Teacher Leader Article Reviews

The following are reviews of articles that can be found in the Mathematics Toolkit from the October 2014 NYSED Common Core/TASC Teacher Learning and Leadership Institute. All of the reviews were written by NYS Teacher Leaders from across New York State.

Article Review Tyler Holzer

Article: "Orchestrating Discussions," *Mathematics Teaching in the Middle School*, Vol. 14, No. 9, 2009.
Authors: Smith, M.K., Hughes, E. K., Engle, R. A., Stein, M. K.

Summary:

This article examines strategies teachers can use to structure whole-class, student-led discussions that convey important mathematical ideas. Writing with the belief that discussions about cognitively challenging mathematical tasks are key to developing students' conceptual understanding of a topic, the authors acknowledge that structuring these types of discussions can be challenging. "Giving students too much or too little support," they point out, "can result in a decline in the cognitive demands of the task." As such, they offer a five practices model for whole-class discussions that they believe can help teachers to plan for and structure the discussion of high-level tasks.

The five practices are:

- **anticipating** student responses
- **monitoring** student work and engagement with the problem
- **selecting** students to present their work
- **sequencing** student responses
- **connecting** student responses.

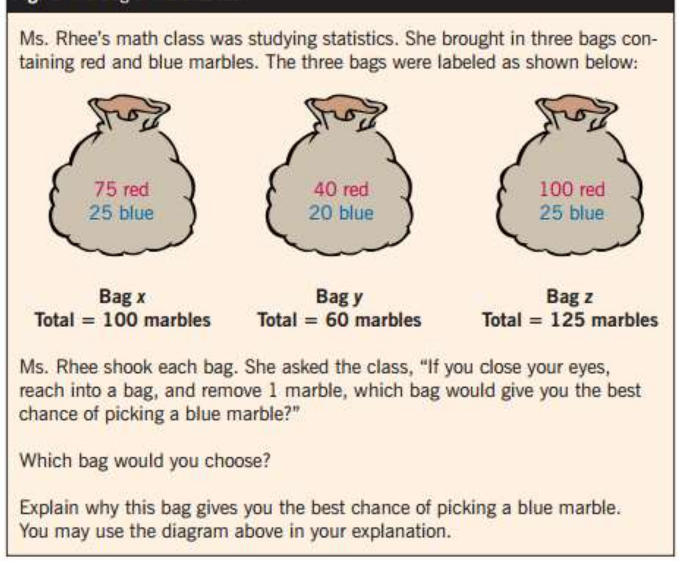


Fig. 1 The Bag of Marbles task

These practices are intended to help teachers manage and steer discussions of problem solving. To show how these practices work, the authors look at the sample problem to the right, as well as student responses to the problem. They use student responses that are both correct and incorrect, complete and incomplete, and point out that almost all student work can be fit into the discussion, if it is chosen carefully.

The first practice, *anticipating*, requires that the instructor think about and solve the problem in as many different ways as they can. By anticipating student responses, the instructor can ask effective questions that support each student's method, and he or she can begin to plan the discussion based on the range of approaches and responses. This connects to the second practice, *monitoring*, which involves circulating around the classroom and checking in on each student's progress. This helps the teacher to decide "who and what to make focal" in the discussion.

The third practice, *selecting*, involves choosing specific approaches that a teacher wants to discuss with the whole group. Selecting students, the authors indicate, "is guided by the mathematical goal for the lesson and the teacher's assessment of how each contribution will contribute to that goal." After the teacher has selected the students she wants to focus on, she moves on to the next practice: *sequencing*. At this stage, the teacher makes careful choices about the order in which to talk about student work. The authors emphasize how crucial this stage is, and they acknowledge that there is no one correct way to sequence the different problem-solving activities for a given activity. When teachers are sequencing work, they say, it is most important that they stay focused on the overarching goals of the lesson. Any sequence is fine as long as it is purposeful.

The last practice is *connecting*. Whole-class discussions should not function like a show-and-tell. Rather, the main purpose is for students to compare and contrast their solution methods and to see important mathematical connections. The authors close by pointing out that this five practice "roadmap" for whole-class discussions can help teachers orchestrate discussions that move students "toward more powerful mathematical understandings."

Connections to the Institute:

Much of the work that we have been doing at the institute has centered around making sure that our students' voices are heard and that their strategies, mistakes, and solutions are inspected. We have analyzed student work in great detail and looked at ways to respond to each student's work in a productive way—both for the benefit of that student, but also for the benefit of the class as a whole. This article offers a straightforward and pragmatic list of practices for structuring whole-class discussions.

The article is intended for middle-school teachers, but it's evident that the five practices model has value in the adult math classroom as well. Many of our students come to class with preconceived ideas about their own abilities and about what math is. One of the best ways to address these preconceived ideas is to get our students talking. When our students talk about math and participate in discussions with their classmates, they develop their own conceptual understanding of a topic and they add new math skills to their toolkit.

Furthermore, with the implementation of the Common Core State Standards, our students are expected to construct arguments and evaluate reasoning. Whole-class discussions give students the opportunity to present an idea and talk about it with their classmates. When other students, or the teacher, presses them on their ideas and asks further questions, that student has to defend their reasoning and explain their thinking. During whole-class discussions, the students who are not presenting their work are assessing the reasoning and methodology of the work being demonstrated. In having productive, focused whole-class discussions, we are helping our students become better thinkers and problem-solvers.

Impact on Teaching:

The ideas set forth in this article have had a major impact on my teaching. Although I have been using student work to start discussions for a while now, this article gave me a clear structure for similar activities in the future. It reminded me that problem-solving activities are most effective when they are connected to an overarching goal. As such, the way we structure these activities has a major impact on how well—and how efficiently—our students attain that goal. A well-structured whole-group discussion is also better for the students. If the teacher plans carefully, anticipates solution methods, and involves all the students, then the class as a whole will be more engaged with the discussion.

Article Review Kathryn Leenay

Article: "Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics"
Author: Randall I. Charles

This article presents the top 21 Big Ideas need in order to develop comprehensive mathematical understanding. The emphasis of the article is on transferrable understanding of the larger concepts. The author provides a breakdown of each idea with examples. I found this article very helpful because my questions always come down to how to get the big ideas across so that when students enter the TASC exam they are able to find some entry point to the problem presented. The article represents much of what we did at the institute because it allows a teacher to present a "big idea" and then use different approaches to solving problems that are as I call them "shades of the same idea." Similar to what we learned at the institute, Charles emphasizes the reality that there is no ONE way to solve a problem. He also acknowledges that a person can still solve the problem if they are weak in some procedural categories.

This article has had a large impact on the way I have been teaching the last couple of weeks. I am preparing seven young men (16-20) for the TASC exam scheduled for April. The level of knowledge is wide. Although all seven students have demonstrated basic procedural fluency in fractions, ratios, isolating a variable and basic computation, they all get "stuck" when I present the problem as a word problem or in an "altered" state. Last week, I was going over how variables can be used to substitute for an unknown when a student stopped me to say, "How can x= 35-y be used to change the expression x-y= 13 to (35-y) – y = 13?" We were doing a sum/difference problem to create two equations, two unknowns. Half the class starting saying "because x is really just 35-y and you're just putting in another value of x in terms of y." This, of course, was correct but even more confusing for the poor student. What was "value, terms!!!" What I realized, however, was that he didn't understand the purpose or function of an = sign. I had never **really** explained that when we did the distribution property and solving for a variable. That night I read this article.

The third Big Idea presented by Charles is:

> **Equivalence: Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.**

This made the substitution method that I was trying to inflict on my poor students real life. I went back the next day with this "Big Idea" on the board. We went over what it means – how it relates to the ratios we had been working with, how it gives reason to why we isolate variables, and so on. We went back to the problem we had done the day before and it started to make sense to my student.

It wasn't a perfect process, but it definitely allowed a couple of switches to be turned on. I will continue to introduce a "big idea" and refer back to them as we move rapidly (sadly this is the way it is) through an intense TASC math curriculum. I will refine how I introduce the big ideas and then I will post them on my wall in the future. The experience has taught me that we really can draw connectors and make relationships as we plow through all this material.

Article Review Kathryn Lent

Article: "Teaching Algebra"
Author: Richard Kalman

Summary

This article discusses the experiences of Richard Kalman, the Executive Director of the Mathematical Olympiads for Elementary and Middle Schools. He was also a former math teacher for over 30 years.

Mr. Kalman's teaching career was built on using rich, engaging problems to teach content. One key for this approach is allowing students to create their own ways to solve problems. Students will then take ownership of the strategies they use, their confidence will be bolstered and their mathematical prowess will increase. He acted as the moderator, allowing students to explain the strategy chosen for that particular problem. This inspired students to volunteer and be active participants in math class.

