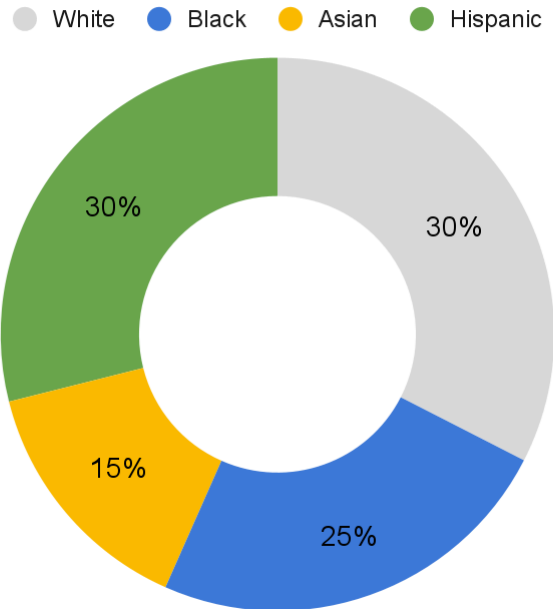
- 5) Check the [answer key](#). How close were your estimates?

Source: <https://www.census.gov/quickfacts/fact/table/US/PSTO45219>

Race and Ethnicity of Frontline Workers in New York City

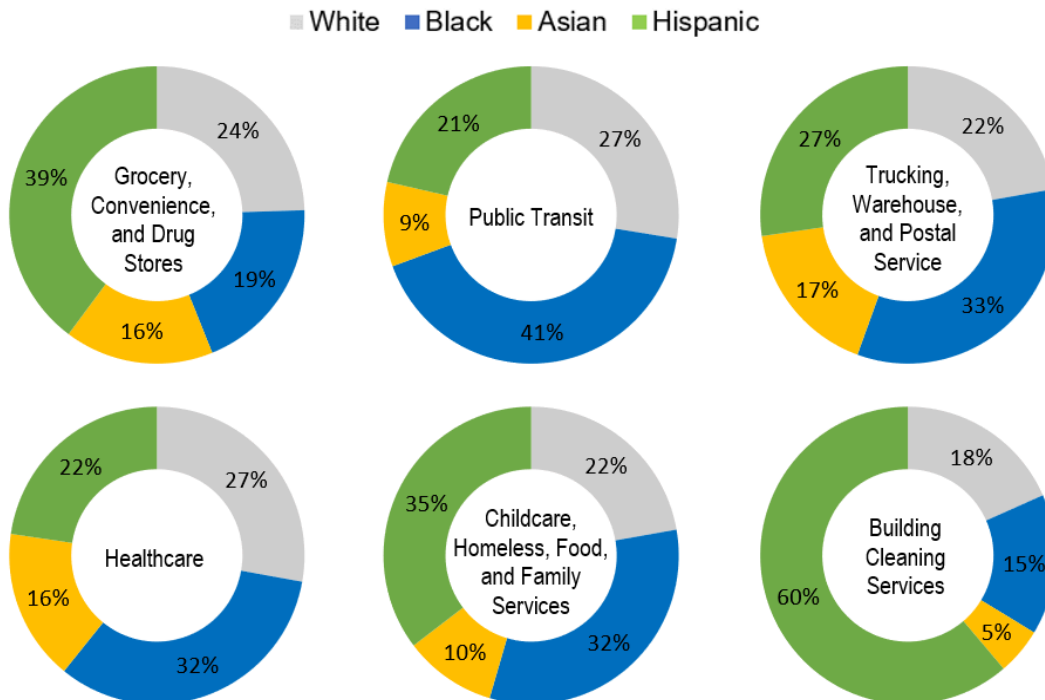
Let's compare different populations of people. About 8 million people live in New York City. On the right, you can see the approximate percentages of New York City's population by race and ethnicity.

New York City Population by Race and Ethnicity
(Approximate Percentages)



6) Now look at the chart at the bottom of the page. What do you notice?

