

Being Counted: Probability & Statistics

Fast Track GRASP Math Packet

Part 1



Version 1.9

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Table of Contents

Overview	4
Assessment Questions	5
Answer Key	8
Recommendations	10
Welcome!	11
Vocabulary	12
Skills Practice	14
Skip Counting	14
<i>Skip Counting - Answer Key</i>	18
Introduction to Statistics and Probability	19
Understanding Ratios	22
Introducing Ratios	22
Part-Whole Ratios	25
Part-to-Part Ratios	26
Rates	28
Chairs and Tables	30
Cooking with Ratios	32
Equivalent Ratios	36
<i>Understanding Ratios - Answer Key</i>	40
Using Ratios	47
Comparing Ratios	47
Ratios, Decimals, and Percents	56
Using Rates to Solve for a Missing Value	63
A Typical Day	66
Ratios Written in Different Ways	68
<i>Using Ratios - Answer Key</i>	69
Introduction to Probability	80
<i>Introduction to Probability - Answer Key</i>	85
Experimental Probability	86
Tossing Tacks	86
Shooting Free Throws	93
The Colors of M&Ms	94
Flipping Coins	96
<i>Experimental Probability - Answer Key</i>	98

Being Counted: Probability & Statistics (Part 1)

Theoretical Probability	103
Rolling Dice	103
Sample Space	106
Probability and Multiple Choice Questions	120
<i>Theoretical Probability - Answer Key</i>	<i>121</i>
Vocabulary Review	128
Glossary	128
Using Graphic Organizers to Learn Vocabulary	133
Sources	136
Versions	137

Overview

In this packet, you will explore basic concepts in probability (the math of studying chance) and statistics (the math of collecting, analyzing, and visually representing data).

Prerequisites	There are no prerequisites to this packet. As long as you are able to read this packet independently, you don't have to study any other math packets first.
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In Part 1, you will study the following topics:

- Understanding part-whole and part-to-part ratios
- How to use equivalent ratios and ratio tables
- Expressing ratios with fractions, percents, and decimals
- An introduction to probability (experimental probability and theoretical probability)

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Gathering data with statistical questions
- Analyzing data with measures of center and spread (mean, median, mode, range)
- Comparing data using two-way frequency tables
- Displaying data (dot plots, tables, histograms, bar graphs, scatter plots, line graphs)
- Statistics in the world

In addition to the learning the topics above, you will find the following materials to help you:

- High School Equivalency Test Practice Questions. You will practice all the concepts you've learned from this packet (parts 1 and 2) to work on these questions. The answer key for this section explains the correct answers, and also some of the wrong answers.
- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.
- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question below. When you are finished with the questions, read our recommendations.

Question 1

The Centers for Disease Control and Prevention (CDC) project that 2 out of 5 Americans will develop diabetes in their lifetime. Which of the values below is equivalent to the ratio 2 out of 5? Select all that apply and show your work.

- A. 40%
- B. 6:15
- C. $\frac{8}{20}$
- D. 2.5
- E. 0.4

Question 2

Calculate the probability of the following events, when rolling a single six-sided die:

- What is the probability of not rolling a 4?
- What is the probability of rolling a number greater than 2?
- What is the probability of rolling an odd number?
- What is the probability of rolling a 7?



Question 3

A street vendor offers customers the choice of a vegan burger or beef burger combo meal that comes with one side: french fries, a small salad, or coleslaw. The combo also includes a soda, ice tea, or bottled water. How many different combo meals result from choosing one type of burger, one side, and a drink? In the empty space on the right, draw a diagram or create an organized list to show all possible combinations.

Question 4

Students in a class tracked the number of miles they walked in three days. The dot plot below represents the data they collected.



What is the median?

What is the mean?

What is the mode?

What is the range?

Question 5

The table shows the gender and color of 10 puppies in a litter.

Gender and Color of Puppies		
	Male	Female
Black	4	2
Brown	1	3

Part I: If a puppy selected at random from the group is brown, what is the probability it is a male?

A. $\frac{1}{10}$

C. $\frac{1}{3}$

B. $\frac{1}{4}$

D. $\frac{1}{2}$

Part II: Which of the following statements is not true?

- A. Most of the female puppies are black.
- B. 50% of the puppies are female.
- C. Most of the puppies are black.
- D. 40% of the puppies are brown.

Answer Key

Question 1

Choices A, B, C, & E are equivalent to the ratio 2 out of 5. This part-whole relationship can be represented as 2:5. If both numbers of the ratio are multiplied by 3, the equivalent ratio of 6:15 is created. The fraction form of this ratio is $\frac{2}{5}$ and can be converted to a percent, 40%, a decimal, 0.4, or the equivalent fraction of $\frac{8}{20}$.

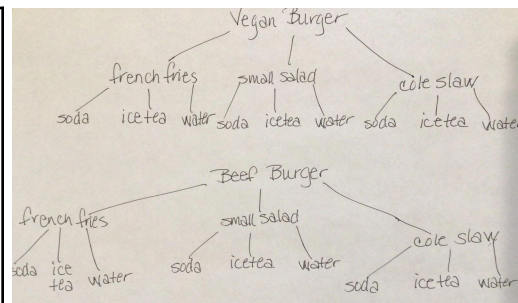
Question 2

- What is the probability of not rolling a 4? **Five out of 6 or $\frac{5}{6}$**
- What is the probability of rolling a number greater than 2? **Four out of six or $\frac{4}{6}$ or two out of three or $\frac{2}{3}$**
- What is the probability of rolling an odd number? **Three out of six or $\frac{3}{6}$ or one out of two or $\frac{1}{2}$**
- What is the probability of rolling a 7? **Zero. There is no chance of rolling a 7.**

Question 3

This problem can be solved with the equation $2 \times 3 \times 3 = 18$ (2 choices of burgers x 3 choices of sides x 3 choices of drinks).

Vegan burger, french fries, soda Vegan burger, french fries, ice tea Vegan burger, french fries, water	Beef burger, french fries, soda Beef burger, french fries, ice tea Beef burger, french fries, water
Vegan burger, small salad, soda Vegan burger, small salad, ice tea Vegan burger, small salad, water	Beef burger, small salad, soda Beef burger, small salad, ice tea Beef burger, small salad, water
Vegan burger, cole slaw, soda Vegan burger, cole slaw, ice tea Vegan burger, cole slaw, water	Beef burger, cole slaw, soda Beef burger, cole slaw, ice tea Beef burger, cole slaw, water



Question 4

The median is 5. If all of the numbers in the data set are placed in order from smallest to greatest, 5 would be the number in the middle.

The mode is 5. It is the number that appears most often.

The mean is 4.85. If all the numbers in the data set are added together and divided by the number of total values (20), the result is 4.85.

The range is 6. This is the difference between the greatest number of miles walked, 9, and the smallest number of miles walked, 3.

Question 5

Part I: Choice B. The total number of brown puppies is 4 but only 1 is a male. If a puppy is selected at random from only the brown puppies, there would be a one-in-four chance of selecting a male puppy. This can be represented as a fraction: $\frac{1}{4}$.

Part II: Choice A. Of the female puppies, 3 are brown and 2 are black so it is not true to say that “Most of the female puppies are black.” B is true because 5 out of the 10 puppies are female, C is true because 6 out of 10 puppies are black, and D is true because 4 out of 10 of the puppies are brown.

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1, 2, or 3: The student may choose to work through the whole packet, both Part 1 and Part 2.
- Student has some difficulty with Question 4 or 5: If a student comfortably answers Questions 1 & 2, but has some difficulty with Question 3 or 4, the student may feel confident enough to skip Part 1 of the packet and go directly to Part 2.
- Student comfortably answers all five questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Being Counted: Probability & Statistics, Part 2*, for test questions that require students to interpret a variety of data representations before they take the GED exam.

This assessment asks students to demonstrate understanding of:

Question 1 (*from Being Counted: Probability & Statistics, Part 1*): Proportional reasoning and equivalent ratios, fractions, decimals, percents (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.3.c)

Question 2 (*from Being Counted: Probability & Statistics, Part 1*): Finding the probability of an event will occur (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.8.b)

Question 3 (*from Being Counted: Probability & Statistics, Part 1*): Sample space and combinations (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.8.a)

Question 4 (*from Being Counted: Probability & Statistics, Part 2*): Measures of central tendency and interpreting data on a dot plot (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.7.a, and Q.6.b)

Question 5 (*from Being Counted: Probability & Statistics, Part 2*): Using a two-way frequency table to determine probability and part-whole relationships (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.8.b)

Welcome!

Congratulations on deciding to continue your learning! We are happy to share this study packet on probability and statistics. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we see on high school equivalency exams. If you study these topics carefully, while also practicing other math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanations and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet. There are some strategies for learning new words described in the next few pages.

We hope you share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: <http://www.acces.nysed.gov/hse/hse-prep-programs-maps>

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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CUNY Adult Literacy and High School Equivalency Program

Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

chance • likely • outcome • probability • random • ratio • sample • trial

In this first activity, you will think about each word and decide how familiar you are with it. For example, think about the word “cube.” Which of these statements is true for you?

- I know the word “cube” and use it in conversation or writing.
- I know the word “cube,” but I don’t use it.
- I have heard the word “cube,” but I’m not sure what it means.
- I have never heard the word “cube” at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✓ (checkmark). Then write your best guess at the meaning of the word in the right column. If it’s easier, you can also just use the word in a sentence.

Here’s an example of how the row for “cube” might look when you’re done:

Word	I know the word and use the word	I know the word but don’t use it	I have heard the word, but I’m not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
cube	✓				like a box, all the sides are the same length

Complete the table on the next page.

Being Counted: Probability & Statistics (Part 1)

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
chance					
likely					
outcome					
probability					
random					
ratio					
sample					
trial					

Skills Practice

Skip Counting



1) How many pennies do you count?

There are actually many different ways to count the pennies. If you show the image to a friend, they might count them differently than you do.