He also "helped students think 'non-algebraically' in the hopes that they would visualize the problems conversationally and be better prepared to understand the techniques required." His belief was that by utilizing this method, students would build the firmest possible foundation for their futures, embed the thinking process in their minds, and acquire greater clarity and more intuitive comprehension. By providing students with opportunities to look at many problems differently and in varying ways, the greater their understanding will be. He also feels that "problem solving, not memorization, is the cornerstone of all mathematics." We need to encourage students to think mathematically, to reason their way to a solution, and not just be happy with an answer. This will prepare them for their futures.

How does it connect to our work at the institute?

Well, I can sum it up this way – Doesn't this sound just like Mark? Isn't this our experience with Mark and then our more extensive experience with both Mark and Eric? I was laughing to myself while I was typing the above thinking, yes, that is Mark and Eric!!! And now, this is how I am trying to get my teachers to see and how they should be teaching!

How did it impact your teaching?

This question for me is how did it impact my professional development presentation for my HSE teachers? As an administrator, I am not in the classroom teaching on a daily basis. This article gave credence to the methods that Mark and Eric have been modeling for us.

Describe happened when you tried the activity. What worked? What would you do differently next time? What did the experience teach you?

There were a few problems discussed in this article. I tried the CD and DVD Sale. There were teachers who dove into finding an algebraic solution, and used different methods. There was a team of two who discussed it verbally, as in the article. And there were some who drew it out on paper or made a chart. Once all of the teachers were satisfied with their solutions, we had a lively discussion of what worked, what did not and what the best approach was. There were many opinions! The best part of this, it got people thinking outside of the box.

I would not do anything differently next time, at least not at this time. This experience taught me that my staff and I can have a lot of fun while learning a tremendous amount of information. It provided a very collegial moment for all of us.

Article Review Todd Orelli

Article: "The Thinking of Students: Cookies"
Author: Edward S. Mooney

Summary
In this article, Edward S. Mooney describes three basic strategies to solve the "Cookie Problem" – guess and check, algebra, and start with the mean.

> The Problem:
> Tim ate 100 cookies in 5 days. Each day he ate 6 more than the day before. How many cookies did he eat on the first day?

Several student solutions to the problem are presented and Mooney deconstructs their solutions. Of note, Mooney states, "Guessing and checking is a legitimate strategy to solving many mathematics tasks. However, being systematic in the process of guessing is helpful to see patterns in data." Mooney also describes two statistical ideas that lead to students using the mean to help find a solution. First, the mean is a balancing point, and second, students sometimes recognize that the data must be symmetrical since the number of cookies eaten each day increase by a consistent amount.

Institute Connections
In "The Thinking of Students," Mooney promotes many of the same approaches that we discussed and likewise promoted in the October Institute. Namely, he promotes using a problem as a platform for student problem solving, classroom discussion and teaching. This is opposed to teaching procedures first and then giving students problems to solve after the procedures have been outlined. Furthermore, he promotes using a problem that allows for many solution pathways as well as a low barrier to entry. By using a problem such as this, he anticipates that most any student in the class will be able to at least start the problem and make some headway without having to have relatively advanced prior knowledge to do so.

As far as content, the problem that this article centers on is in line with the ideal of teaching algebraic thinking without algebra. Through this problem, we can introduce algebraic concepts as well as concepts of central tendency.

Impact
The article impacted my teaching by highlighting the connection between central tendency and consecutive integer problems. Through the Cookie Problem I was able to create a classroom discussion in which we touched on both topics, thus helping to solidify student understanding through spiraling.

Experiment
When I tried the activity, I was surprised to find that this question was quite difficult for many of my students. I had anticipated that many of my students would not seek an algebraic solution, but would be able to find the correct answer through guess-and-check. To my dismay, more students than I would have thought became frustrated and gave up before I, or another student, assisted them in taking the next step. What's more, many of my colleagues had a similar performance. They came up answers other than 8, for good reasons, but they failed to check to see if their answer

solved the problem. For example, some people would do $100/6 \approx 16$ and then just stick with 16 as their answer.

Overall, using this problem was incredibly rewarding. Students were sincerely challenged and eager to discuss solutions to the problem. Each student made personal growth, even if it was just using guess and check to solve the problem. Regardless of their solution method, an algebraic modeling of the problem made sense to all students after we had an in-depth analysis of why the answer to the problem is 8 cookies. From that experience, modeling this problem with an algebraic equation was easier for everyone to digest. As a result, we could, as a class, have better success with modeling other problems algebraically and using algebra to solve consecutive integers problems.

A big challenge for my students was explaining their work. This process was very beneficial for them and what I realize I need to do differently next time, and in general, is make more of an effort to model and practice what a sufficient solution explanation looks and reads like. To the problem's directions I have added: "Your explanation should be complete enough that someone reading it can inspect what you did and why you did it."

Teachings

Form this activity I learned:

- Almost none of my students felt comfortable enough with writing algebraic equations to choose that as an initial solution strategy for this problem.
- Many of my students attempted calculations with little concern as to what those calculations were calculating. For example, a student would find $100/6 = 16.7$, but they would have no idea what that number represented. Then, they would perform further operations on that number.
- Several students did not attempt to check if their answer solved the problem.
- Having students explain their work helps students expose to themselves whether or not performing certain operations makes sense or has a strategic purpose.
- We need to work on writing in math class and writing explanations to their solutions in general.

Article Review Danielle Fridstrom

Article : "Fostering Relational Thinking while Negotiating the Meaning of the Equals Sign"
Authors: Marta Molina and Rebecca C. Abrose, Teaching Children Mathematics September (2006)

Summary

The article discusses the common misconception among students that the equals sign indicates a stimulus to generate an answer. Additionally the authors relate student understanding of the equals sign with relational thinking. They conducted a series of in class explorations with third graders to determine how student concepts of the equal sign would change while exploring various formats of number sentences, if students would develop relational thinking while developing an understanding of the equal sign, and if this new understanding would persist over time. The authors worked as guest teachers with a group of 18 students over five sessions between November and May. They incorporated a number of tasks such as open number sentences, true/false number sentences, and student generated true number sentences. At the first session no student gave more than one correct answer to the six open number sentences provided. Discussion alone was not sufficient to displace this misconception for most students, which required much time, exploration and discussion with their peers. By the last session 12 of 15 students demonstrated an understanding of the equals sign for five of seven sentences. The authors were only partially successful in initiating relational thinking, though they posit that more examples of the type $11 + 3 = 4 + \_\_$ and extended discussion of patterns would foster more growth of relational thinking.

How does this resource connect to our work at the institute?

The article examines developing deep understanding of the meaning of the equal sign over memorization of algorithms. Concurrently they aimed to support the development of relational thinking, which would provide students with the pattern recognition skills to more successfully navigate equivalencies, without relying so dependently on computational skills alone (multiple pathways to solving problems). Additionally one can see that this investigation took place over an extended time (5 sessions over six months) allowing students time to thoroughly explore a concept (depth over breadth).

How did it impact your teaching?

This article and activity refreshed for me the importance of conducting my own frequent classwide pre-assessments for basic mathematical concepts. Clearly I have become too complacent relying on those initial and periodic TABE exams for pre assessment of student abilities. Unfortunately there are a wide range math concepts that are not captured on the TABE, or flawed questions that do not capture the most common student misconceptions (such as fraction questions utilizing tenths when assessing subtraction with borrowing, which allows the "carry the one" error to work by chance). Checking for deeper foundational understanding of basic concepts, such as equivalence, are vital to support students as they move towards algebraic reasoning.

Describe what happened when you tried the activity. What worked? What would you do differently next time? What did the experience teach you?

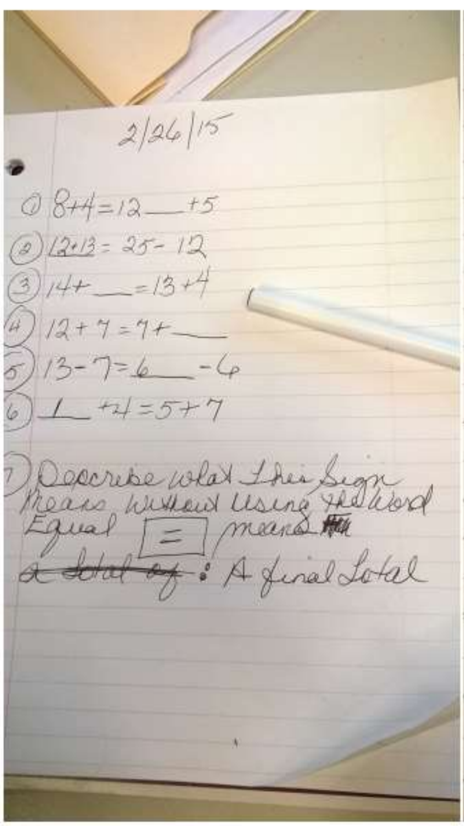
I was drawn to this article as I have noticed (and often been surprised by) my students' misunderstanding of the equal sign as a prompt to generate an answer. This presents problems in many areas, from generating equivalent fractions, to balancing algebraic equations. While I certainly have addressed this misconception when it has arisen, I had not explicitly explored how widespread this misconception is amongst my pre-HSE students.