New York City Frontline Workers by Race and Ethnicity

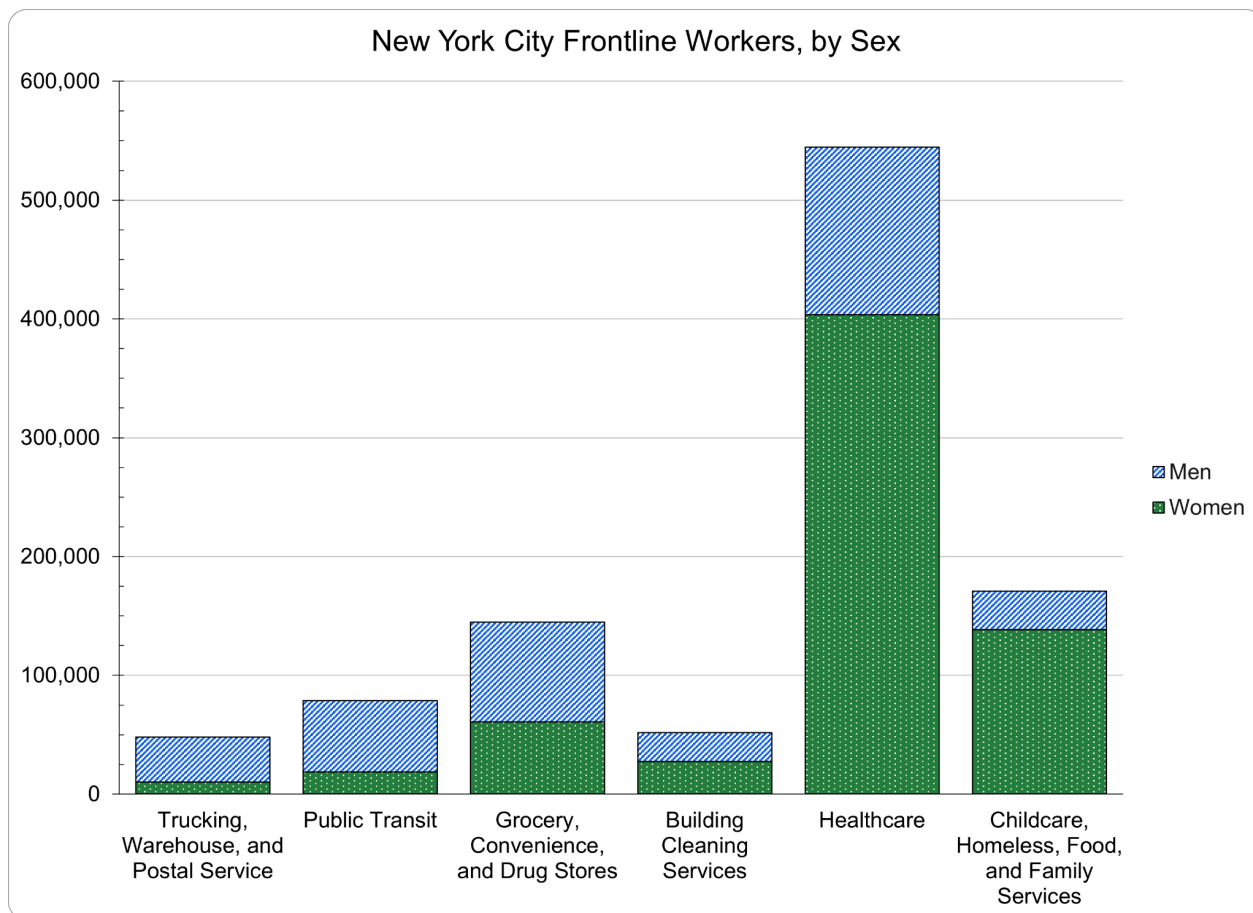


Female Frontline Workers

According to 2019 Census estimates, 52.3% of the population of New York City is female.

- 8) Write three ratios based on the table on the right.
- 9) Now look at the bar graph below. What do you notice when you compare the table of data with the bar graph?

New York City Frontline Workers, by Sex			
Industry	Women	Men	% Women
Trucking, Warehouse, and Postal Service	10,381	37,638	22%
Public Transit	18,788	60,037	24%
Grocery, Convenience, and Drug Stores	60,732	83,968	42%
Building Cleaning Services	27,375	24,448	53%
Healthcare	403,546	140,912	74%
Childcare, Homeless, Food, and Family Services	138,360	32,594	81%
Total	659,182	379,597	63%



Proportions in the Real World - Answer Key

Bread and Wages

- 1) Answers will vary.
- 2) 1970. People making federal minimum wage could buy 6.4 pounds of bread for an hour's work.
- 3) 2010. People making the federal minimum wage could only buy 2.4 pounds of bread for an hour's work.
- 4) Answers will vary.
- 5) Answers will vary.

A Typical Day

6)

Statements	T or F
Jakima typically works 8 hours per day.	T
All of Jakima's activities on the graph add up to 20 hours.	F
Jakima works more hours than she sleeps.	T
$\frac{1}{3}$ of Jakima's day is spent at work.	T
Jakima studies $\frac{1}{8}$ of a typical day.	F
For Jakima, the ratio of working at her job to cooking is 4:1.	T
Jakima uses approximately 10% of a typical day taking care of her family.	T

7) Answers will vary.

Essential Workers: Pay Them, Protect Them, Empower Them

- 1) Answers will vary.
- 2) Answers will vary.
- 3) Answers will vary.

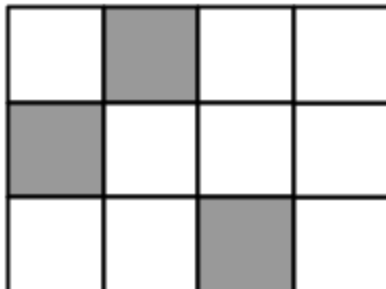
Is It in Proportion?

- 4) Ndeye and Moise's pay is not in proportion. Moise is making \$20/hour. Ndeye is only making \$15/hour. If they have the same job, it does not seem fair that Ndeye is making less money per hour. It's possible that Moise has worked at the job longer and received raises. What do you think? What other explanations could there be? Is it fair? (On average, women in the United States make \$0.82 for every \$1.00 that men make: <https://www.pewresearch.org/short-reads/2023/03/01/gender-pay-gap-facts/>)
- 5) People who identify as white are about 76% of the total US population.
- 6) People who identify as Black are about 13% of the total population.
- 7) People who identify as female are about 51% of the population.
- 8) Answers will vary.
- 9) There are many you might notice. Here are a few things we noticed:
30% of New Yorkers identify as Hispanic, but 60% of the people who clean buildings are Hispanic.
25% of New Yorkers identify as Black, but 41% of workers in public transit are Black.
30% of New Yorkers identify as white, but only 24% of grocery, convenience, and drug store workers are white.
- 10) Answers will vary.
- 11) There are many different ratios you could write. Can you write a part-to-part ratio? How about a part-whole ratio?
- 12) Answers will vary.

Practice Test Questions

1) What percent of the figure below is shaded?