One way is to count each penny one by one.



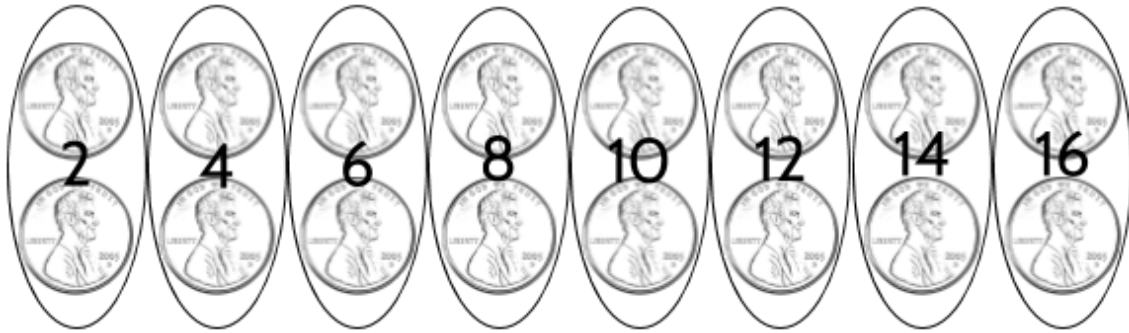
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Counting one by one is a good way to make sure that you counted all the pennies. It can be a little slow, but it is fast enough for small groups of pennies.

When we have larger numbers of things to count, counting by ones is less practical. For example, if you wanted to count all the pennies in a jar, you would probably use another method.



You might count the pennies by twos.



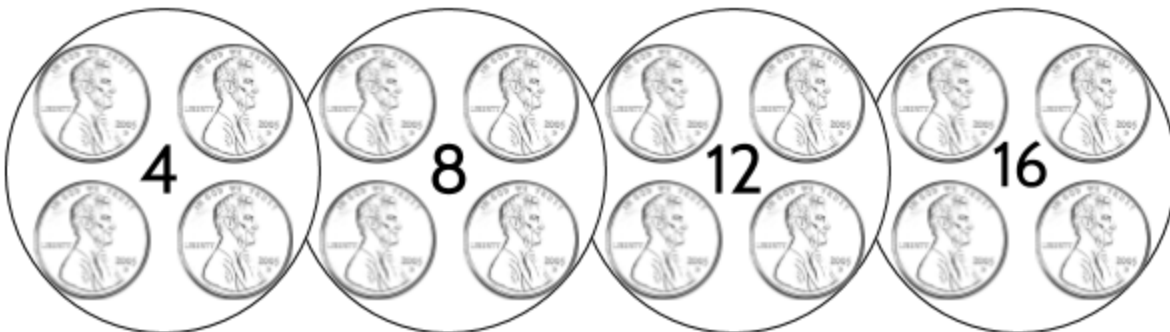
2, 4, 6, 8, 10, 12, 14, 16

2) Continue each sequence counting by twos.

14, 16, 18, _____, _____, _____, _____, 30

48, 50, _____, _____, _____, _____, _____, 62

You could also count by fours.



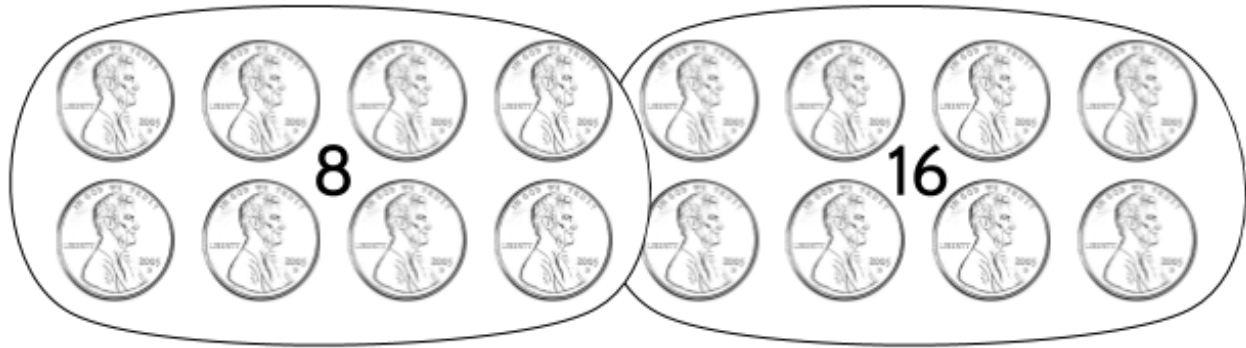
4, 8, 12, 16

3) Continue each sequence counting by fours.

12, 16, 20, _____, _____, _____, _____, 40

60, 64, _____, _____, _____, _____, 84

You could even count these pennies by eights.



8, 16

4) Continue each sequence counting by eights.

8, 16, 24, _____, _____, _____, _____, 72

80, 88, _____, _____, _____, _____, 128

Counting by different numbers (twos, threes, fours, etc.) is called *skip counting*. Skip counting is a way to practice patterns. It will be helpful in many kinds of math.

5) Fill in the blanks in the skip counting series below.

Twos	0	2								18		
Threes	0	3							24			
Fours	0	4						28				
Fives	0	5					30					
Sixes	0	6				30						
Sevens	0	7					42					
Eights	0	8						56				
Nines	0	9							72			
Elevens	0	11								99		

Being Counted: Probability & Statistics (Part 1)

Did you notice that the skip counting series above is very similar to the multiplication table?
Check your work from the previous page using the multiplication table below.

×	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11
2	0	2	4	6	8	10	12	14	16	18	20	22
3	0	3	6	9	12	15	18	21	24	27	30	33
4	0	4	8	12	16	20	24	28	32	36	40	44
5	0	5	10	15	20	25	30	35	40	45	50	55
6	0	6	12	18	24	30	36	42	48	54	60	66
7	0	7	14	21	28	35	42	49	56	63	70	77
8	0	8	16	24	32	40	48	56	64	72	80	88
9	0	9	18	27	36	45	54	63	72	81	90	99
10	0	10	20	30	40	50	60	70	80	90	100	110
11	0	11	22	33	44	55	66	77	88	99	110	121

6) Complete the twelves row.

12	0	12										132
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Skip Counting - Answer Key

- 1) 16
- 2) ..., 20, 22, 24, 26, 28, ...
..., 52, 54, 56, 58, 60, ...
- 3) ..., 24, 28, 32, 36, ...
..., 68, 72, 76, 80, ...
- 4) ..., 32, 40, 48, 56, 64, ...
- 5) ..., 96, 104, 112, 120, ...

Twos	0	2	4	6	8	10	12	14	16	18	20	22
Threes	0	3	6	9	12	15	18	21	24	27	30	33
Fours	0	4	8	12	16	20	24	28	32	36	40	44
Fives	0	5	10	15	20	25	30	35	40	45	50	55
Sixes	0	6	12	18	24	30	36	42	48	54	60	66
Sevens	0	7	14	21	28	35	42	49	56	63	70	77
Eights	0	8	16	24	32	40	48	56	64	72	80	88
Nines	0	9	18	27	36	45	54	63	72	81	90	99
Elevens	0	11	22	33	44	55	66	77	88	99	110	121

6)

12	0	12	24	36	48	60	72	84	96	108	120	132
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Introduction to Statistics and Probability

Will it rain tomorrow? Who do we think will win the election? Does eating red meat increase the risk of getting cancer? What is the expected lifespan for men vs. women in the United States? As a society, we try to answer questions like these by collecting information from different sources, analyzing the information and making predictions.

In the news you will see many items based on research and *data* (information). For example, think about how much sleep you get and how you learn. A scientific study showed that students who slept fewer hours on average didn't do as well at school¹. Other studies have shown that lack of sleep can also cause weight gain and depression. Another study showed that most high school students don't get enough sleep². Since more than 90 percent of high schools start before 8:30 a.m., it is hard for young people to get enough sleep.³

Note: The footnotes at the bottom of the page show where these statistics came from. You should always look for sources when someone is using statistics to convince you of something. If they do not share where they got their information, there is a chance it is not true.

Sample source: Sleep and Student Performance at School (Journal of School Health, Sept 2005)

This footnote means that the information comes from an article called "Sleep and Student Performance at School," which was published in September 2005 in a publication called the Journal of School Health. You could confirm that the information in the scientific study is true by looking up the article on the Internet or in the library.

For the reasons above, many people believe that schools should start later. In order to decide whether you agree with a proposal to delay the start time for your local school, you might want to look at the research yourself. You could see how the scientific studies were conducted, how the data was collected, and make sense of the data yourself so that you can be informed about decisions that are made in your community. Young people getting enough sleep is just one issue. Life in the modern world is filled with decisions based on data and statistics.

¹ Sleep and Student Performance at School (Journal of School Health, Sept 2005)

² Prevalence of Sleep Duration... High School Students (Preventing Chronic Disease, 2007–2013)

³ Results from the School Health Policies and Practices Study (Centers for Disease Control, 2014)

Data is not only for experts and researchers. You could do your own research and collect your own data. Spend a month recording the number of hours of sleep you get each night. Then do a puzzle each day (Sudoku, word jumble, crossword, etc.) and record how long it takes you to complete each puzzle. Does the amount of sleep affect how quickly you complete a puzzle?

In this packet, you will study statistics and probability, two fields of study that help people use data to understand the world and make decisions.

Data

Data is information that is collected, analyzed, and often used to tell a story. Data is collected in many places.

If you work at the Department of Motor Vehicles, you collect data from people who apply for a driver's license, including date of birth, height, hair color, and eye color.

Or you might work for the U.S. Census, which counts everyone living in the United States. In this job, you would collect data such as the number of people living in a household, along with their gender, ethnicity, and age. Census data tells the story of who lives in the U.S.

If you work as a nurse, you collect health data from your patients. This includes body temperature, blood pressure, heart rate, and breathing rate. These pieces of data are called *vital signs*, since they tell us the story of a person's vital (life-sustaining) functions.