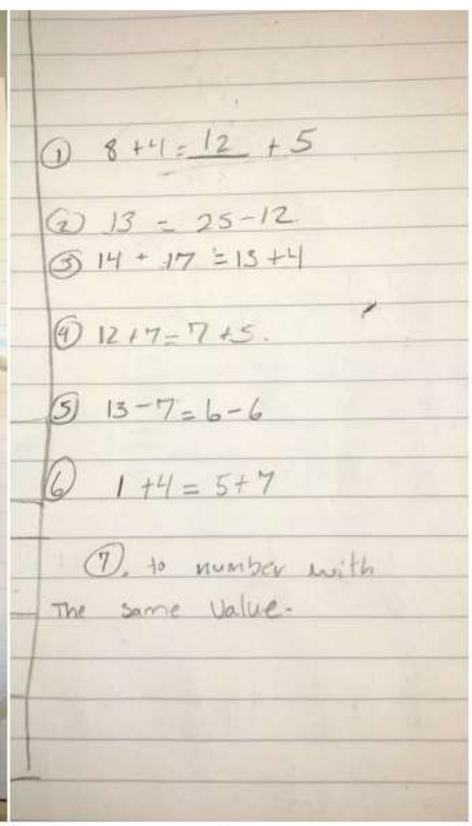
I chose to challenge my students with the following open number sentences from table 2 the article.

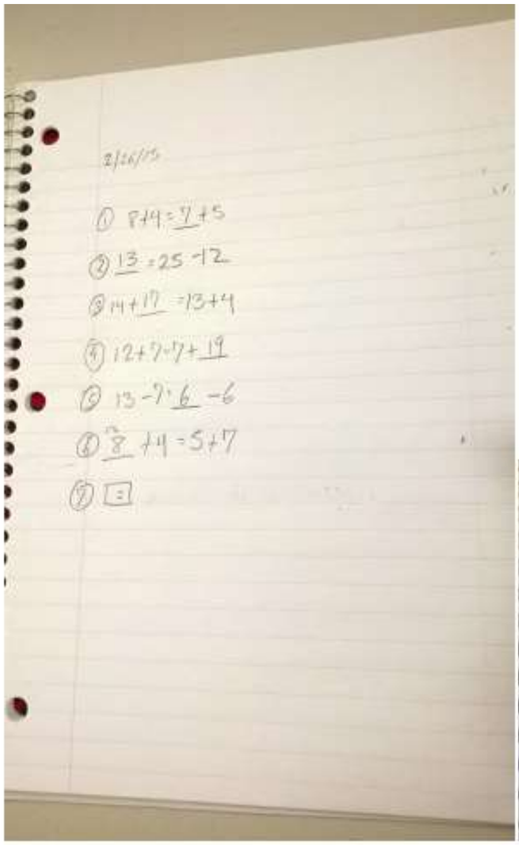
1)	8 + 4 =___ + 5
2)	___= 25 - 12
3)	14 + ___ = 13 + 4
4)	12 + 7 = 7 + ___
5)	13 - 7 = ___ - 6
6)	___ + 4 = 5 + 7
7)	Describe what this sign = means without using the word “equal”

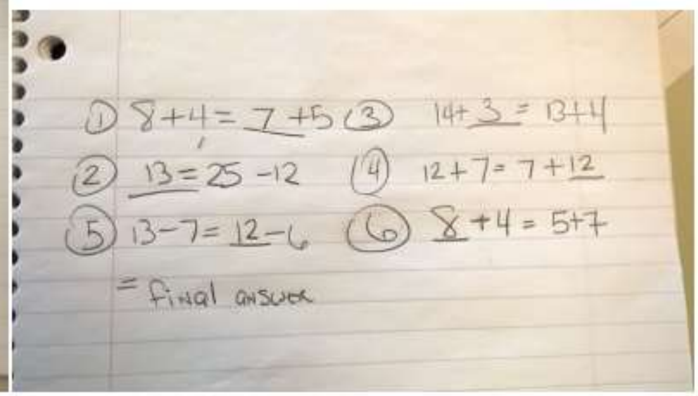
About Question #7 - Describe what this sign = means without using the word “equal”
I added the written prompt “describe the meaning of the sign = without using the word equal”, as I was curious to see how students would define it. I also felt this would be a good starting off point for discussion of student concepts prior to reviewing their answers. I expected some of my students would respond to the presence of the equal sign as a stimulus for an answer. However I did not have a good sense of how prevalent this misconception would be, as about 1/3 of my students were new to my class that week. While visual models were not included in the activity as written in the article, I felt this may be needed for some students. I borrowed a math balance from a fellow teacher to better support students who may need a visual model.

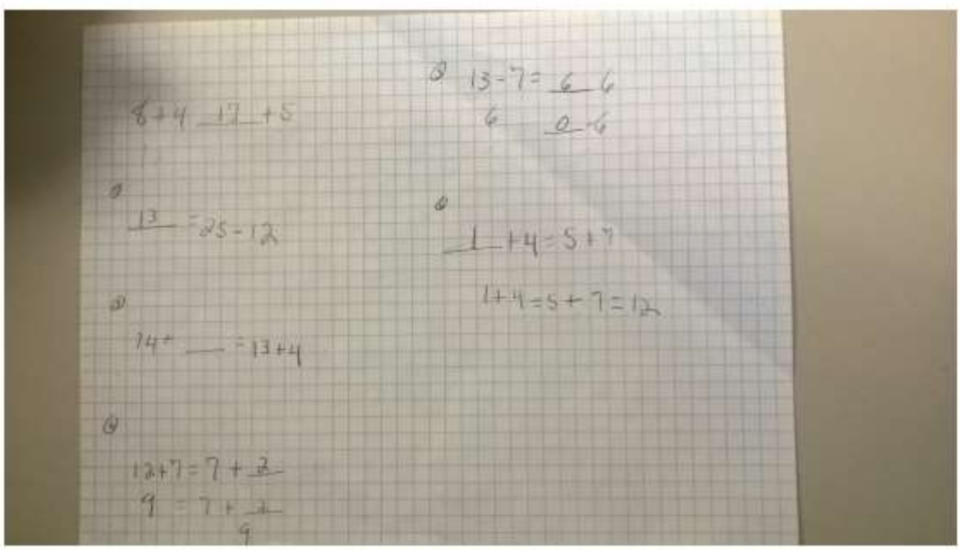
Below are images of some students work:











Over 1/2 of both my morning and afternoon class initially demonstrated the "generate an answer" interpretation of the equals sign. The most common form this took was performing the operation directly preceding or following the equals sign (ignoring the rest of the expression on that side). Some students chose to add all the numbers on both sides and place the sum wherever the blank was. More students experienced success at "backwards" equations (__= 25 12) than those with expressions on both sides of the equal sign. Two of my new students who started this week expressed confusion by the presence of the equals sign in the middle of the sentence (much like in the article) and were initially reluctant to write anything. Most students avoided defining the equals sign, though when prompted to the most common definition was "final answer". This includes two students who completed the number sentences correctly. Other answers were "a total of" and "the sum". A small minority of the class stated "equivalence", "the same as" or "the same value". One student who incorrectly completed most of the number sentences then correctly defined the equals sign when prompted, at which point he identified his error and went back adjust his answers (the second picture of student answers was the point where he stated "Wait a minute....!" before he went back and corrected them).

When we shared out student responses, I chose to start with the definitions that students generated before moving to reviewing the number sentences. As there was disagreement and confusion over answers to the number sentences I modeled their responses on the math balance, which several students who were really struggling to grasp the concept found very helpful. Those who had successfully solved the math sentences were initially tapped to share and explain why, moving towards those students with initial misconceptions revising their solutions and explaining their reasoning. Students were also challenged to identify patterns in the way numbers were changing; though few were inclined to revise their reasoning this way, preferring to solve one side for an answer first before comparing to the other side.

This was a very helpful activity for me as it identified a serious gap in understanding for a large number of my students. My students were able to articulate the concept of both sides of the equal sign having the same value after the activity, as well as later on when discussing their independent work with algebra or equivalent fractions. It is too soon to tell if this new concept will persist or the old misconception will creep back, though I plan to periodically challenge this. I saw little evidence of students developing relational thinking from this initial activity however, which the authors also acknowledged they had only partial success with after 5 sessions. As I have not yet challenged students with other tasks in the article, such as true/false or student generated true number sentences, students may need more exposure to these types of tasks in multiple contexts to begin to develop that skill. I plan to incorporate more challenges of these type, as well as an activity I saw in another PD that used playing cards to generate true number sentences.

Bibliography:
Molina, Marta, and Rebecca C. Abrose. "Fostering Relational Thinking While Negotiating the Meaning of the Equals Sign." Teaching Children Mathematics September (2006): pg 11117. Web. <http://sddial.k12.sd.us/esa/grants/sdcounts/sdcounts0809/fall08/Equal_Sign.pdf>

Article Review Kathryn Lent

Article: "Characteristics of Japanese Mathematics Lessons"
Author: Akihiko Takahashi

Summary

This article discusses Japanese mathematics lessons, focusing on elementary grades. The key is problem solving and the instructional approach is structured problem solving. With this approach, they are hoping to create an interest in math and to stimulate creativity through collaboration by the students.