- A. 10%
- B. 25%
- C. 30%
- D. 300%



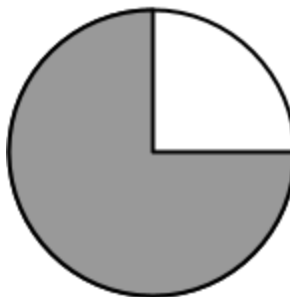
2) What fraction of the figure below is shaded? Circle all correct answers.

- A. $\frac{1}{3}$
- B. $\frac{2}{3}$
- C. $\frac{6}{9}$
- D. $\frac{3}{9}$



3) What percent of the circle below is shaded?

- A. 25%
- B. 50%
- C. 75%
- D. 80%



Proportional Reasoning (Part 2)

- 4) The city charges \$1.50 for each half-hour of parking at a meter. If Ndidi parks for 4 hours, how much money will she pay?
- A. \$1.50
 - B. \$3.00
 - C. \$6.00
 - D. \$12.00
- 5) If pencils cost \$0.20 each, how many pencils can Maria buy with \$3.00?
- A. 7
 - B. 8
 - C. 10
 - D. 15
- 6) If 8 oranges cost \$6.00, how much would 24 oranges cost?
- A. \$48.00
 - B. \$18.00
 - C. \$12.00
 - D. \$2.00
- 7) If 6 grapefruits cost \$4.50, how much would 2 grapefruits cost?
- A. \$1.50
 - B. \$2.25
 - C. \$9.00
 - D. \$27.00

Proportional Reasoning (Part 2)

- 8) A store charges \$27.00 for a case of diet soda.
Each case contains 3 boxes of diet soda.
Each box contains 6 bottles of diet soda.
What is the cost of each bottle of diet soda?
- A. \$1.50
 - B. \$3.00
 - C. \$4.50
 - D. \$9.00
- 9) Consider the prices of bananas at the following stores:
- Uptown Groceries (\$1.60 for 2 pounds)
 - Downtown Products (\$2.70 for 3 pounds)
 - Quick Stop Goods (\$3.20 for 4 pounds)
- Which of the following statements is true?
- A. Downtown has the cheapest price for bananas.
 - B. The price of bananas at Quick Stop is more expensive than Uptown.
 - C. Uptown and Quick Stop are selling bananas at the same rate.
 - D. Uptown bananas are the most expensive.
- 10) Jazae drove from her home outside Binghamton to Lake George, which is about 195 miles. The trip took her 3 hours. What was her average speed?
- A. 55 miles per hour
 - B. 65 miles per hour
 - C. 195 miles per hour
 - D. 585 miles per hour

Proportional Reasoning (Part 2)

- 11) If you make orange juice with a ratio of 2 cans of orange concentrate to 3 cans of water, how many cans of orange concentrate would you need to use with 9 cans of water?
- A. 5
B. 6
C. 8
D. 18
- 12) The Butiko grocery store charges \$3 per pound for apples. Which of these tables below shows this relationship?

A.

Pounds	Total Cost (dollars)
0	0
2	6
4	12

B.

Pounds	Total Cost (dollars)
0	3
1	6
2	9

C.











Pounds	Total Cost (dollars)
0	3
2	3
4	3

D.

Pounds	Total Cost (dollars)
0	0
6	2
12	4

Proportional Reasoning (Part 2)

Answer the following two questions based on the chart below.

	Urban Forest (acres)	Size of Borough (acres)
Bronx		
Brooklyn		
Manhattan		
Queens		
Staten Island		

Data from the Natural Areas Conservancy map of New York City: <https://naturalareasnyc.org/map>

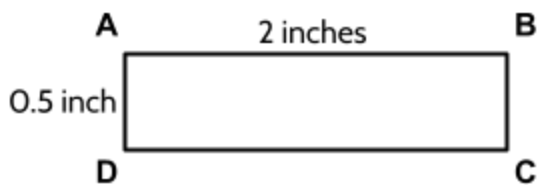
 = 500 acres  = 10,000 acres

- 15) Approximately how many acres of urban forest does Queens have?
- A. 500
 - B. 1000
 - C. 2000
 - D. 10,000
- 16) About what percentage of the Bronx is covered with urban forest?
- A. 0.2%
 - B. 1%
 - C. 7%
 - D. 25%

Proportional Reasoning (Part 2)

17) According to the scale drawing below, what is the actual size of side \overline{AB} ?

- A. 1 foot
- B. 2 feet
- C. 4 feet
- D. 8 feet



Scale: $\frac{1}{4}$ inch = 1 foot

18) A contractor is building a house using a building plan with a scale of $\frac{1}{2}$ inch = 1 foot. If the length of a wall drawn in the plan is 12 inches, how long would the actual wall be when it is built?

- A. 1 foot
- B. 12 feet
- C. 24 inches
- D. 24 feet

19) Two cities that are 150 miles apart are 5 inches apart on a map. What is the scale used on the map?

- A. 1 in = 3 mi
- B. 1 in = 30 mi
- C. 1 in = 150 mi
- D. 1 in = 750 mi

20) The actual distance between Springfield and Martinsville is 54 miles. On a map, Springfield and Martinsville are 3 centimeters apart. On the map, Martinsville and Quincy are 12 centimeters apart. What is the actual distance between Martinsville and Quincy?