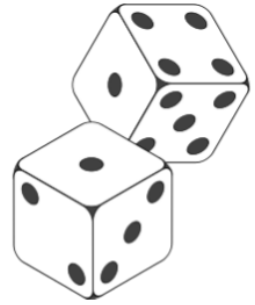
We live in a world of data. Daily, we collect different kinds of data: the number of steps we take, the number of calories we eat, and the amount of money we spend. And we constantly provide data to others, especially to Internet companies who keep track of our "likes" on social media, our purchases, and which advertisements are successful in encouraging us to buy products.



João Batista Neto (September 2015) [wikimedia.org](https://commons.wikimedia.org/wiki/File:Types_of_Data.png)

Probability

Probability is the study of how likely it is that something will happen. At the time of this writing, there is a 100% chance of rain tomorrow between 5pm and 8pm, when I travel home from work. The weather forecasters have decided that it definitely will rain tomorrow. I should probably wear rain boots and bring an umbrella, right? What if there was a 40% chance of rain? I might risk it, because I don't like carrying an umbrella. I'm using probability to make a decision. We can explore probability by flipping coins, rolling dice, and thinking about the future.



Statistics

Statistics is the study of numbers in the world. Statistics can be defined as the science of collecting, reviewing, and analyzing data. One place where we see statistics is in sports. In baseball, we measure statistics such as batting average and number of home runs. In football, passing yards and rushing yards are important statistics. In basketball, a player's free throw percentage is a common statistic for measuring the player's skill. For example, in the 2018-2019 season, Stephen Curry of the Golden State Warriors shot 287 free throws and made 263 of these shots. This is a 91.6% free throw percentage, which was one of the best in the league. The free throw percentage is a *statistic*, or a fact that is a result of analyzing data. In this case, the data was collected from basketball games over the course of a season.

Mathematicians use statistics to analyze events in the past or things that are happening now, and they use probability to understand the past and what could happen in the future. Probability and statistics can be found in games, sports, the stock market, scientific research, and many other areas of interest.

Note: You will probably want a calculator while working on some sections of this packet. We recommend using the TI-30XS calculator, if possible, since this is what you will use when you take the HSE exam.

Use the calculator only if you need it, so that you can practice doing calculations in your head, and with paper and pencil.

Understanding Ratios

Introducing Ratios

Ratios are a tool we can use to analyze data, understand probability, and report statistics. We can use ratios to make predictions based on what we know.

Consider this statistical question:

How does the number of left-handed people in our adult education class compare with the number of left-handed people in the general population?

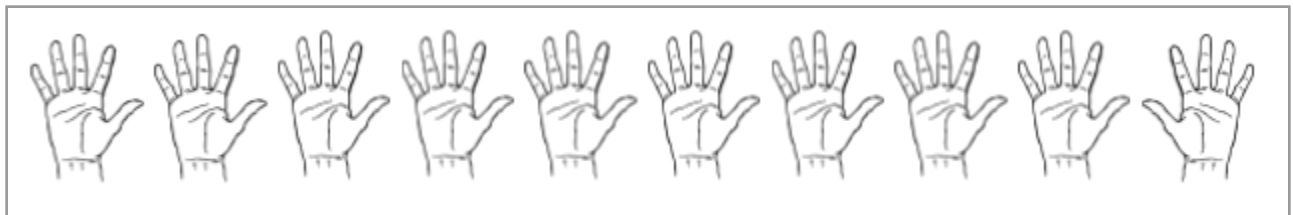
In the diagram below, each hand represents one person.



indicates a left-handed person.



is a right-handed person.



- 1) How many right-handed people are shown in the diagram above?

How many people in the diagram are left-handed?

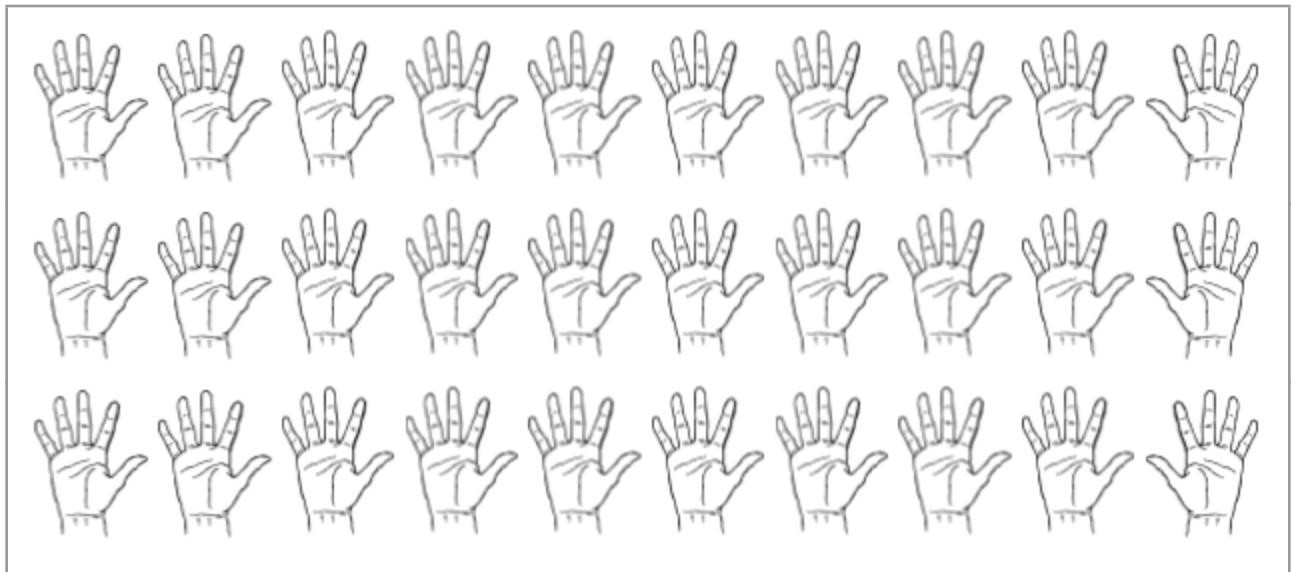
How many total people are there?

Approximately 1 out of every 10 people is left-handed.⁴ This means that if you find yourself on an elevator with 10 random people, one of them is likely to be left-handed.

The number of left-handed people in a population can be represented with a *ratio*, which shows the size of one value in comparison to other values. In the diagram above, there is 1 left-handed person out of 10 total people. This is a ratio of 1:10, which is read as “one to ten.”

- 2) Which choice shows the correct ratio of *right*-handed people to total people above?
- A. 1:9
B. 1:10
C. 9:10
D. 10:10
- 3) If we can expect 1 left-handed person in a group of 10 people, how many lefties would you expect in a group of 30 people?

Use the diagram to predict how many left-handed people there would be.



⁴ Hardyck, C., & Petrinovich, L. F. (1977). Left-handedness. *Psychological Bulletin*.

Being Counted: Probability & Statistics (Part 1)

The ratio of 1 left-handed person out of every 10 people is a statistic that can be used to make estimates about the number of left-handed people in different populations. To make predictions with ratios, we can use a *ratio table*. An example is shown below.

4) Fill in the missing blanks in the ratio table.

Left-handed people	1	2	3	4	5	6	7	8	9	10
Total population	10		30							

5) In a population of 50 people, how many people would you expect to be left-handed?

6) In a population of 200 people, how many people would you expect to be left-handed?

7) How many right-handed people would you predict in a population of 200 people?

8) If there are 12 left-handed people in a population, how many total people do you think there might be?

9) How many left-handed people would you expect in a population of 225 people?

Part-Whole Ratios

In humans, approximately 9 people out of every 10 people are right-handed. This is called a *part-whole ratio*, since part of the population (the number of right-handed people) is being compared with the whole population (the total number of people).

Part-whole ratios are often written as a fraction ($\frac{9}{10}$) but can be written with a colon (9:10) as well.

$$\frac{\text{part}}{\text{whole}} = \frac{9 \text{ right-handed people}}{10 \text{ total people}} = 9 \text{ right-handed people out of 10 people} = 9:10$$

Here are some other situations that involve part-whole ratios:

- Out of 20 M&Ms, 3 are yellow
- 11 “heads” out of 20 coin flips
- 12 bilingual students out of a class of 20 students
- 42 correct answers out of 50 answers on a test

Imagine we flipped 20 pennies and laid out the results on the right. With a part-whole ratio, we could compare the number of “heads” to the total number of flips:

$$\frac{11 \text{ "heads"}}{20 \text{ total flips}} = 11:20 = 11 \text{ “heads” out of 20 flips}$$

We could also use a different part-whole ratio to compare the number of “tails” to the total number of flips:

$$\frac{9 \text{ "tails"}}{20 \text{ total flips}} = 9:20 = 9 \text{ “tails” out of 20 flips}$$



10) Which choice correctly shows the part-whole ratio below?

Out of 20 M&Ms, 3 are yellow.

A. $\frac{3}{17}$

C. $\frac{17}{20}$

B. $\frac{3}{20}$

D. $\frac{17}{3}$

Part-to-Part Ratios

The part-whole ratio of $\frac{1 \text{ left-handed person}}{10 \text{ total people}}$ compares the number of left-handed people to the total population. If you wanted to compare the number of left-handed people to the number of right-handed people, you would use a *part-to-part ratio*. Here are two examples:

For every 9 right-handed people, there is 1 left-handed person. This is a ratio of 9:1. or For every 1 left-handed person, there are 9 right-handed people. This is a ratio of 1:9.

A *part-to-part ratio* compares part of the population (left-handed people) to another part of the population (right-handed people). The ratio can be written in two ways (9:1 or 1:9).