This is very similar to what Mark has done with us right from the start. The math lesson begins by having the students work individually, using their own toolkit of math strategies to solve the problem(s). Then the teacher leads a classroom discussion so the students may see how others have approached solving the problem and compare these approaches with their own. As we did, the students have an opportunity to develop their own problem solving abilities and add to their toolkits. Starting at such a young age prepares them for the higher levels of mathematics and also takes some of the fear out of math!

How does it connect to our work at the institute?

Again, I have to repeat myself here from article number one; doesn't this sound just like Mark? Isn't this our experience with Mark and then our more extensive experience with both Mark and Eric? This is what I am teaching my teachers to do with their students.

How did it impact your teaching?

This question for me is how did it impact my professional development presentation for my HSE teachers? As an administrator, I am not in the classroom teaching on a daily basis. This article supports moving from a teacher-centered classroom to a student-centered classroom. Using lecture style to teach math (and I think back to my classroom days) may be easier for the teacher, but it is not very effective for the students. Understanding concepts, strategies and procedures are not maximized. Don't just have your students listen to you talk, have them become actively involved in learning! Structured problem solving allows for students to create their own methods to solve problems. The teacher appears to take a less active role in the classroom, but the important work done by the teacher is in the preparation of the lesson. Lessons must be carefully designed and executed so that students may use strategies recently added to their toolkits.

Describe happened when you tried the activity. What worked? What would you do differently next time? What did the experience teach you?

There was one problem discussed in this article, finding the area. Again, there were teachers who dove into finding an algebraic solution, and used different methods. There were some who drew it out on paper. Once all of the teachers were satisfied with their solutions, we had a lively discussion of what worked, what did not and what the best approach was. There were many opinions! The best part of this, it got people thinking outside of the box, and looking forward to reading the whole article.

I would not do anything differently next time, at least not at this time. This experience taught me that my staff and I can have a lot of fun while learning a tremendous amount of information. It provided a very collegial moment for all of us.

Article Review Patricia Helmuth

Article: "Using Students' Work as a Lens on Algebraic Thinking"
Author: Mark Driscoll and John Moyer

A summary of the article

This article explores the work of the ***Linked Learning in Mathematics Project,*** where teachers investigated the efficacy of examining student work so as to: (1) understand student thinking and (2) improve instruction. The goal was to provide students with opportunity to develop patterns of algebraic thinking and habits of mind that serve to inform them how to solve problems by: doing and undoing, recognizing patterns and building rules to represent functions, and abstracting from computation. The problems that teachers unraveled with students, and with each other, were chosen because they fostered one or more of the aforementioned habits of mind, they were open-ended, and they lent themselves to extension activities.

Teachers first worked through the Crossing the River problem in a professional development workshop, solving the problem themselves and comparing solution pathways, and then they examined student work of the same problem. As teachers studied student work of the *Crossing the Rive*r problem, they specifically looked for evidence that students were able to link the solution pathway that they used to solve the problem, with building a rule that can be represented by a function. With a deeper understanding of the learning process and frequent pitfalls in student thinking, along a list of questions that teachers can ask students (or students can ask themselves) as the work through this type of problem, teachers were more fully informed on how to proceed with instruction in their classrooms.

How does it connect to our work at the institute?

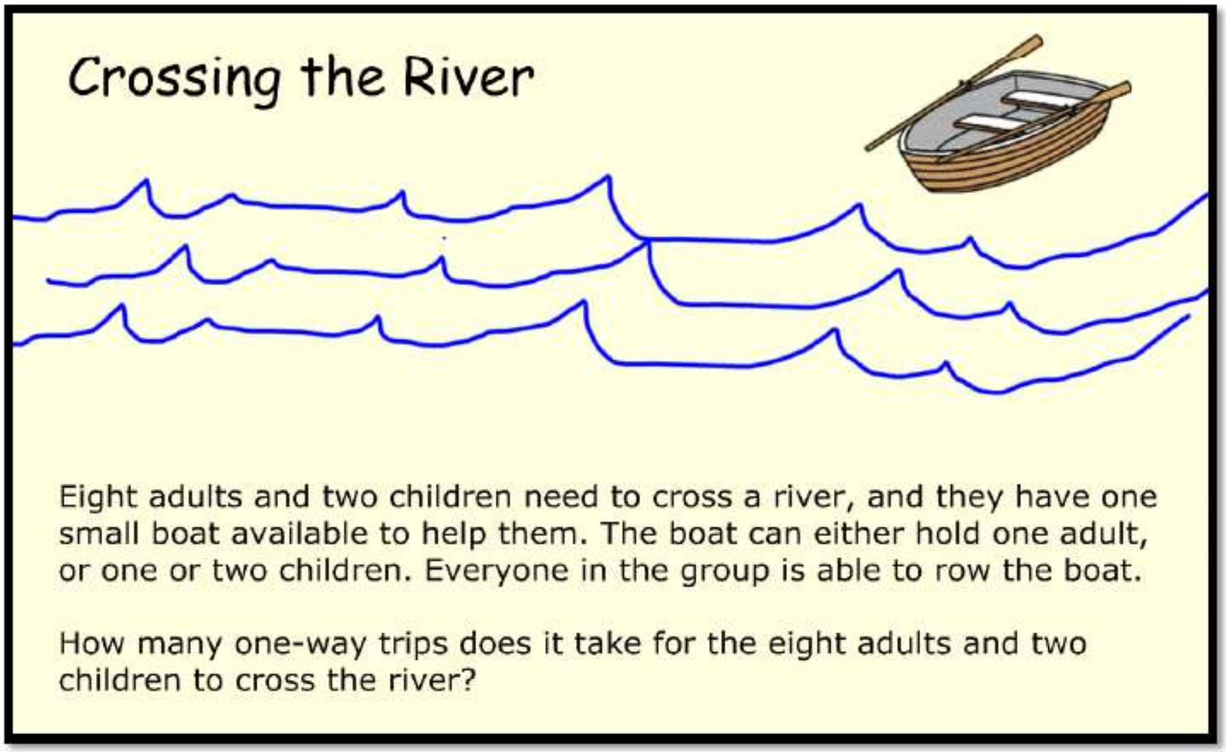
This article exemplifies our training at the October Institute and serves as a good one-stop reminder of how to structure a PD workshop, as well as how to improve our instructional strategies and nudge our students to develop more complex algebraic thinking through choosing effective open-ended problems, asking effective questions during classroom activities, and examining student work.

How did it impact your teaching?

The article helped me to identify what I wanted students to get out of doing the problem. It was more than just being able to solve the problem or to think algebraically. The problem is specifically designed to help students to recognize a pattern and build a rule to represent a function. Thus, I prepared the lesson with this, and the cited questions list in mind, so I could make finding and identifying a rule that represents a function the main objective of the lesson.

Describe what happened when you tried the activity. What worked? What would you do differently next time? What did the experience teach you?

Given the layers of this problem, I wanted to make sure that students understood the conditions before they jumped into trying to figure it out. When I asked students to deconstruct the problem, there was much discussion about how many people could fit into the boat, which I hadn't anticipated. During the discussion one student announced, "That boat can fit two adults", pointing to the SMARTboard as he said this. (See pic below for SMARTboard page I had prepared for the lesson.)



This led to a discussion about the scale of the boat. We had no way of knowing from the picture how big the boat really is. The conclusion was reached that it could be a kiddie boat that was too small for two adults, and all seemed satisfied with that explanation. This discussion, though, really wasn't relevant or necessary for the problem at hand, so at best it was a sideways tangent and at worst it served as additional information that only served to confuse students. Next time I will do something different with the SMARTboard page before I present this problem to students. Perhaps I can eliminate the boat altogether and put the images of 8 adults and two children on one side of the river and/or I can find a different rowboat that more accurately depicts the conditions of the problem. If I add pictures of people and demonstrate that they can be moved, students might then want to use it as a visual model. Or perhaps I should just eliminate the SMARTboard altogether. After students were in agreement with how many people could fit in the boat at one time, they started discussing how it would be possible to get everyone across the river, given the conditions of the boat. There was much lively discussion here and I captured a 2 minute segment of it in an audio file that can be accessed at:

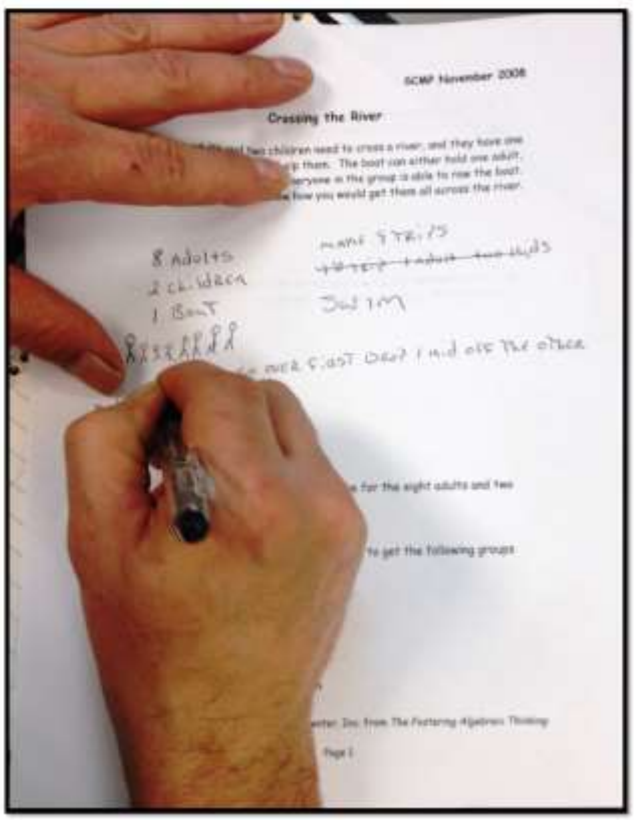
https://drive.google.com/file/d/0BzquayYciy7VdXd2WDg5eWZhU3c/view?usp=sharing

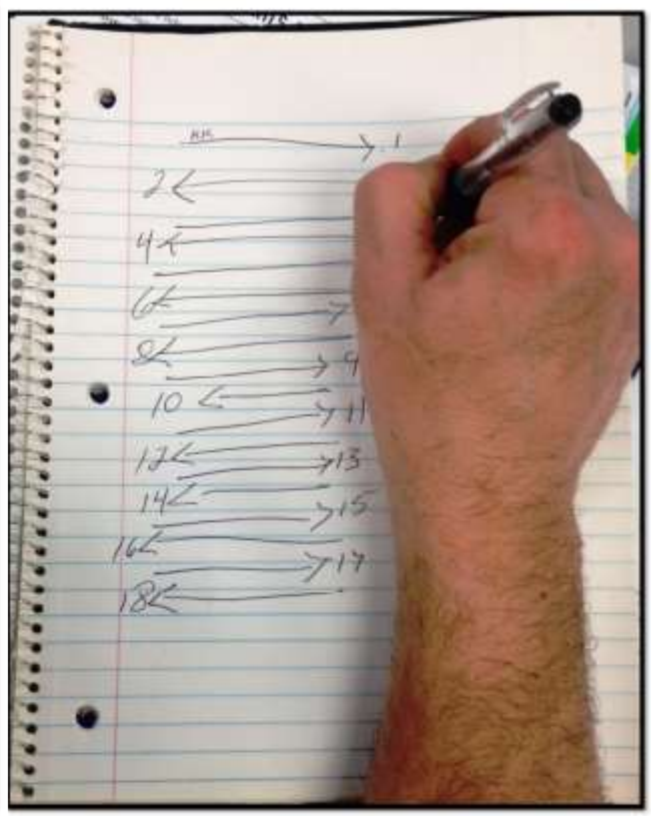
(You'll need a Google account to access this file and you will need to download it to your pc.)

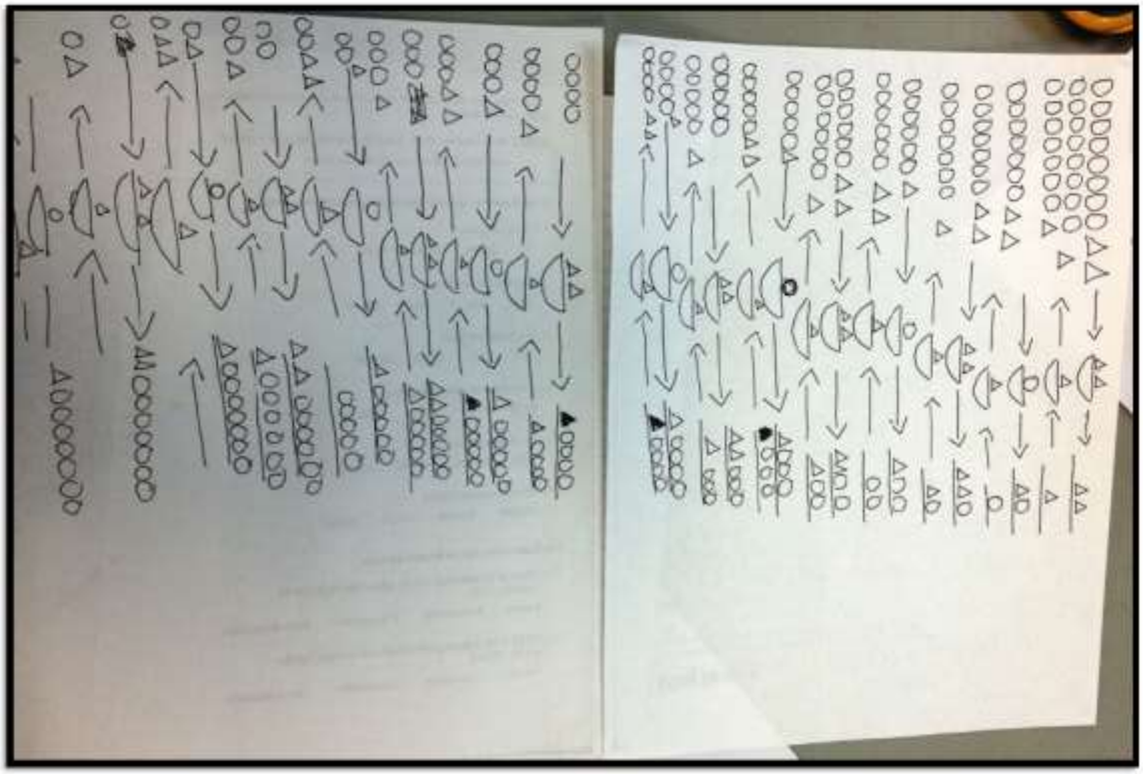
In the audio file, one student can be heard looking for a pattern. At first he notices a pattern of "every other trip is one kid", but was unable to use that pattern to make a rule that would solve the problem. After working deeper into the problem, he realized that it took 4 trips to get one adult to the other side and that the 5th trip was with two children, so he concluded that ***every*** 5th trip would be two children, but disproved that theory quickly as he continued to work through the next 3 or 4 trips. About that time, he connected the 4 trips for one adult to the rule of #of Adults x 4, but he still hadn't accounted for the one last trip across made by the two children. He needed to work through the entire sequence in order to find this last one-way trip and the rule that would represent *any* number of adults with two children. Students created various visual models to represent the situation so that they were better able to decode the problem.

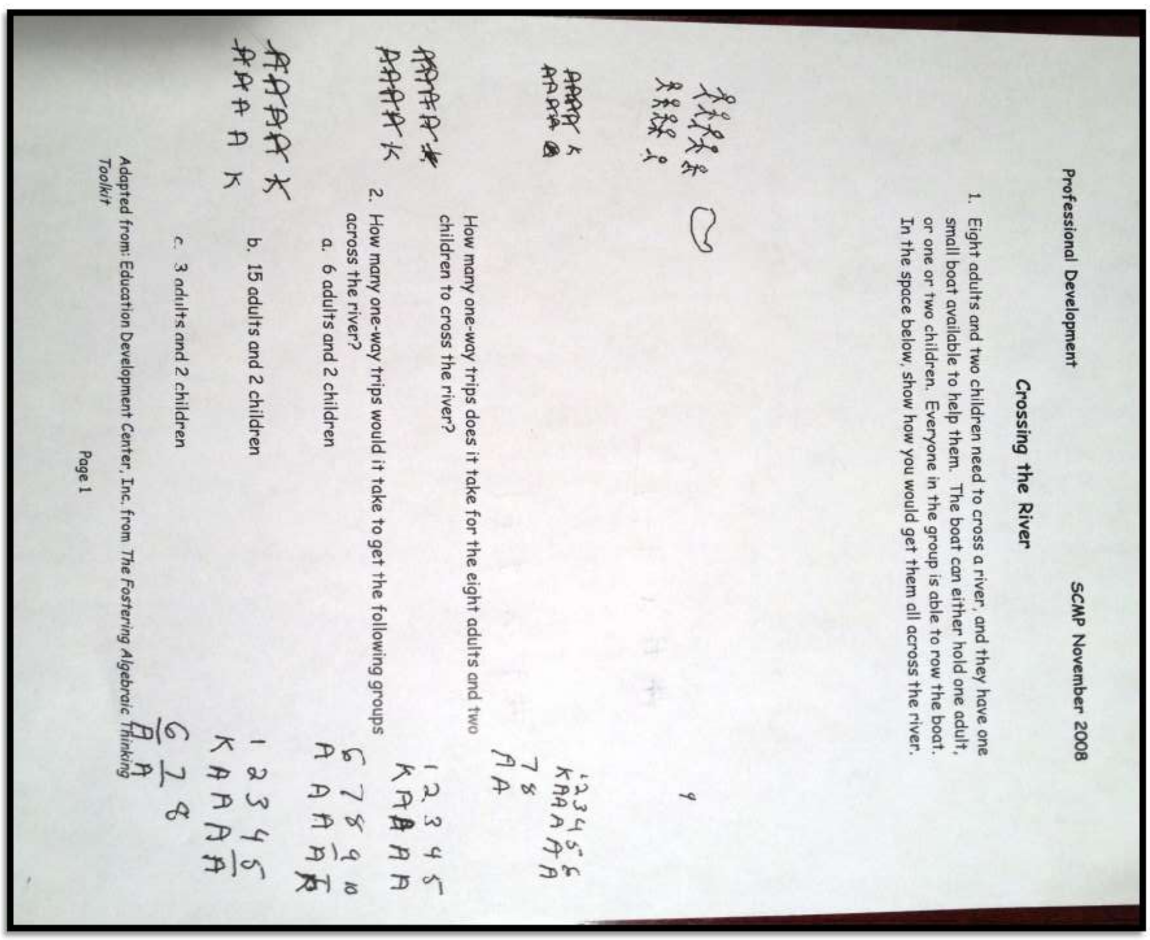
What is briefly described above, and what you can see in the pictures that follow, took about an hour. As class was coming to a close we defined the rule that would represent the function that would solve for *any* number of adults: 4(x) + 1. We defined this as a linear equation that could be graphed on a coordinate grid. We then briefly explored what would happen if the amount of children was increased to 3, but time ran out before we could dig deeper into that discussion.