- A. 63 miles
- B. 162 miles
- C. 216 miles
- D. 648 miles

Proportional Reasoning (Part 2)

- 21) A scale drawing of a house uses a scale of 0.5 inches = 2 feet. What is the length, in inches, of a line on the scale drawing that represents an actual length of 5 feet?
- A. 1.25 inches
B. 2.5 inches
C. 10 inches
D. 20 inches
- 22) Julia uses a photocopier to enlarge a business logo. The original dimensions of the logo were 2" by 3". Which of the following could not be the dimensions of the enlargement?
- A. 4" by 6"
B. 4" by 5"
C. 6" by 9"
D. 8" by 12"
- 23) A hospital claims that in their wards, the nurse to patient ratio is always at least 3:10. Which of the wards below does not live up to the hospital's claim?
- A. A ward with 5 nurses and 16 patients
B. A ward with 6 nurses and 18 patients
C. A ward with 8 nurses and 20 patients
D. A ward with 10 nurses and 35 patients
E. A ward with 12 nurses and 40 patients
- 24) A job training program accepted $\frac{4}{5}$ of the people who applied. If 60 people were accepted, how many people applied?
- A. 12
B. 48
C. 75
D. 80
E. 125

Source of questions 19-21: *Will This Be on the Test?* packet and question 22 is from a Will This Be on the Test? December 2023 blog entry. Both resources were created by the SABES Mathematics and Adult Numeracy Curriculum & Instruction Center with funding from the MA Department of Adult and Community Education. Full resource available at <https://sabes.org/content/will-be-test-inspiration-tackling-standardized-test-questions-conceptually-and-creatively>

Proportional Reasoning (Part 2)

- 25) Amy and Kate are shopping online for kitty litter and have found the following prices at five different online stores. Which store has the lowest price per pound for kitty litter?

Website	Deal
Pet Supplies Unlimited	8 pounds for \$27
Cat Life	16 pounds for \$40
Raining Cats and Dogs	10 pounds for \$35
Love Your Pet	15 pounds for \$37
PetCare.com	12 pounds for \$36

- A. Pet Supplies Unlimited
B. Cat Life
C. Raining Cats and Dogs
D. Love Your Pet
E. PetCare.com
- 26) If a train travels at an average of 70 miles per hour, how many miles does it travel in 4 hours?
- A. 140 miles
B. 210 miles
C. 280 miles
D. 350 miles
- 27) On a map, 1 centimeter represents 50 miles. How many miles are represented by 5 centimeters?
- A. 1
B. 5
C. 50
D. 250

Proportional Reasoning (Part 2)

- 28) The table below shows different trips by a cyclist who rides her bike for exercise on the weekends.

Hours	Distance (kilometers)
2	36
3	
6	108

How many kilometers could the cyclist ride in 3 hours?

- A. 18 kilometers
 - B. 54 kilometers
 - C. 72 kilometers
 - D. 144 kilometers
- 29) A typical shower in the United States uses about 2 gallons of water per minute. If 1 gallon equals 16 cups, how many cups of drinking water are used in a 15-minute shower?

- A. 30
- B. 60
- C. 240
- D. 480

- 30) Molly runs $\frac{1}{3}$ of a mile in 4 minutes. If Molly continues at the same speed, how many minutes will it take her to run two miles?

Enter your answer on the grid to the right.

	⊗	⊗	⊗	
⊙	⊙	⊙	⊙	⊙
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

Proportional Reasoning (Part 2)

31) The table below shows the price of different quantities of rice.

Rice (pounds)	6	8	10	12
Dollars	\$4.68	\$6.24	\$7.80	\$9.36

How much does one pound of rice cost?

- A. \$0.46/lb.
- B. \$0.62/lb.
- C. \$0.78/lb.
- D. \$1.56/lb.

32) The graph shows how much water is used in a typical shower.

Part I:

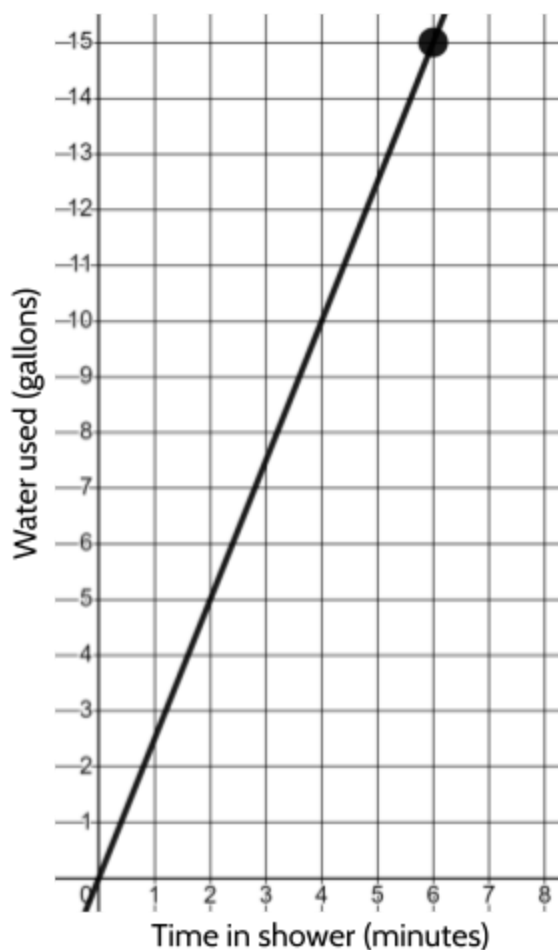
What is the significance of the point (6, 15) on the graph?

- A. Every shower uses 15 gallons of water.
- B. Every shower lasts 6 minutes.
- C. A 15-minute shower uses 6 gallons of water.
- D. A 6-minute shower uses 15 gallons of water.