Part-to-part ratios are usually written with a colon (9:1), but they can also be written as a fraction ($\frac{9 \text{ right-handed people}}{1 \text{ left-handed person}}$).

$$\frac{\text{part}}{\text{part}} = \frac{9 \text{ right-handed people}}{1 \text{ left-handed person}} = 9 \text{ right-handed people for every 1 left-handed person} = 9:1$$

Here are some other situations that involve part-to-part ratios:

- 4 blue M&Ms for every 3 yellow M&Ms
- 11 “heads” for every 9 “tails”
- 12 bilingual students for every 8 students who speak only one language
- 42 correct answers for every 8 incorrect answers

Look at the results of our coin flip again. With a part-to-part ratio, we can compare the number of “heads” to the number of “tails.”

11) How can we write this part-to-part ratio for coins? Choose all that apply.

- A. $\frac{11 \text{ "heads"}}{9 \text{ "tails"}}$
- B. 11:9
- C. 9:11
- D. $\frac{11 \text{ "tails"}}{9 \text{ "heads"}}$



Practice your ratio skills with the situation below.

Polly bought a bag of Polygon Pieces and emptied the candies onto a table. They were in the form of triangles (*t*), quadrilaterals (*q*), pentagons (*p*), and hexagons (*h*).



12) Mark the following statements true or false.

Statements	T or F
There are 2 triangles for every 1 quadrilateral.	T
The ratio of pentagons to triangles is 7:6.	
The ratio of hexagons to total candies is $\frac{4}{16}$.	
There are 7 pentagons out of 20 total pieces of candy.	
For every 2 hexagons, there are 3 triangles.	
The ratio of quadrilateral to total candies is 3:17.	
The ratio of triangles to total candies is $\frac{3}{10}$.	

13) Write your own true statement using ratios from Polygon Pieces diagram above.

Rates

A *rate* is a ratio that compares different kinds of things. For example, if apples cost \$2.00 per pound, we are comparing dollars and pounds, with a rate of two dollars for every one pound.

Here are some other examples of rates:

- \$15 per hour
- 80 heartbeats per minute
- 4 oranges for \$3.00
- 50 miles/hour
- 10,000 steps per day
- \$7.50 for a 6-pack of soda

The word *per* means *for each*, so an hourly wage of “\$15 per hour” means the same thing as “\$15 for each hour.” For each 1 hour that you work, you are paid 15 dollars. The slash symbol / also means “per” or “for each.” 50 *miles/hour* is another way to write “50 miles per hour.”

Complete the following tables.

14) Rate: \$15/hour

Hours worked	1	2	4		20	
Pay	\$15			\$150		\$600

15) Rate: 50 miles/hour

Hours driving	1	3		10	15	
Miles traveled	50		300			650

16) Rate: 80 heartbeats per minute

Minutes	1	2		10		60
Heartbeats			400		2400	

Being Counted: Probability & Statistics (Part 1)

Fill in the blanks.

17) Rate: 10,000 steps per day

Days	1	3	4		20	
Steps	10,000			100,000		270,000

18) Rate: \$7.50 for a 6-pack of soda

Cans of soda	6	3	1	12		
Cost					\$16.25	\$2.50

19) Rate: 4 oranges for \$3.00

	4	2	1	7		20
	\$3.00				\$7.50	

20) Rate: _____ calories per day

Days	2	1	4	7		20
Calories	4,000				10,000	

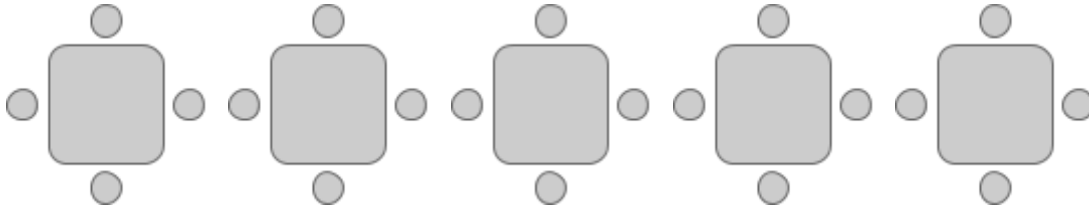
21) Rate: 20 Mexican pesos = 1 U.S. dollar

U.S. dollars	1	2		0.5		
Mexican pesos			200		5	1

Chairs and Tables

Consider the following situation.

An event planning company is loading up tables and chairs for a large wedding. Each table has room for 4 chairs. The chairs and tables come in a set, so the company always brings 4 chairs for every table.



22) If the company brings 8 tables, how many chairs would they bring?

23) If the company brings 28 *chairs*, how many *tables* would they have?

24) Fill in the blanks to complete these sentences.

If 1 table has 4 chairs, then 10 tables would have _____ chairs.

... and 15 tables would have _____ chairs.

... and _____ tables would have 72 chairs.

... and _____ tables would have 160 chairs.

Being Counted: Probability & Statistics (Part 1)

There should always be a ratio of 1 table for every 4 chairs at the company's events. We can use a ratio table to make sure the furniture is kept in the same ratio of 1:4.

25) Fill in the missing blanks in the ratio table.

Tables	1	2	3	4	5	6	7	8
Chairs	4	8					28	

26) If there were 12 tables, how many chairs would you have?

27) Which choice shows the ratio of tables to chairs when there are 16 tables?

A. 4:12

C. 16:48

B. 4:64

D. 16:64

When they pack up after the wedding, the event planning company does an inventory count of all the furniture they brought. Then they add the tables and chairs to get total pieces of furniture.

28) Fill in the missing blanks in the ratio table.

Tables	1	2	3	4	5	6	7
Chairs	4	8					
Total Furniture	5						

29) If there were 15 tables, how many total pieces of furniture were there?

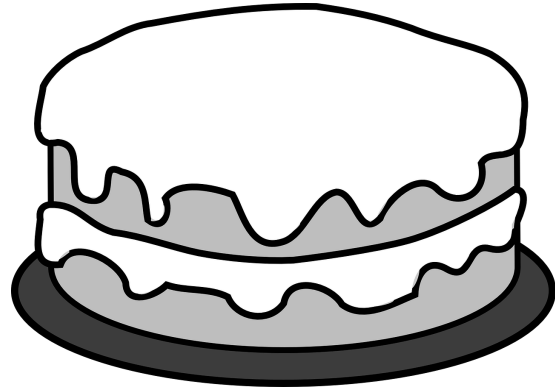
30) If there were 45 total pieces of furniture, how many chairs were there?

Cooking with Ratios

Many people who like to bake know a famous recipe called the 1-2-3-4 cake.

To make this cake, you need:

- 1 cup of butter,
- 2 cups of sugar,
- 3 cups of flour, and
- 4 eggs



Imagine that you own a bakery and the 1-2-3-4 cake is one of the baked goods you have for sale. Since you sell a lot of cakes, you have to increase the amounts in the recipe so that you can make more than one cake at a time.

31) Look at the following table.

cups of sugar	2	4	16	20
cups of flour	3	6	24	30

What do you notice?

32) How many cakes can you make with 4 cups of sugar?

33) How many cakes can you make with 24 cups of flour?

Baking requires attention to detail. It is important to measure carefully and use the correct amounts of ingredients such as butter, sugar, flour, and eggs. When you make changes to a recipe, you can use ratios to make sure that each amount is correct so that the final product is equally delicious. Understanding ratios can help you become an expert baker.

As we increase the number of cakes we make for the bakery, we can use ratio tables to make sure that we follow the original ratio of 2 cups of sugar for every 3 cups of flour. The new ratios in the table are equivalent (equal in value) to the original ratio of 2:3. We create equivalent ratios by multiplying the amount of each ingredient by the same number.

Here is the sugar and flour table again, starting with the ratio $\frac{2 \text{ cups of sugar}}{3 \text{ cups of flour}}$. The equivalent ratios of $\frac{4 \text{ cups of sugar}}{6 \text{ cups of flour}}$ and $\frac{16 \text{ cups of sugar}}{24 \text{ cups of flour}}$ were created with the table.

cups of sugar	2	4	16
cups of flour	3	6	24

Diagram annotations: An arrow from 2 to 4 is labeled $\times 2$. An arrow from 4 to 16 is labeled $\times 8$. An arrow from 3 to 6 is labeled $\times 2$. An arrow from 6 to 24 is labeled $\times 8$.

Both numbers in the ratio $\frac{2}{3}$ were multiplied by 2 to make 2 cakes.

$$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Then both parts of $\frac{2}{3}$ were multiplied by 8 to make 8 cakes.

$$\frac{4 \times 4}{6 \times 4} = \frac{16}{24}$$

34) Which of the following ratios would follow the 1-2-3-4 cake recipe for 12 cakes?