I learned that you cannot rush through these kinds of activities. I guess I already knew that but even allotting an hour wasn't sufficient, because the hour didn't give us enough time to graph the function nor expand on what would happen if we changed the number of children to 3. How would that affect the rule that represents a function? How would you graph that on a coordinate grid? What if we changed the number of children to 4? I also learned much from listening to that short audio file. As it's hard for me to keep up with or hear every nuance of a conversation in real-time, since my learning style does not favor auditory learning, the audio file of my students' discussion helped me to get more insight into student thinking. I plan to play the file for my students, so they can reflect on what and how they contributed to the discussion. This may enable them to think about how to improve on their group interactions and make group discussions more productive.









Professional Development SCMP November 2008

Crossing the River

1. Eight adults and two children need to cross a river, and they have one small boat available to help them. The boat can either hold one adult, or one or two children. Everyone in the group is able to row the boat. In the space below, show how you would get them all across the river.

How many one-way trips does it take for the eight adults and two children to cross the river?

2. How many one-way trips would it take to get the following groups across the river?
   a. 6 adults and 2 children
   b. 15 adults and 2 children
   c. 3 adults and 2 children

Adapted from: Education Development Center, Inc. from *The Fostering Algebraic Thinking Toolkit*

Page 1

Article Review Celia Volbrecht

Article: "Using Students' Work as a Lens on Algebraic Thinking"
Authors: Mark Driscoll and John Moyer

I chose to review and read the article on "*Using Students Work as a Lens on Algebraic Thinking*" by Mark Driscoll and John Moyer. After reading this article for a second time, I now understand how algebraic thinking develops for students by introducing math problems to "see" how they express patterns to make a solution. The article supports the concept that students can develop pattern strategies through drawings, manipulatives, charts, and guess and check, etc. Students must have the opportunities to investigate and communicate the patterns they are "seeing" before they move towards algebraic equations.

Today my students had the opportunity to solve the arch problem. I wanted to see how the students approached and developed their solutions. Students grasped the concept of figuring the pattern for figure 4 and 5 and could explain it to others in the classroom. When they had to describe what the 10th figure would look like, students took different approaches. When a few students were "stuck," I interjected questions such as "what other strategy could you use?" Would a chart be helpful or can you continue to draw figures?" The students could "see" the pattern from their own math solution. As I moved from student to student, I asked students to explain their strategy to me. "Can you write your explanation on paper?" It took a while to get them to write and explain their solution on paper. I realized that "*the art of questioning*" makes a difference in how my students respond. They became more comfortable with the process when my questions helped them "see" their pattern, and how they arrived at their solution.

Students were able to figure out the number of squares in the 99th figure. One student suggested I write out higher patterns on the Smartboard. Some students commented at that point, "I got it!" We had a 10 minute break. During that time, a few students worked on the bonus question. They wanted me to say if the answer was correct. I encouraged them to wait until class was back from break and share their solutions. As a result, there was quite a bit of discussion among the students as to how they arrived at their answers (right or wrong). This was the most productive part of the lesson as students showed and explained through drawings of patterns and thinking backwards to arrive at their solutions. Students, who had a wrong solution, were able to "see" where they needed to change and adapt their pattern perceptions! (another "*Aha Moment*!")

I included Tykesha's work to show how she wrote figures 1-99 and then showed how many squares corresponded to each figure. She was pleased with herself to see how a solution was arrived at. She is quiet in class and sits by herself concentrating on her work; yet, when I asked students to share their answers, she was willing to explain hers.

I chose Jessica's work because her approach to the solution was understood by everyone. Her approach was different from the others in class. Someone suggested I show her pattern on the Smartboard. I think the visual on the board helped several to "see" the pattern. She developed a sequence for every 10^{th} figure. She said, "You are adding 20 squares." Therefore, she categorized by counting every ten figures and adding 20 squares.

Ex. 10 figures = 23 squares
20 figures = 43 squares
30 figures = 63 squares
40 figures = 83 squares
90 figures = 183 squares

She also used estimation to reach the 99^{th} figure.

Chris's work showed his pattern solution by "seeing" 3 squares for the base and then add 2 squares for each new figure. He also said it was easier for him to do an equation

2(Figure) + 3 = number of squares

Ex. 2(F) + 3 = # squares

(2) (4^{th} figure) + 3 = # squares

2(4) + 3 = 11 squares

Reflection:

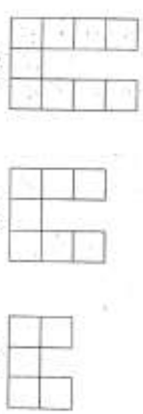
I believe students benefitted from this math problem. Students began to articulate how they arrived at their solutions. Students became more comfortable in sharing their ideas and methods. For some students, it was their first time realizing that their approach may differ from others in class. (*"another Aha Moment")* I will continue to encourage students to consider alternative approaches in their problem solving. I realized that the practice of pattern perception needs to continue in my classroom before introducing algebraic equations. The pattern and structure of algebraic thinking allows my adult learners to understand math. The students were engaged throughout this whole process.

Tykesha
2.23.15

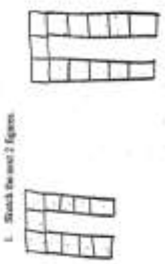
Arch Problem

Observe the figures below.

1st figure 2nd figure 3rd figure



1. Sketch the next 2 figures.



Tykesha

2. Complete the chart

Figure	Number of Squares
1	5
2	7
3	9
4	11
5	13

3. In a few sentences, describe what the 10^{th} figure would look like.

The 10th figure would have 11 blocks going straight down connected with one block in the middle and another 11 blocks going straight down

4. Explain how you would figure out the number of squares in the 99^{th} figure.

I wrote down numbers 1-99 I used what they gave me from the number squares and added 2 to each figure

201

5. In a few sentences, write a way to describe how to figure out how many squares are in any stage of the pattern.

I wrote down numbers 1-99 I used what they gave me from the number squares and added to 2 to each figure

Bonus Question

6. Which figure will have 175 squares in it?

86 figure

Iykish
2.23.15

28 · 23/99

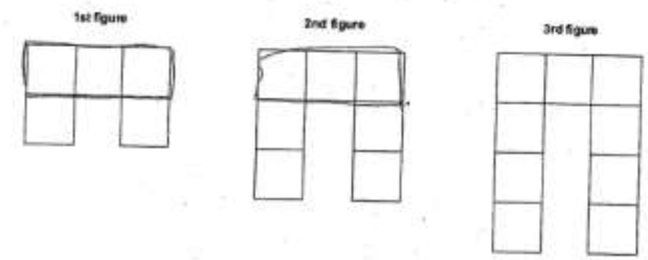
6

11	25	31	65	51	105		
12	27	32	67	52	107		
13	29	33	69	53	109		
14	31	34	71	54	111		
15	33	35	73	55	113		
16	35	36	75	56	115		
17	37	37	77	57	117		
18	39	38	79	58	119		
19	41	39	81	59	121		
20	43	40	83	60	123		
21	45	41	85	61	125		
22	47	42	87	62	127		
23	49	43	89	63	129		
24	51	44	91	64	131		
25	53	45	93	65	133		
26	55	46	95	66	135		
27	57	47	97	67	137		
28	59	48	99	68	139		
29	61	49	101	69	141		
30	63	50	103	70	143		
71	145	80	163	89	181	98	199
72	147	81	165	90	183	99	201
73	149	82	167	91	185	100	203
74	151	83	169	92	187		
75	153	84	171	93	189		
76	155	85	173	94	191		
77	157	86	175	95	193		
78	159	87	177	96	195		
79	161	88	179	97	197		

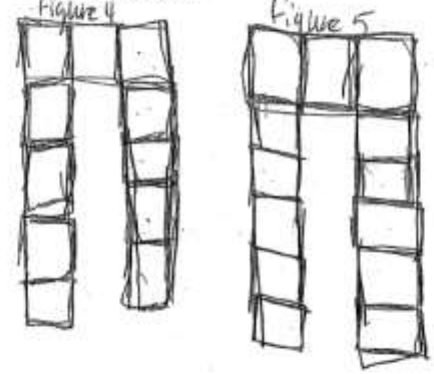
Christian

Arch Problem

Observe the figures below.