Part II:

How many gallons are used in a 10-minute shower?

- A. 25 gallons
- B. 15 gallons
- C. 10 gallons
- D. 4 gallons



Proportional Reasoning (Part 2)

- 33) In New York City, the property tax rate for many homes is \$8.70 per \$1,000 of property value⁴. What is the tax on a property valued at \$200,000?
- A. \$870
 - B. \$1740
 - C. \$8700
 - D. \$17,400
- 34) In February 2024, 7 in 10 Americans surveyed said they were concerned about rising prices for everyday purchases. In the summer of 2022, 83% of Americans surveyed said they were concerned about rising prices for everyday purchases.

Part I: What is the difference in percent between the two ratios?

- A. 3%
- B. 13%
- C. 73%
- D. 76%

Part II: Based on these ratios, which statement is true? In 2024...

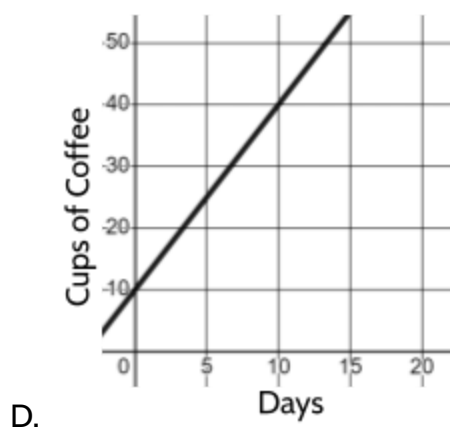
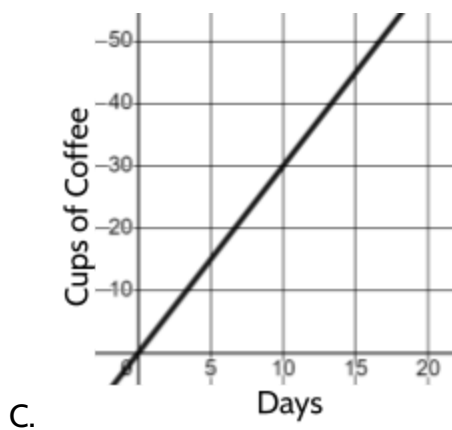
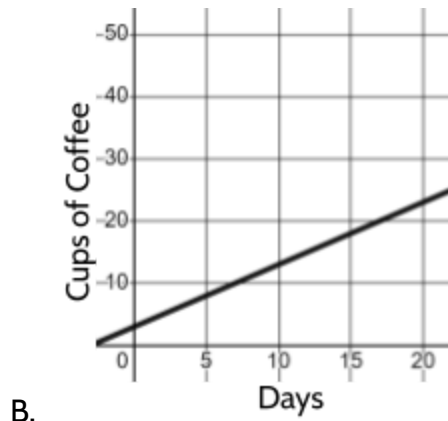
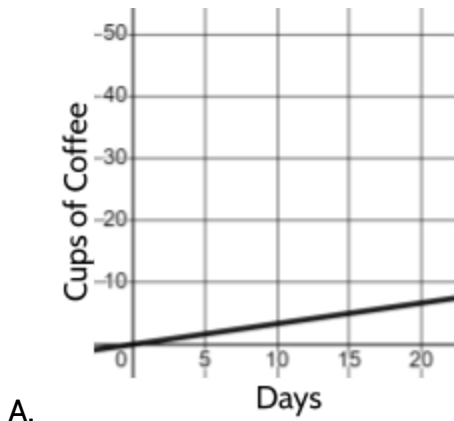
- A. ...fewer Americans were concerned about rising prices of everyday purchases compared with 2022.
- B. ...more Americans were concerned with rising prices of everyday purchases compared with 2022.
- C. ...there was no change in the number of Americans concerned about rising prices of everyday purchases.
- D. ...all Americans were concerned about rising prices of everyday purchases.

⁴ <https://cbcny.org/nyc-effective-tax-rates>

Proportional Reasoning (Part 2)

35) Fredy drinks 3 cups of coffee every day.

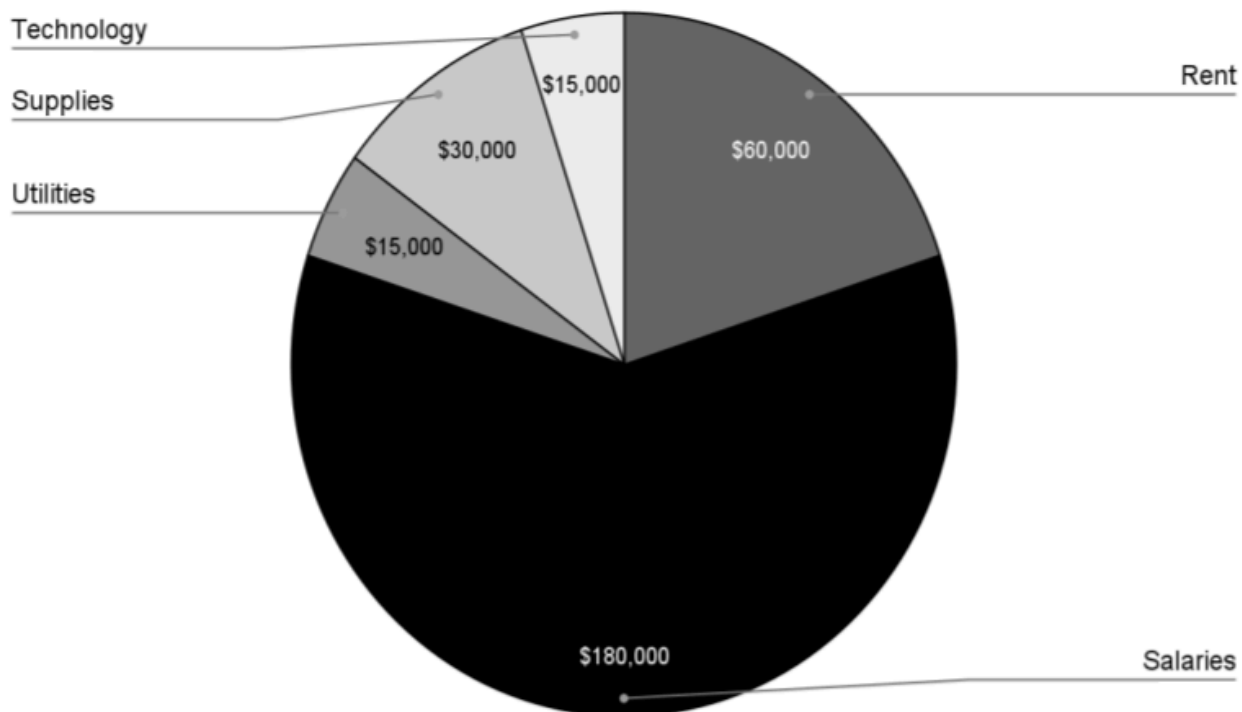
Which graph represents the relationship between the total number of cups of coffee Fredy drinks and the number of days?