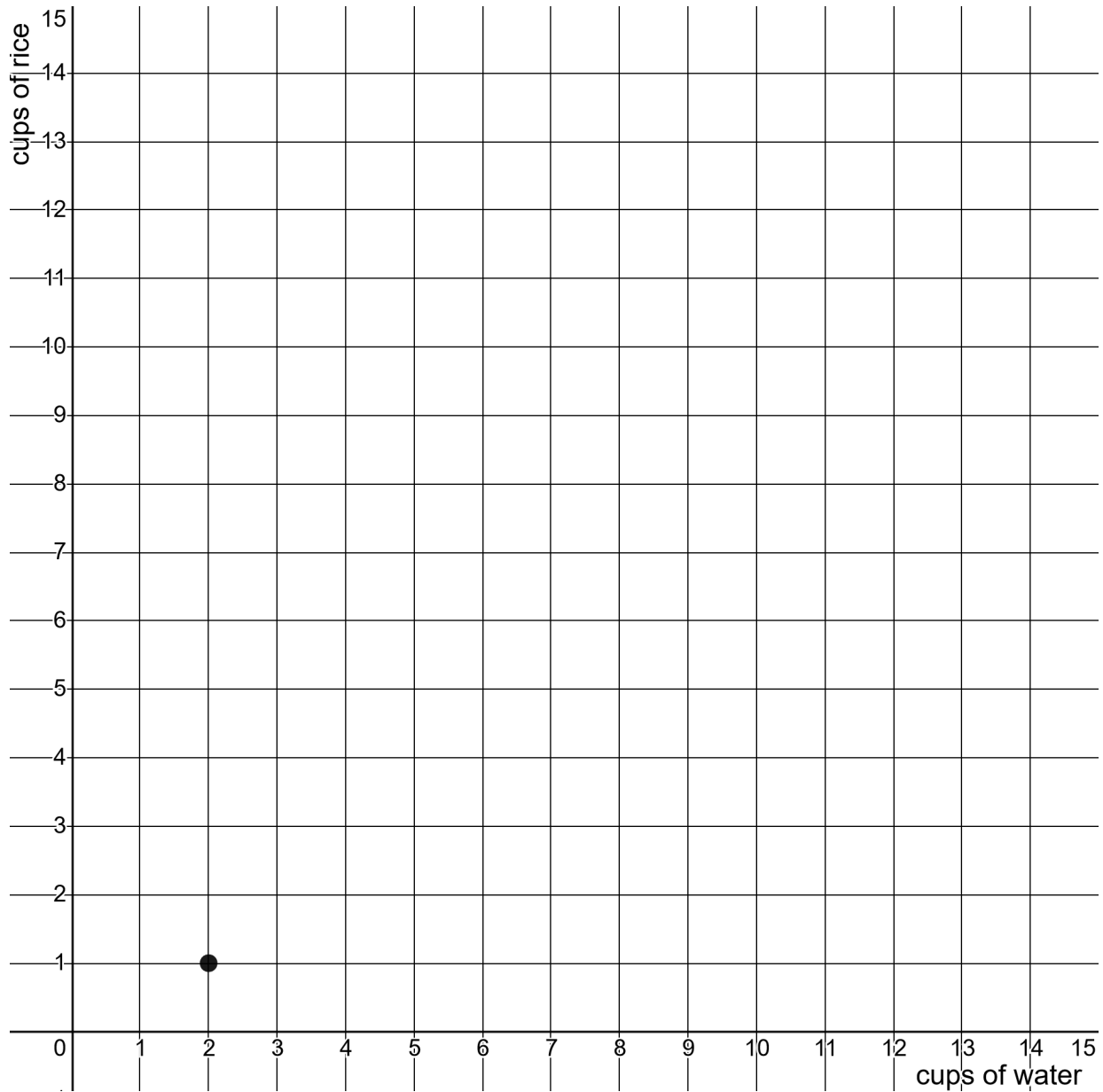
- A. 12 cups of sugar and 18 cups of flour
- B. 18 cups of sugar and 12 cups of flour
- C. 24 cups of sugar and 36 cups of flour
- D. 36 cups of sugar and 24 cups of flour

You can also use ratios to cook rice. A recipe calls for two cups of water for every one cup of rice. Use this ratio of 2:1 to guide your reasoning below.

- 35) Complete the table below by determining which of the recipes are correct. Then predict how the rice will turn out.

Cups of Water	Cups of Rice	Does the recipe follow the correct ratio? YES or NO	How will the rice taste?
2	1	Yes	Perfect
3	2		
5	2.5		
6	3		
7	2		
9	4.5		
12	4		
13	10		
14	7		

- 36) Now graph all the points and see whether your predictions were correct. The first recipe has been placed on the chart.



- 37) Look at the graph after you draw all the points. What do you notice?

Equivalent Ratios

The top number in a fraction is called the *numerator*. It tells you the value of the “part.” The bottom number in a fraction is called the *denominator*. It tells you the value of the “whole.” When you multiply or divide the

numerator (part) → $\frac{1}{10}$ left-handed people
denominator (whole) → all people

numerator and denominator by the same number, the value of the fraction doesn’t change. The new fraction will be equivalent to the old fraction. For example, $\frac{2}{20}$ is equivalent to $\frac{1}{10}$.

38) Fill in the missing values in the ratio table.

1	2			
10	20			

$\times 2$ $\times 5$ $\div 2$ $\times 3$
 $\times 2$ $\times 5$ $\div 2$ $\times 3$

39) Which of these equations is not true?

A. $\frac{1}{10} = \frac{2}{20}$

B. $\frac{10}{100} = \frac{5}{50}$

C. $\frac{2}{20} = \frac{15}{150}$

D. $\frac{1}{10} = \frac{10}{20}$

Multiply the top number and bottom number by any number.

40) Create five equivalent fractions for $\frac{3}{4}$. (One has been done for you.)

3	6				
4	8				

41) Create five equivalent fractions for $\frac{7}{10}$.

7					
10					

- 42) Multiply or divide the numerator and denominator by any number to create five equivalent fractions for $\frac{1}{2}$.

$$\frac{1}{2} \quad \frac{2}{4} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}}$$

- 43) Create five equivalent fractions for $\frac{15}{6}$.

$$\frac{15}{6} \quad \frac{5}{2} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}}$$

- 44) Which of the following ratios is not equivalent to $\frac{1}{4}$?

A. $\frac{4}{16}$

C. $\frac{7}{35}$

B. $\frac{6}{24}$

D. $\frac{9}{36}$

How do you know?

- 45) Fill in the missing parts of the fractions to create equivalent ratios.

$$\frac{2}{10} \quad \frac{4}{\hspace{1cm}} \quad \frac{\hspace{1cm}}{45} \quad \frac{5}{\hspace{1cm}} \quad \frac{\hspace{1cm}}{55} \quad \frac{20}{\hspace{1cm}}$$

- 46) Which ratio is equivalent to $\frac{2}{10}$ and the other fractions above?

A. 1:10

C. 2:5

B. 1:5

D. 5:10

A percent is a part-whole ratio “out of” 100. *Per-* means “for every” and *-cent*⁵ means “100,” so the word *percent* literally means “for every 100.”

- 47) Fill in the missing information. The ratios in each row are equivalent. For example, $\frac{10}{100}$ and $\frac{20}{200}$ are equivalent.

“out of 10”	“out of 50”	“out of 100”	Percent	“out of 200”
$\frac{1}{10}$	$\frac{\quad}{50}$	$\frac{10}{100}$	10%	$\frac{20}{200}$
$\frac{\quad}{10}$	$\frac{\quad}{50}$	$\frac{40}{100}$	%	$\frac{\quad}{200}$
$\frac{6}{10}$	$\frac{\quad}{50}$	$\frac{\quad}{100}$	%	$\frac{\quad}{200}$
$\frac{\quad}{10}$	$\frac{\quad}{50}$	$\frac{\quad}{100}$	%	$\frac{30}{200}$

- 48) Write two different ratios that are equivalent to 30%.

A.

B.

- 49) Which ratio below is not equivalent to 40%?

A. $\frac{4}{10}$

C. $\frac{4}{50}$

B. $\frac{20}{50}$

D. $\frac{40}{100}$

- 50) We learned earlier that about 1 out of every 10 people is left-handed. What percent of people are left-handed?

A. 1

B. 2

C. 9

D. 10

⁵ By the way, you can find “cent” in many words related to 100. A century is 100 years. A cent (penny) is a hundredth of a dollar. A centimeter is a hundredth of a meter. A centennial is the 100th anniversary of an event. A centipede (supposedly) has 100 legs. A centenarian is a person who has lived to be 100 years old!

Being Counted: Probability & Statistics (Part 1)

A ratio can also be expressed with decimals. A percent compares a number “for every 100.” When a ratio is written as a decimal, it compares a number “for every 1.”

To practice this, think about money. Let’s consider $\frac{1}{2}$ of a dollar, which is 50 cents. This can be written as \$0.50 and is equivalent to two quarters, five dimes, or 50 pennies.

$$50\% \text{ of 1 dollar} = \frac{2 \text{ quarters}}{4 \text{ quarters}} = \frac{5 \text{ dimes}}{10 \text{ dimes}} = \frac{50 \text{ pennies}}{100 \text{ pennies}} = 0.50$$

51) Complete the table. The ratios in each row are equivalent.

Number of Pennies	Percent	“out of 100”	Written as money (decimal)
50	50%	$\frac{\quad}{100}$	0.50
	%	$\frac{\quad}{100}$	0.25
80	%	$\frac{\quad}{100}$	
	8%	$\frac{\quad}{100}$	

52) Fill in the missing information. The ratios in each row are equivalent.

“out of 5”	“out of 10”	“out of 50”	“out of 100”	Percent	Decimal
$\frac{3}{5}$	$\frac{\quad}{10}$	$\frac{\quad}{50}$	$\frac{\quad}{100}$	60%	0.6
$\frac{\quad}{5}$	$\frac{2}{10}$	$\frac{\quad}{50}$	$\frac{20}{100}$	%	
$\frac{\quad}{5}$	$\frac{\quad}{10}$	$\frac{40}{50}$	$\frac{\quad}{100}$	%	

53) Which of the values below is equivalent to the ratio 3 out of 4? Select all that apply.

A. 3:4

D. 75%

B. $\frac{3}{4}$

E. 0.75

C. $\frac{75}{100}$

F. 7.5

Understanding Ratios - Answer Key

1) There are 9 right-handed people shown in the diagram. There is 1 left-handed person. There are 10 people in total.

2) 9:10

3) 3 (You might circle the three left hands, or 1 out of every 10 people.)

4)

Left-handed people	1	2	3	4	5	6	7	8	9	10
Total population	10	20	30	40	50	60	70	80	90	100

5) 5

6) 20

7) 180

8) 120

9) 23 is a good estimate. 22.5 is exact, but half of a person isn't possible, so we would probably round up to 23.

10) B

11) A, B, and C are all correct answers.

12)

Statements	T or F
There are 2 triangles for every 1 square.	T
The ratio of pentagons to triangles is 7:6.	T
The ratio of hexagons to total candies is $\frac{4}{16}$.	F
There are 7 pentagons out of 20 total pieces of candy.	T
For every 2 hexagons, there are 3 triangles.	T
The ratio of squares to total candies is 3:17.	F
The ratio of triangles to total candies is $\frac{3}{10}$.	T

13) Answers will vary.