1. Sketch the next 2 figures.



Fx2+3=5 Christian

2. Complete the chart

Base 0 3

+1 Figure	+2 +0 3 Number of Squares
1	5
2	7
3	9
4	11
5	13

3. In a few sentences, describe what the 10th figure would look like.

it would have the same 3 base blocks, but the two colums would extend down 23 blocks.

4. Explain how you would figure out the number of squares in the 99th figure.

with a base of 3, each figure adds two new blocks, so add two to each figure number, until reaching the 99th & ontop of the base 3

$99 \times 2 + 3 = 201$

5. In a few sentences, write a way to describe how to figure out how many squares are in any stage of the pattern.

the base pattern has 3 block, after each adisional figure you add two blocks. So take the figure, multiply by 2 and add 3 (the base)

Bonus Question

6. Which figure will have 175 squares in it?

figure 86 $175 - 3 = 172 \div 2 = 86$ — # of squares added each fig.

I subtracted 3 (base figure) and divided by two. equaling 86.

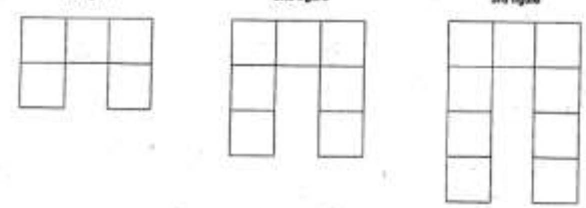
NYSED Common Core/TASC Math Teacher Learning & Leadership Institute

Jessica

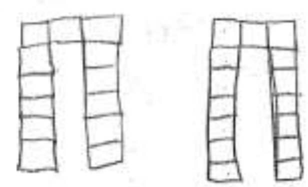
Arch Problem

Observe the figures below.

1st figure 2nd figure 3rd figure



1. Sketch the next 2 figures.



Jessica

2. Complete the chart

Figure	Number of Squares
1	5
2	7
3	9
4	11
5	13

3. In a few sentences, describe what the 10th figure would look like.

Each Figure adds one Square to each Side so the 10th figuare would have 23 Squares

6 - 15   10-23
7- 17
8- 19
9- 21

4. Explain how you would figure out the number of squares in the 99th figure.

Since each figure adds 20 Squares to the 10th figure. I would add 20 Squares to every 10th figure

20 = 43 Squares   70 = 143   Every fifth figure adds 10 then add 2 Squares untill I reached 99th figuare.
30 = 63 Squares   80 = 163
40 = 83   90 = 183
50 = 103   95 = 193   97 = 197
60 = 123   96 = 195   98 = 199   99th 201

5. In a few sentences, write a way to describe how to figure out how many squares are in any stage of the pattern.

I Found that the pattern added 10 Square for every five figuare and 20 Squares for every tenth Figure. So I would get as close to answer that method, then add or Subtract until I reached the number I needed.

Bonus Question

6. Which figure will have 175 squares in it?

Figure 86 would have 175 squares. using my Chart I Knew figure 80 had 163 Squares and Figure 85 would have 173 Square so adding two more Squares in figure 86 would give me 175 Squares.

Jessica

6) 10 - 23   60 - 123
20 - 43   70 - 143
30 - 63   80 - 163 ←
40 - 83   90 - 183
50 - 103   100 - 203

Every 5th figure adds 10 Squares and every 10th Figures adds 20 Squares Figure 80 has 163 so I add 5 more figures. So Figure 85 has 173, I add one more figure, because each figure adds 2 square. I got Figure 86 has 175 squares

Article Review Christina Giorgio

Article: "A World of Difference… Classrooms Abroad Provide Lessons in Teaching Math and Science"
Author: James Hiebert and James W. Stigler

A summary of the article:

The Third International Mathematics and Science Study (TIMSS) conducted an international comparison of 8th grade classrooms in Asia and several European countries. They studied how the classrooms were organized, the kind of math problems that were given, and how these problems were worked on by the students.

Higher achieving countries like Hong Kong, Japan, Switzerland all taught mathematics using different methods. Problems designed to teach skills, versus those designed to teach conceptual understanding varied as well. United States students were also given both types of problems. When working on conceptual understanding, the teachers in the US stepped in and did the work for the students. Students weren't given the opportunity to explore or discuss the mathematical and conceptual relationships.

If the U.S wants to develop higher achieving students, the methods of teaching need to be changed. Teachers need to avoid stepping in and giving the answers to the students. They must allow students an opportunity to think and reason about mathematical concepts, and discuss the relationships.

Changing the methods of teaching won't happen overnight. They must be done gradually by incorporating small changes into their daily and weekly routines.

Three changes, mentioned in the article, which can be made to teaching to improve student's achievement, are listed below:

1). Teachers need to spend time focusing on planning lessons and reflecting on their effectiveness.

2). Teachers need to be given examples of alternative teaching methods.

3). Teachers must have the opportunity to study student responses. They need to learn how to analyze and interpret student's thinking, so they can adjust and make significant changes in their teaching strategies.

In order for the US to compete with some of the higher achieving countries, teachers need to make some changes to their teaching methods. The emphasis needs to be on developing lessons that deepen the student's conceptual understanding and allows sharing of ideas through class discussions.

2). How does it connect to our work at the institute?

As I was reading this article, I thought I was sitting in one of Mark's workshops and he was giving me the information. I knew I wasn't in one of his workshop, but everything written in the article completely connects to everything Mark has been doing with us at the institute. The three changes that were mentioned in the article are the same as the steps we need to do in our problem solving projects ....Plan, analyze, reflect.

I have heard this quote several times during our institute sessions, "Make small changes, Do it again". The article emphasizes that change won't happen overnight. Change is a gradual process that takes time. At the institute, we are learning to make changes to our teaching strategies that will be beneficial to our students. We are learning to choose problems that are non-routine that can be solved using multiple strategies. These types of problems will enable the students to become better problem solvers and higher achievers.

The article was very informative and relates well to everything we are doing at the institute. I enjoyed reading it; I have shared it with some of my colleagues.

3). How did it impact your teaching?

Being told to change the way I have been teaching for the past 20 years is extremely difficult, but seeing proof that your method may not be producing higher achieving students, really makes you think. I have been following the advice from both the article and the institute, make small changes and do it again. Every week, I have been giving the students a non-routine math problem. The students work independently on the problem as I walk around the room to observe their strategies. I ask questions, redirect students that may have gotten side tracked, and ask them to verbalize their thinking. We have a discussion about the multiple strategies that were used, challenges that were faced, and how can this problem relate to similar problems. The students have become more comfortable and confident working on these non-routine problems. Students of all ability levels can be successful with some or all parts of the problems. I have made some changes to my teaching methods, and I think we all have benefited from them. We still have a long way to go, but it's a start.

4). If the article describes an activity, try it out in your classroom. Describe what happened when you tried the activity. What worked? What would you do differently next time? What did this experience teach you?

The only problem in the article was--"Find a pattern for the sum of the interior angles of a polygon"

The intent is for the students to explore the relationship among the measures of angles in figures with different numbers and sides and detect a pattern in the way that the sums can be calculated. The article showed two different teaching approaches to one concept. In the first approach, the students were given the opportunity to explore the concept. In the second approach, the teacher jumped in and gave the students the answer, and didn't allow them to make the connection.

I took the original question and made a problem for my class to try and here's what happened

I handed the paper to my students. They read it and most were confused about some of the terms being used. All of the students knew what triangles, squares, parallelograms, and rectangles were, but many didn't know rhombus, or hexagon. This lead to a discussion about shapes and the number of different sides they had. We talked about pentagons, hexagons, and octagons, decagons and how many sides they each had. Everyone knew the sum of the interior angles of a triangle was 180 degrees and that squares and rectangles had 4, 90 degree angles. I asked the students if there was a tool that is used to measure angles. Wilmer, and engineer in his country, knew there was but couldn't think of the name of it. He was able to describe what it looked like to the rest of the class. A few of the students who were in my class last year remembered the name, and came up with the word protractor. Paula, a former student, explained to the class how it can be used to measure angles. I passed out the protractors to give the students an opportunity to use them with this problem.