Proportional Reasoning (Part 2)

The following two questions are based on the chart below.

Adult Learning Center Yearly Budget

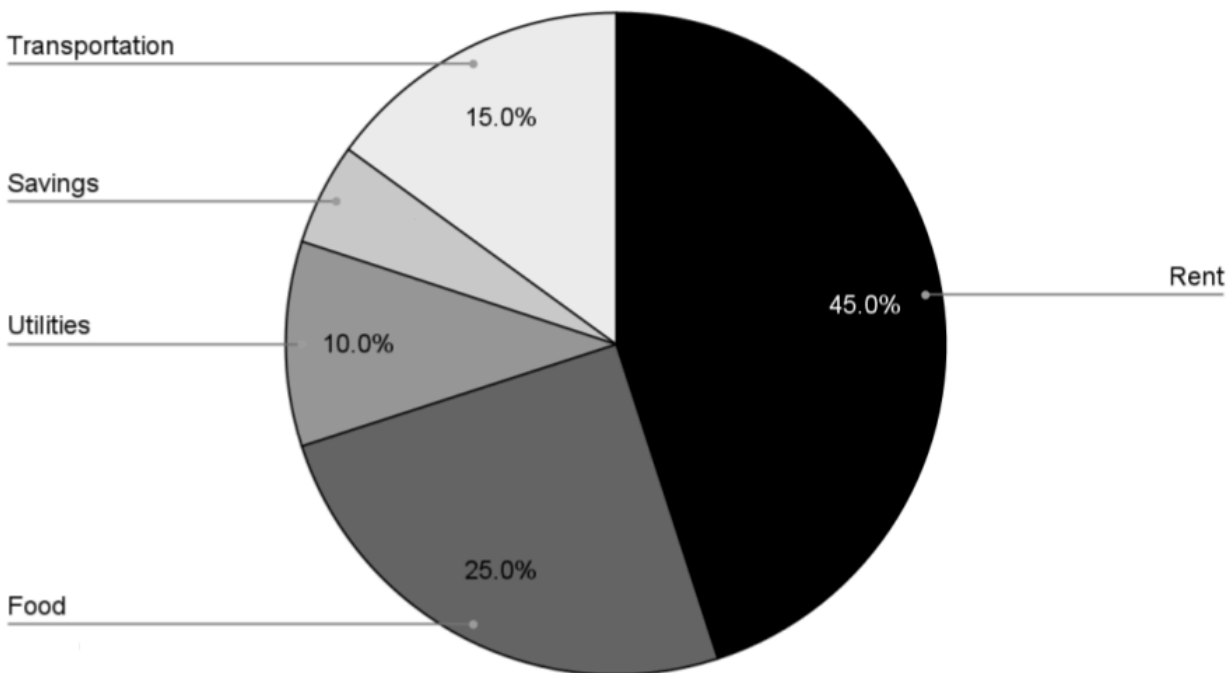


- 36) What percent of the total yearly budget are salaries?
- A. 18%
 - B. 40%
 - C. 60%
 - D. 100%
- 37) If Rent goes up by \$15,000 and no money is spent on Technology, what percent of the yearly budget would Rent be?
- A. 15%
 - B. 25%
 - C. 60%
 - D. 75%

Proportional Reasoning (Part 2)

The following two questions are based on the graph below.

Antoinette's Monthly Budget



- 38) If Antoinette spent \$4,000 for her total month budget, how much did she spend on food?
- A. \$500
 - B. \$1000
 - C. \$1500
 - D. \$2000
- 39) What percentage of her monthly budget was Antoinette able to put into savings?
- A. 5%
 - B. 10%
 - C. 15%
 - D. 20%

Proportional Reasoning (Part 2)

- 40) The table below can be used to find out how many gallons of gasoline a car will need to travel a given number of miles.