14)

Hours worked	1	2	4	10	20	40
Pay	\$15	\$30	\$60	\$150	\$300	\$600

15)

Hours driving	1	3	6	10	15	13
Miles traveled	50	150	300	500	750	650

16)

Minutes	1	2	5	10	30	60
Heartbeats	80	160	400	800	2400	4800

17)

Days	1	3	4	10	20	27
Steps	10,000	30,000	40,000	100,000	200,000	270,000

Being Counted: Probability & Statistics (Part 1)

18)

Cans of soda	6	3	1	12	13	2
Cost	\$7.50	\$3.75	\$1.25	\$15.00	\$16.25	\$2.50

19)

Oranges	4	2	1	7	10	20
Cost	\$3.00	\$1.50	\$0.75	\$5.25	\$7.50	\$15.00

20) Rate: 2,000 calories per day

Days	2	1	4	7	5	10
Calories	4,000	2,000	8,000	14,000	10,000	20,000

21)

U.S. dollars	1	2	10	0.5	0.25 or $\frac{1}{4}$.05 or $\frac{1}{20}$
Mexican pesos	20	40	200	10	5	1

22) 32

23) 7

24) 40, 60, 18, 40

25)

Tables	1	2	3	4	5	6	7	8
Chairs	4	8	12	16	20	24	28	32

26) 48

27) D

28)

Tables	1	2	3	4	5	6	7
Chairs	4	8	12	16	20	24	28
Total Furniture	5	10	15	20	25	30	35

29) 75

30) 36

31) Answers will vary.

32) 2 cakes. 4 cups of sugar and 6 cups of flour is needed to double the recipe.

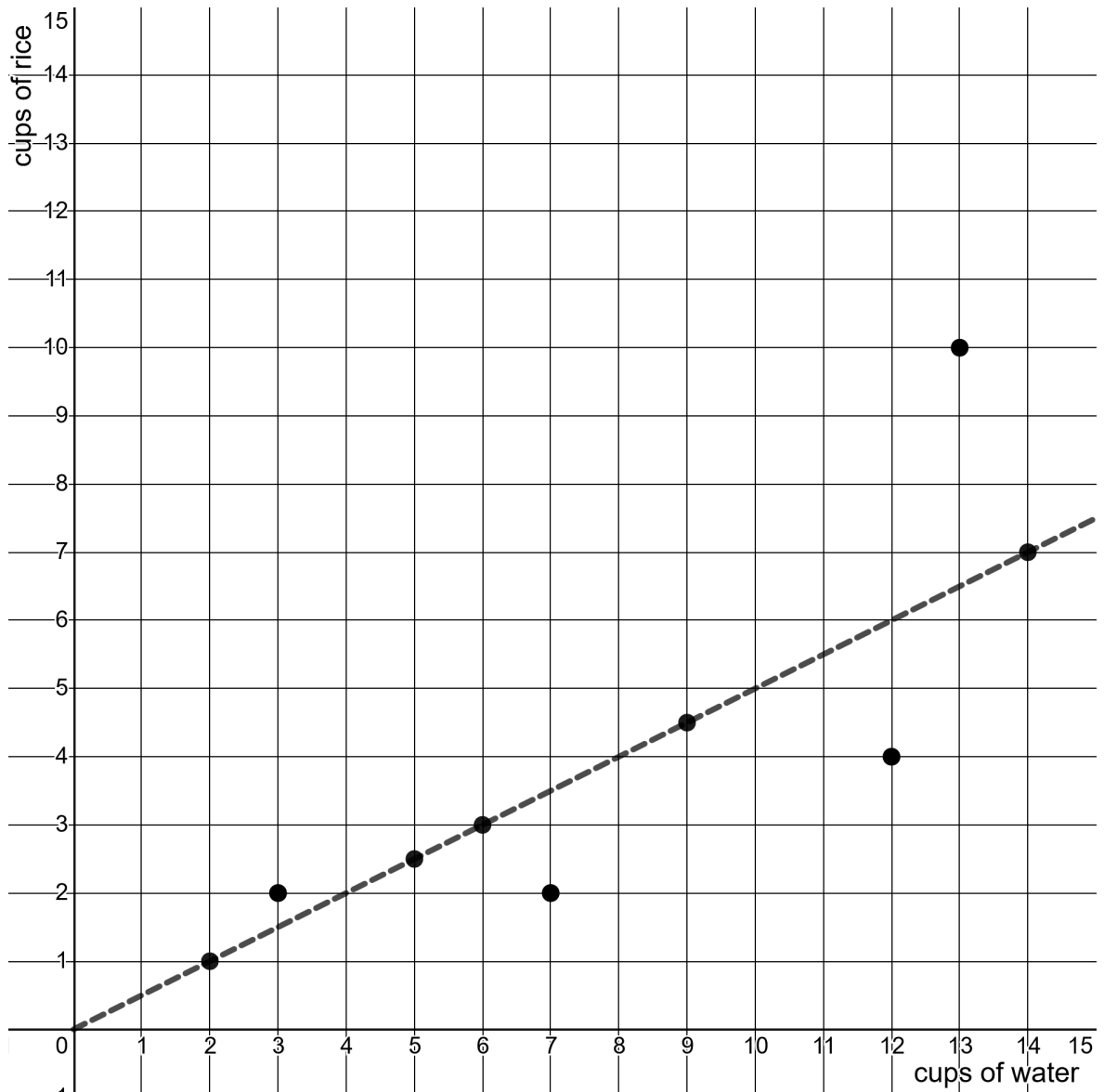
33) 8 cakes. 3 cups of flour $\times 8 = 24$ cups of flour, and 2 cups of sugar $\times 8 = 16$ cups of sugar. This is the equivalent of multiplying the flour and the sugar by 8 in order to make 8 cakes.

34) C

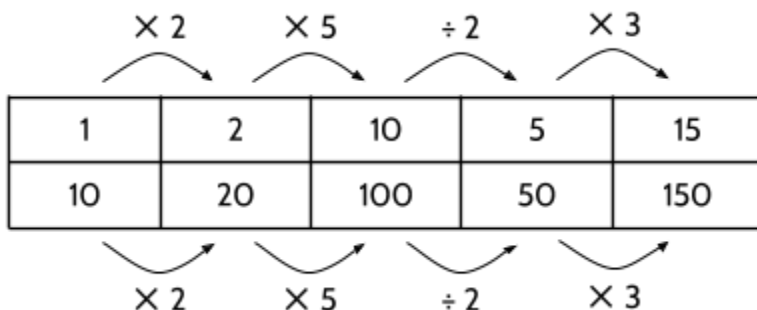
35)

Cups of Water	Cups of Rice	Does the recipe follow the correct ratio? YES or NO	How will the rice taste?
2	1	Yes	Perfect
3	2	No	Dry
5	2.5	Yes	Perfect
6	3	Yes	Perfect
7	2	No	Wet
9	4.5	Yes	Perfect
12	4	No	Wet
13	10	No	Dry
14	7	Yes	Perfect

36)



- 37) Each of these ratios are equivalent and result in well-cooked rice: 2:1, 5:2.5, 6:3, 9:4.5, and 14:7. If you connect each of the dots for the recipes that follow the 2:1 ratio, you will get a straight line. The dots that don't follow the ratio are not on the line. You might notice that the dots above the line are for recipes that are too dry. Dots below the line are recipes that are too wet.



38)

39) D

40) Answers will vary. As long as the top number and the bottom number are multiplied by the same number, the new ratio will be equivalent to the old ratio.

41) Answers will vary.

42) Answers will vary.

43) The first two fractions are $\frac{15}{6}$ and $\frac{5}{2}$. After that, answers will vary.

44) C. $\frac{1}{4}$ does not equal $\frac{7}{35}$. If you multiply 1 (the top number) by 7, you get the 7 (the top number in the second fraction). However, if you multiply 4 (the bottom number) by 7, you get 28, not 35 (the bottom number in the second fraction). $\frac{7}{28}$ is equivalent to $\frac{1}{4}$.

45) Since 2 multiplied **by 5** is 10, each of the other fractions should follow the same ratio. The top number multiplied **by 5** should equal the bottom number.

$$\frac{2}{10} \quad \frac{4}{20} \quad \frac{9}{45} \quad \frac{5}{25} \quad \frac{11}{55} \quad \frac{20}{100}$$

46) B

47)

"out of 10"	"out of 50"	"out of 100"	Percent	"out of 200"
$\frac{1}{10}$	$\frac{5}{50}$	$\frac{10}{100}$	10%	$\frac{20}{200}$
$\frac{4}{10}$	$\frac{20}{50}$	$\frac{40}{100}$	40%	$\frac{80}{200}$
$\frac{6}{10}$	$\frac{30}{50}$	$\frac{60}{100}$	60%	$\frac{120}{200}$
$\frac{1.5}{10}$	$\frac{7.5}{50}$	$\frac{15}{100}$	15%	$\frac{30}{200}$

Being Counted: Probability & Statistics (Part 1)

48) There are other possible answers, but here are a few: $\frac{3}{10}$, $\frac{15}{50}$, $\frac{30}{100}$, and $\frac{60}{200}$.