After a lengthy discussion about shapes and angles, I asked the students to use the information we just talked about to try the problem.

What worked?

The students immediately began drawing figures of the shapes they knew, and labeling the angles. (Triangles, squares, rectangles) As I was walking around the room, I saw that the students were using different strategies to find the sum of the interior angles. I liked the way Amy divided the figures in to shapes she was comfortable with, rectangles and triangles. All the students had addition skills, so the computations weren't difficult for them. Most could figure out the sums of the 3,4,5,6, and 8 sided figures, but seeing a pattern or developing a rule was challenging. Some began to get frustrated, but I didn't jump in and give the answer. I, instead, started using questions to draw out the responses. This approach worked for some students, but others just wanted me to give them the answer. My class is slowly realizing that the answers will not be told to them; it will be drawn from them through questioning. Questioning and discussing worked well with this problem.

What would you do differently?

If I tried this problem again, I would introduce the protractors ahead of time. I had manipulatives, but trying to use a protractor on such a small object didn't work so well. I would give them paper and scissors to cut out shapes, so they could measure the angles more easily.

What did this experience teach you?

When working with adult students, never take anything for granted and don't assume. I assumed all the students would know the names of the 5, 6, 8, and 10 sided figures. What I assume will be an easy task for the students, may really be a huge challenge for them. I still have a lot to learn about planning lessons, introducing concepts to my class, analyzing student's work and reflecting about the success of the lesson. It will take time, but my class and I are moving in the right direction.

Samples of work by Amy, Walter, and Ajsa

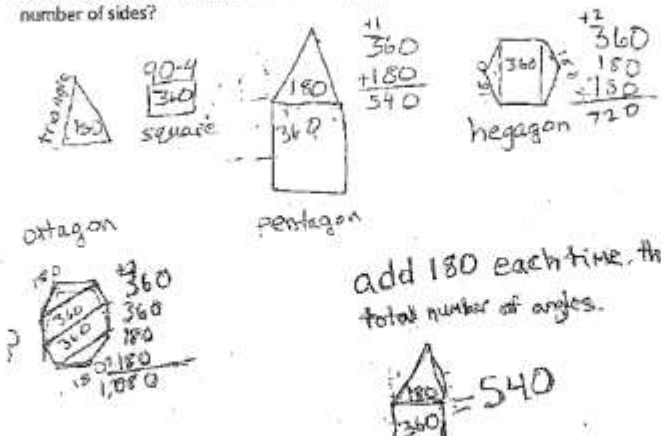
Name: Amy F.

A polygon is a plane figure with at least 3 straight sides and angles. Examples are: triangle, 180 rhombus, parallelogram, and hexagon.

360 360

1).Find a pattern for the sum of the interior angles of a polygon. Start with a three sided figure, then a four sided figure, and so on. ....Please show all your work

2).Once you see a pattern, can you develop a rule for the sum of the angles if you knew the number of sides?



add 180 each time, then multiply the total number of angles.

180
360
= 540

3 sides = 180°
4 sides = 360°
5 sides = 540°
6 sides = 720°
8 sides = 1,080°

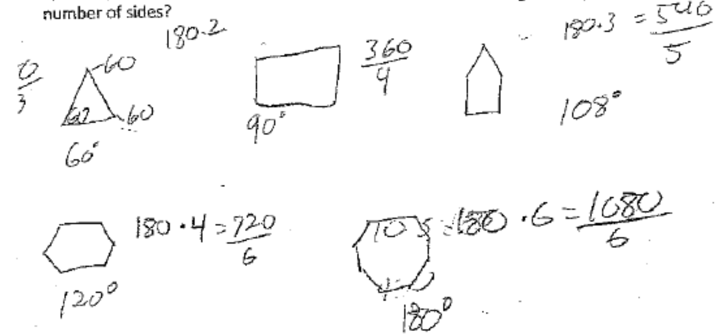
add 180 every time
180+180=360

Name: Walter

A polygon is a plane figure with at least 3 straight sides and angles. Examples are: triangle, rhombus, parallelogram, and hexagon.

1).Find a pattern for the sum of the interior angles of a polygon. Start with a three sided figure, then a four sided figure, and so on. ....Please show all your work

2).Once you see a pattern, can you develop a rule for the sum of the angles if you knew the number of sides?



$\dfrac{180^\circ(\#\text{ of sides} - 2)}{\#\text{ of sides}}$

10 sides
$180^\circ(10-2)$
$180(8)$
$\dfrac{1440}{10}$
$\boxed{144^\circ \text{ per Ang}}$

Sum of Interior Angles

$180^\circ(\#\text{ of sides} - 2) = \text{Int. Angle.}$

$180^\circ(10\text{ sides} - 2)$
$180^\circ(8) =$
$\boxed{1440\text{ total}}$

Aisa

Name:

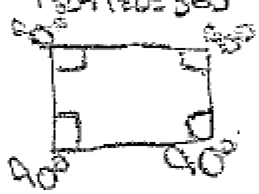
A polygon is a plane figure with at least 3 straight sides and angles. Examples are: triangle, rhombus, parallelogram, and hexagon.

1).Find a **pattern for the sum** of the interior angles of a polygon. Start with a three sided figure, then a four sided figure, and so on. ....Please show all your work

2).Once you see a pattern, can you develop a rule for the sum of the angles if you knew the number of sides?

$\frac{180°}{3} = 60°$

180+180=360



$\frac{360}{4} = 90°$

360+180° = 540

90+90=180

$540-180=\frac{360}{3}=120$

-50

70

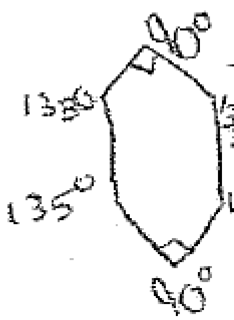
120+25=145

12 sides 12-2=10

Y=180(x)

Y=180(10)

Y=1,800°



540
+180
720

90+90=180

720
−180
$\frac{540}{4} = 135$

Y=(X−2) x 180

X represents the number of sides of the figure.

Article Review Al Pfaeffle

Article: "Talk Your Way into Writing"
Authors: DeAnn Huinker and Connie Laughlin

The Toolkit resource I chose to review is titled "Talk Your Way into Writing", written by DeAnn Huinker and Connie Laughlin. I decided on this article as it connects thinking with talking and writing in the math classroom. Yes, thinking is going on in our math classrooms, but are our students talking and writing about math as well? I gained a lot of insight into math instruction from our October Institute. Of the many concepts and ideas that I took home, I felt that implementing more talking and writing into my math classroom would help improve the overall learning experience of my students.

The article describes a strategy called "think-talk-write" for improving written communication of mathematics. According to the author's, more "talk" in the classroom fosters collaboration and helps build a better learning community amongst students. This learning community of dialogue allows students to modify existing understandings and construct new meaning of a given math idea.

Student-to-student dialogue is most significant in the fact that is reveals the thoughts of peers. Access to others' thoughts allows for validation, extension, and clarification of one's own thoughts. As students talk and share their thoughts they become aware of what they already know and what more they need to know in order to learn. This strategy can not only benefit a student content-wise, but it can also increase student confidence.

When students are asked to write about mathematics without "talk" involved, they are only utilizing their own knowledge and familiarity. Using the "think-talk-write" strategy allows for exploratory talk where students can share thoughts and ideas with each other before they begin to write.

After studying the October Toolkit it seems as if we were just getting into the topic of discourse towards the end of the Institute. There are many techniques for improving discourse in the classroom and I feel that the "think-talk-write" strategy is a great way to improve discourse in the classroom on many levels.

This strategy is improving on student's verbal and written discourse. Talking about math should be encouraged in the classroom. Whether students talk in pairs or in groups it will still foster the same positive results of listening to one another and sharing ideas. Talking also helps define what a mathematical concept is. I student may be able to do the math but does the student understand the math? Writing can elicit understanding.

After I read this article I decided to use "think-talk-write" as a strategy rather than just a one-time activity. In a way it reminds me of "think-pair-share". The key difference being that "think-pair-share" doesn't always concern itself with writing. My teaching has been highly impacted by this form of discourse strategy. I use it regularly in my classroom and I have not only seen my student's writing improve but also their willingness to share their thoughts with each other and the class. Also, grouping my low-performing students with my high-performing students has shown significant improvements with my low-performing students. This strategy works great overall, but I feel it works best when you group students heterogeneously. Since implementing this strategy, nearly all of my low end students have made NRS gains.

One recent example of using "think-talk-write" was on the topic of coordinate geometry and slope. I grouped my students into 5 groups of 4. I asked everyone to think individually about what finding the slope of a line really means for one minute. I then had each group share ideas with each other. After 5 minutes of group discussion I had everyone individually write down what they had learned in one paragraph. After reviewing student writing, my students who did not "get" what slope was the class before, now were able to not only give me the formula but they were able to explain what it meant. I would suggest supplementing this strategy with some form of formative assessment in order to make sure students are actually understanding the math and not just memorizing and writing.