Gasoline Usage

Distance (miles)	Rate of Use (miles per gallon)						
	20 mpg	25 mpg	30 mpg	35 mpg	40 mpg	45 mpg	50 mpg
500	25.0	20.0	16.7	14.3	12.5	11.1	10.0
1000	50.0	40.0	33.3	28.6	25.0	22.2	20.0
1500	75.0	60.0	50.0	42.9	37.5	33.3	30.0
2000	100.0	80.0	83.3	71.4	50.0	44.4	40.0
2500	125.0	100.0	133.3	114.3	62.5	55.6	50.0
3000	150.0	120.0	216.7	185.7	75.0	66.7	60.0

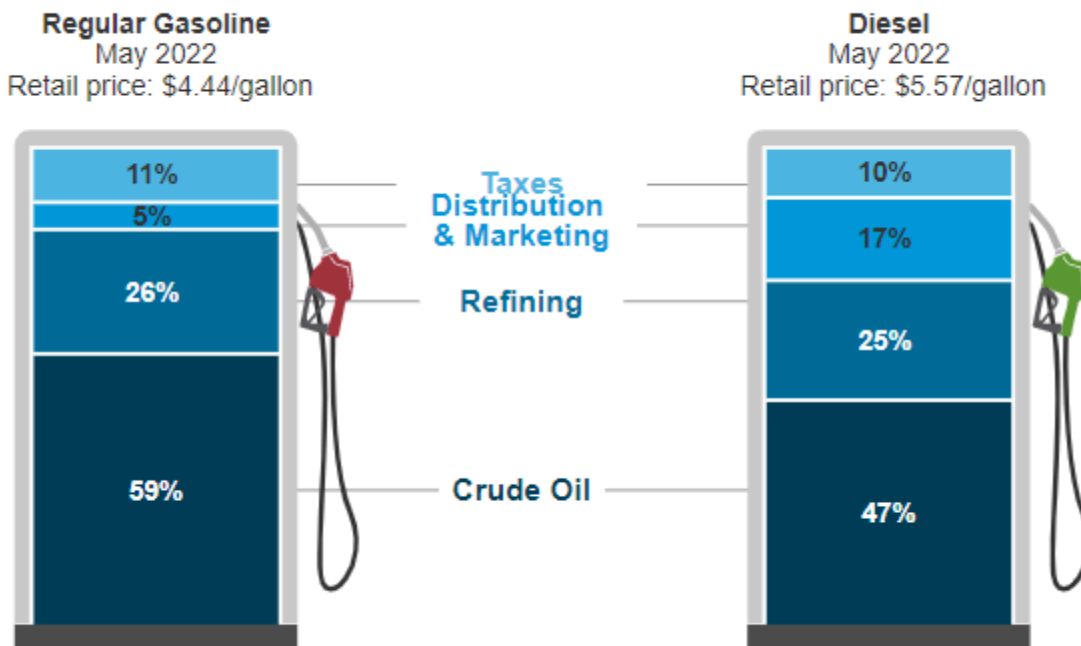
If a car uses gasoline at a rate of 40 miles per gallon (mpg), how much gasoline would be needed to travel 2000 miles?

- A. 37.5 gallons
 - B. 44.4 gallons
 - C. 50.0 gallons
 - D. 71.5 gallons
- 41) The price of Gina's dinner before tax and tip was \$20.00. The restaurant added 10% tax to the bill, then Gina left a \$3.00 tip. How much did Gina pay for her dinner in total?
- A. \$22.00
 - B. \$24.00
 - C. \$25.00
 - D. \$33.00

Proportional Reasoning (Part 2)

The major components of average monthly prices of gasoline and diesel fuel are shown below. Also known as petroleum, crude oil is a fossil fuel that is extracted from the ground through oil drilling. Oil companies such as Exxon and British Petroleum refine crude oil into regular gasoline or diesel. They advertise their fuel and distribute the fuel with pipelines and trucks. Federal and state governments use fuel taxes to pay for transportation infrastructure and encourage protection of the environment.

What we pay for in a gallon of:



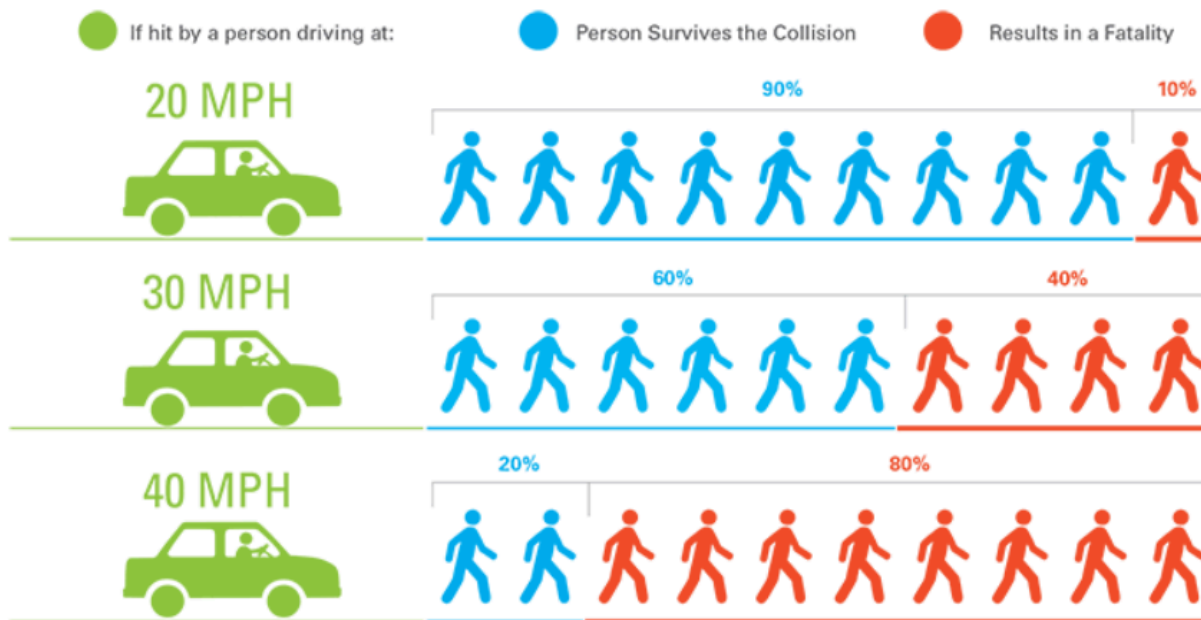
 Data source: U.S. Energy Information Administration, *Gasoline and Diesel Fuel Update*