49) C

50) D

51)

Number of Pennies	Percent	"out of 100"	Written as money (decimal)
50	50%	$\frac{50}{100}$	0.50
25	25%	$\frac{25}{100}$	0.25
80	80%	$\frac{80}{100}$	0.80
8	8%	$\frac{8}{100}$	0.08

52)

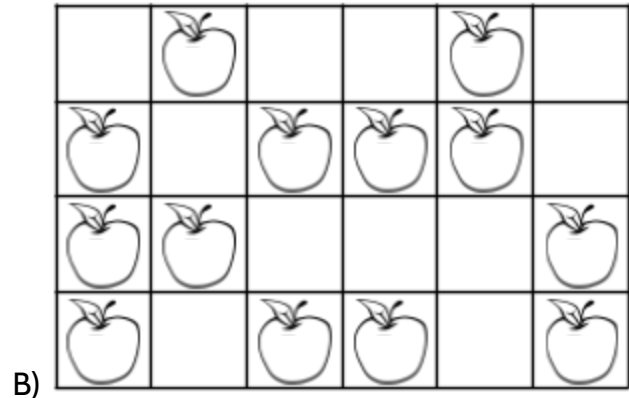
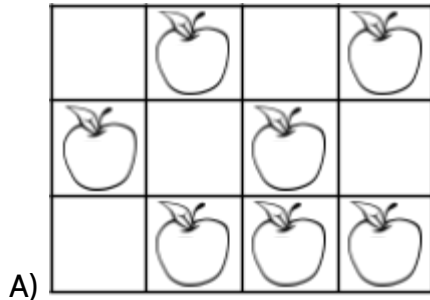
"out of 5"	"out of 10"	"out of 50"	"out of 100"	Percent	Decimal
$\frac{3}{5}$	$\frac{6}{10}$	$\frac{30}{50}$	$\frac{60}{100}$	60%	0.6
$\frac{1}{5}$	$\frac{2}{10}$	$\frac{10}{50}$	$\frac{20}{100}$	20%	0.2
$\frac{4}{5}$	$\frac{8}{10}$	$\frac{40}{50}$	$\frac{80}{100}$	80%	0.8

53) A, B, C, D, E

Using Ratios

Comparing Ratios

The two drawings below show apples compared with area (the total number of squares)⁶.



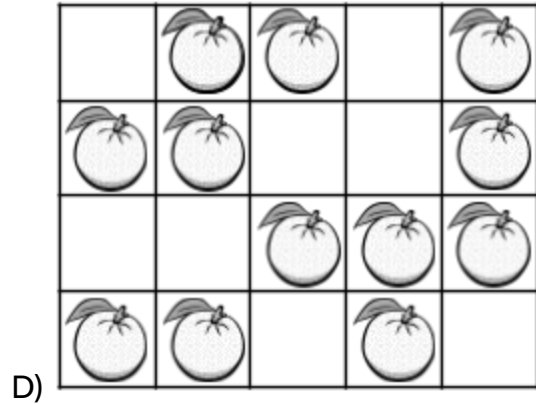
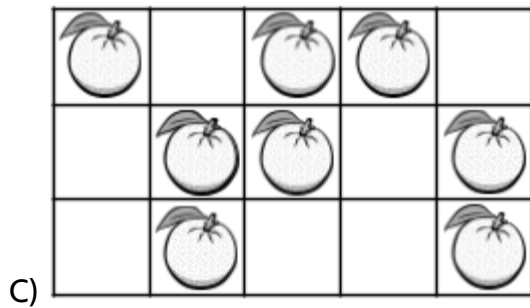
- 1) What do you notice?

- 2) In image A, there is a ratio of 7 apples to 12 squares. This can also be written as 7:12 or $\frac{7 \text{ apples}}{12 \text{ squares}}$. This ratio is equivalent to $\frac{\quad}{24}$ and $\frac{\quad}{48}$.

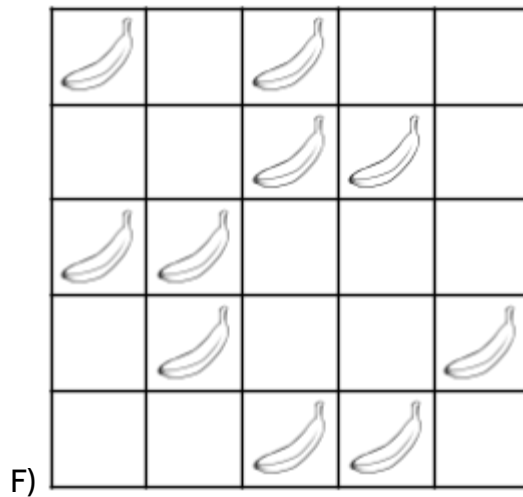
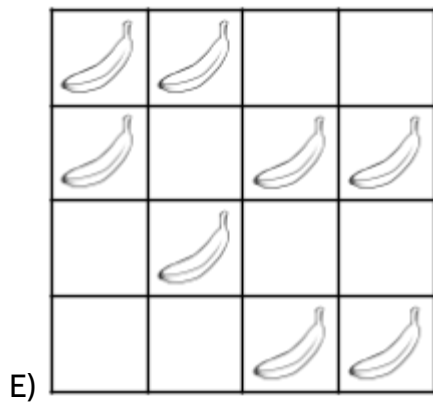
- 3) In image B, there is a ratio of _____ apples to _____ squares. This can also be written as ____ :24 or $\frac{\text{apples}}{24 \text{ squares}}$. This ratio is equivalent to $\frac{\quad}{48}$ and $\frac{\quad}{12}$.

- 4) Which image is more crowded with apples? In other words, which image has a greater number of apples per area?

⁶ Apples per area is similar to population density, which is a topic on the high school equivalency exam. See **Area & Population Density** and **Volume & the Density of Matter** for more practice with density.



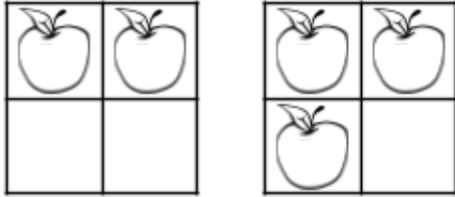
- 5) In image C, the ratio of $\frac{\text{oranges}}{15 \text{ squares}}$ is equivalent to $\frac{\quad}{30}$, $\frac{\quad}{45}$, and $\frac{\quad}{60}$.
- 6) In image D, the ratio of $\frac{\text{oranges}}{\text{squares}}$ is equivalent to $\frac{\quad}{40}$, $\frac{\quad}{60}$, and $\frac{\quad}{80}$.
- 7) Which image above has a greater number of oranges per area? How do you know?



- 8) Which image has a higher density of bananas per area?

- 9) Which is greater, $\frac{2}{4}$ or $\frac{3}{4}$? How do you know?

Did you think of objects and area?



Or maybe you thought about money?

- $\frac{3}{4}$ of a dollar is 3 quarters or \$0.75.
- $\frac{2}{4}$ of a dollar is 2 quarters or \$0.50.

- 10) Which is greater, $\frac{1}{2}$ or $\frac{2}{5}$? How do you know?

Comparing $\frac{1}{2}$ or $\frac{2}{5}$ is not as easy, since they aren't "out of" the same amount. The ratio $\frac{1}{2}$ is 1 out of 2 pieces and the ratio $\frac{2}{5}$ is 2 out of 5 pieces.

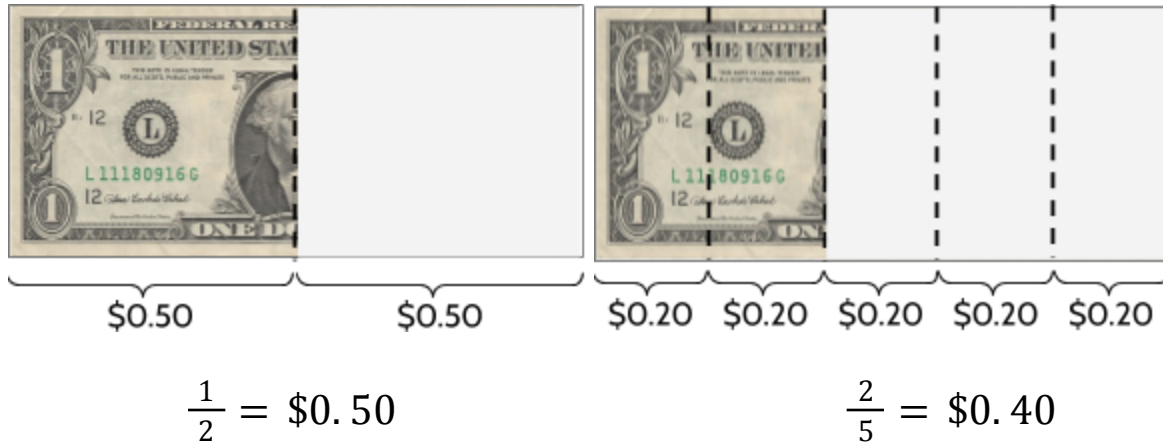
- 11) If it was money, you might ask, "Which is greater, half a dollar or two-fifths of a dollar?"



Explain which fraction is greater using the images above.

Being Counted: Probability & Statistics (Part 1)

Here is one way to see that $\frac{1}{2}$ is greater than $\frac{2}{5}$.

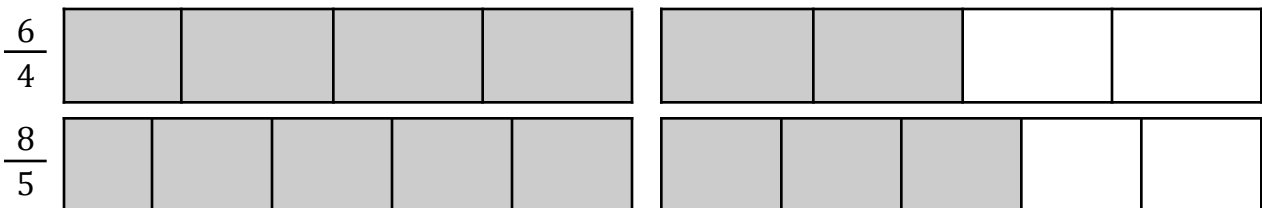


Drawing the fractions as rectangles can also help.



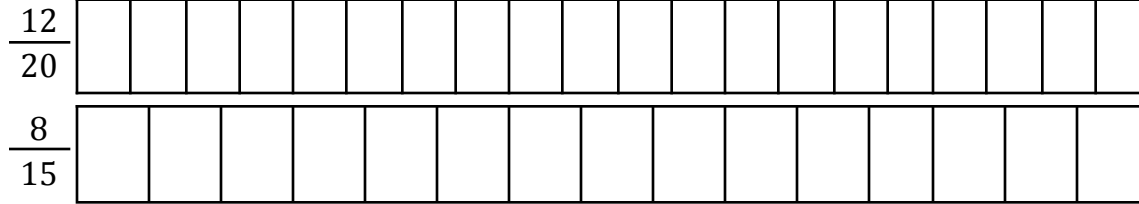
12) What do you notice?