<https://www.eia.gov/petroleum/gasdiesel/>

- 42) What was the approximate tax charged on a gallon of diesel fuel in May 2022?
- A. \$0.90
 - B. \$0.56
 - C. \$0.11
 - D. \$0.10

Proportional Reasoning (Part 2)

Analyze the diagram below, then answer the questions that follow.



Data source: US Department of Transportation, *Vehicle Travel Speeds and Pedestrian Injuries*. March 2000.

- 43) Which of these ratios correctly describes the number of pedestrians who survive when hit by a car traveling at 30 mph?
- A. 2 out of 10 pedestrians survive the collision
 - B. 4 out of 6 pedestrians survive the collision
 - C. 4 out of 10 pedestrians survive the collision
 - D. 6 out of 10 pedestrians survive the collision
- 44) Based on the chart, which of the following statements is true when a person driving a car hits a pedestrian?
- A. The likelihood of pedestrian fatality decreases as speed increases.
 - B. The likelihood of pedestrian fatality increases as speed increases.
 - C. There is no correlation between pedestrian fatality and speed.
 - D. A decrease in speed causes more pedestrian fatalities.

Practice Test Questions - Answer Key

- | | | |
|-------|--------|-----------------|
| 1) B | 16) C | 31) C |
| 2) A | 17) D | 32) I. D, II. A |
| 3) C | 18) D | 33) B |
| 4) C | 19) B | 34) I. B, II. A |
| 5) D | 20) C | 35) C |
| 6) B | 21) A | 36) C |
| 7) A | 22) B | 37) B |
| 8) A | 23) D | 38) B |
| 9) C | 24) C | 39) A |
| 10) B | 25) D | 40) C |
| 11) B | 26) C | 41) C |
| 12) A | 27) D | 42) B |
| 13) C | 28) B | 43) D |
| 14) B | 29) D | 44) B |
| 15) C | 30) 24 | |

Vocabulary Review

You can use this section to look up words used in this math packet.

area (noun): The size of a flat surface, measured by counting how many squares it would take to cover the surface.

equivalent (adjective): Equal in value. Example: $\frac{1}{2}$ and 0.5 are *equivalent*.

multiple (noun): A number that can be divided by another number evenly, with no remainder. 25 is a *multiple* of 5.

number line (noun): A picture of a straight line on which all numbers can be placed. Numbers get bigger as you move to the right and smaller as you move to the left.

fraction (noun): A part of a whole amount. $\frac{1}{2}$ or “one half” is an example of a fraction.

The top number (**numerator**) shows how many parts we have. It is the value of the “part” in a part-whole relationship.

The bottom number (**denominator**) says how many equal pieces the whole is divided into. It is the value of the “whole” in a part-whole relationship.

per (preposition): for each, for every

- The car was traveling 40 miles *per* hour.
- Most showers use 2.1 gallons of water *per* minute.
- The taxi charges \$2.75 *per* mile.
- The air pressure in your car tires should be 32-35 pounds *per* square inch.

percent (noun): A percent is a part-whole ratio which is “out of” 100. *Per-* means “for every” and *-cent* means “100,” so the word *percent* literally means “for every 100.” For example, “5 percent” means “5 for every 100.” We use the symbol % to show percent.

product (noun): The result of multiplication. 4 times 5 gives a *product* of 20.

proportion (noun): a statement that two ratios or fractions are equivalent. This is an example of a proportion: $\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{36 \text{ inches}}{3 \text{ feet}}$. It can also mean a proper or equal share or the size of something compared to something else.

disproportionate (adjective): Larger or smaller size, share, or cost than we expect

Proportional Reasoning (Part 2)

rate (noun): A ratio that compares two different quantities, such as miles and hours (speed) or dollars and pounds (price).

- The sink was leaking at a *rate* of $\frac{1}{2}$ an ounce of water per minute.
- The train traveled at a *rate* of 80 miles per hour.
- The hybrid car gets a gas mileage *rate* of 50 miles per gallon.

ratio (noun): A comparison between two or more numbers using multiplication or division. It is a relationship between two quantities.

A **part-whole ratio** compares part of a quantity to the whole quantity. Example: 9 right-handed people out of every 10 people

A **part-to-part ratio** compares part of a quantity to another part of the quantity. Example: 1 left-handed person for every 9 right-handed people.

sequence (noun): A sequence is a list of things in order. Often, a sequence is a list of numbers in order.

series (noun): A group of things that come one after another

set (noun): A collection of things

term (noun): A number (or other quantity) that is part of an ordered series

unit rate (noun): A unit rate shows how much of something per 1 unit of something else. Examples: \$0.50 per orange, 65 miles per hour, and 25 students for every teacher.

unit price (noun): The cost for one item or for one unit of measure. A unit price is a unit rate that shows the price of something.

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