Let's look at $\frac{6}{4}$ and $\frac{8}{5}$ with drawing of the fractions.



13) What do you notice?

Shade in sections of the figures below to show each fraction.

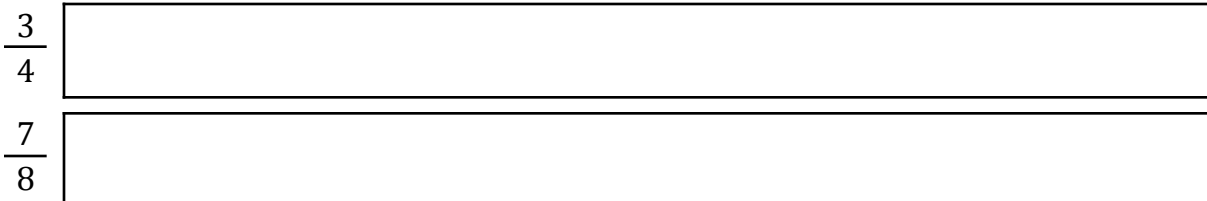


14) Which fraction is greater? How do you know?



15) Which fraction is greater? How do you know?

Draw and shade in sections for the two fractions.



16) Which fraction is greater? How do you know?

- 17) Compare $\frac{1}{4}$ and $\frac{3}{10}$ using money or drawings of fractions. Which is greater?

Ratio tables can also help us compare ratios.

- 18) Complete the following tables.

	$\times 5$	$\times 5$		$\times 2$	$\times 5$	
3				7		
4				10		
	$\times 5$	$\times 5$		$\times 2$	$\times 5$	

How do the ratio tables help us see which ratio is greater?

The ratio table above compares ratios that are both “out of” 20 and “out of” 100. We can see that $\frac{3}{4}$ is greater than $\frac{7}{10}$ because 15 for every 20 is greater than 14 for every 20.

We can also see that $\frac{3}{4}$ is equivalent to $\frac{75}{100}$ and $\frac{7}{10}$ is equivalent to $\frac{70}{100}$. 75 is greater than 70, so $\frac{3}{4}$ is greater than $\frac{7}{10}$.

Try making equivalent ratios in each table until you can compare ratios that have the same number on the bottom row. In other words, create equivalent fractions until you can compare fractions with the same denominator.

- 19) Which is greater, $\frac{2}{5}$ or $\frac{1}{4}$?

	$\times 2$	$\times 2$	$\times 5$
2			
5			
	$\times 2$	$\times 2$	$\times 5$

	$\times 5$	$\times 5$
1		
4		
	$\times 5$	$\times 5$

How do you know?

- 20) Which is greater, $\frac{9}{15}$ or $\frac{6}{8}$?

	$\div 3$	$\times 2$	$\times 10$
9			
15			
	$\div 3$	$\times 2$	$\times 10$

	$\div 2$	$\times 5$	$\times 5$
6			
8			
	$\div 2$	$\times 5$	$\times 5$

How do you know?

In these ratio tables, decide what numbers to multiply or divide by to make the ratios “out of” the same number. You don’t have to use all the boxes in the table.

- 21) Which is greater, $\frac{3}{5}$ or $\frac{6}{8}$?

3				
5				

6				
8				

How do you know?

- 22) Which is greater, $\frac{6}{4}$ or $\frac{8}{5}$?

How do you know?

- 23) Which is greater, $\frac{14}{20}$ or $\frac{9}{15}$?

How do you know?

- 24) Which is greater, $\frac{8}{3}$ or $\frac{10}{4}$?

How do you know?

Compare the following rates using ratio tables.

- 25) How would you rather be paid, \$45.00 for every 3 hours of work or \$80.00 for every 5 hours of work?

Hours worked	3				
Pay	\$45.00				

Hours worked	5				
Pay	\$80.00				

- 26) Which is a better deal, 4 oranges for \$3.00 or 5 oranges for \$4.00?

Oranges	4				
Cost	\$3.00				

Oranges	5				
Cost	\$4.00				

- 27) Which is faster, 200 miles in 4 hours or 90 miles in 1.5 hours?

Miles					
Distance					

Miles					
Distance					

Ratios, Decimals, and Percents

Ratio tables are one way to compare ratios to see which is greater. Decimals and percents are another way to do the same thing. Let's review how we compare $\frac{1}{2}$ and $\frac{2}{5}$ with a ratio table:

28) Fill in the missing blanks.

	$\times 5$	$\times 10$
1		
2		
	$\times 5$	$\times 10$

	$\times 2$	$\times 10$
2		
5		
	$\times 2$	$\times 10$

We can now see that $\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$ and that $\frac{2}{5} = \frac{4}{10} = \frac{40}{100}$. Since $\frac{5}{10}$ is greater than $\frac{4}{10}$, we know $\frac{1}{2}$ is greater than $\frac{2}{5}$. The fact that $\frac{50}{100}$ is greater than $\frac{40}{100}$ proves the same thing.

How do decimals help us compare ratios?

When a ratio is written as a decimal, an amount is being compared to 1. For example, if both ratios are written as fractions with 1 as the denominator, we can see which is greater. But how can we change a ratio like $\frac{2}{5}$ so that the bottom number is 1?

29) Fill in the blanks. You may want to use a calculator.

	$\div 5$
2	
5	
	$\div 5$

If we divide the numerator and the denominator by 5, we get the equivalent ratio $\frac{0.4}{1}$.

Since $5 \div 5$ is 1, the new denominator is 1.

Since $2 \div 5$ is 0.4, the new numerator is 0.4.

$\frac{0.4}{1}$ is equal to 0.4. Since we already know $\frac{1}{2}$ is equal to 0.5, the ratio $\frac{1}{2}$ is greater than $\frac{2}{5}$ because $0.5 > 0.4$.

Note: Any number divided by 1 is itself. For example,

$3 \div 1 = 3$ and $10 \div 1 = 10$. The ratio $\frac{0.4}{1}$ is the same as $0.4 \div 1$, which equals 0.4.

Fill in the blanks in the following ratio tables. You may want to use a calculator.

30)

$$\begin{array}{c} \div 5 \\ \curvearrowright \\ \begin{array}{|c|c|} \hline 7 & \\ \hline 5 & 1 \\ \hline \end{array} \\ \curvearrowleft \\ \div 5 \end{array}$$

decimal: 1.4

$$\begin{array}{c} \div 4 \\ \curvearrowright \\ \begin{array}{|c|c|} \hline 5 & \\ \hline 4 & 1 \\ \hline \end{array} \\ \curvearrowleft \\ \div 4 \end{array}$$

decimal: _____

Which is greater, $\frac{7}{5}$ or $\frac{5}{4}$?

How do you know?

31)

$$\begin{array}{c} \div 8 \\ \curvearrowright \\ \begin{array}{|c|c|} \hline 3 & \\ \hline 8 & 1 \\ \hline \end{array} \\ \curvearrowleft \\ \div 8 \end{array}$$

decimal: _____

$$\begin{array}{c} \div 10 \\ \curvearrowright \\ \begin{array}{|c|c|} \hline 4 & \\ \hline 10 & 1 \\ \hline \end{array} \\ \curvearrowleft \\ \div 10 \end{array}$$

decimal: _____

Which is greater, $\frac{3}{8}$ or $\frac{4}{10}$?

How do you know?

32)

12	
25	1

decimal: _____

9	
20	1

decimal: _____

Which is greater, $\frac{12}{25}$ or $\frac{9}{20}$?

How do you know?

33)

$$\begin{array}{ccc} \div 8 & & \div 20 \\ \curvearrowright & & \curvearrowright \\ \frac{7}{8} = \frac{\quad}{1} & & \frac{17}{20} = \frac{\quad}{1} \\ \curvearrowleft & & \curvearrowleft \\ \div 8 & & \div 20 \end{array}$$

Which is greater, $\frac{7}{8}$ or $\frac{17}{20}$?

How do you know?

decimal: _____ decimal: _____

34)

$$\begin{array}{ccc} \frac{7}{16} = \frac{\quad}{1} \\ \frac{11}{25} = \frac{\quad}{1} \end{array}$$

Which is greater, $\frac{7}{16}$ or $\frac{11}{25}$?

How do you know?

decimal: _____ decimal: _____

35)

$$\begin{array}{ccc} \frac{6}{5} = \frac{\quad}{1} \\ \frac{23}{20} = \frac{\quad}{1} \end{array}$$

Which is greater, $\frac{6}{5}$ or $\frac{23}{20}$?

How do you know?

decimal: _____ decimal: _____

36) Which is greater, $\frac{4}{5}$ or $\frac{6}{8}$? Use decimals to show how you know.

How do percentages help us compare ratios?

Percents are ratios that can be used to compare other ratios. They are part-whole ratios “out of” 100. *Per-* means “for every” and *-cent* means “100,” so the word *percent* literally means “for every 100.” Since we often need to compare ratios that are “out of” different amounts, we can use percentages to make any ratio into an “out of 100” ratio.

Percent (%) and “out of 100” mean the same thing:

$$\frac{25}{100} = 25\%$$

$$\frac{40}{100} = 40\%$$

$$\frac{50}{100} = 50\%$$

$$\frac{62.5}{100} = 62.5\%$$

To use percents to compare ratios, the first step is to convert the ratio into a percent. In other words, you want to change the ratio so that it is “out of 100.” Here is one way to do that.

37) Fill in the blanks. You may want to use a calculator.

original	decimal	percent
	$\div 5$	$\times 100$
4		
5	1	100
	$\div 5$	$\times 100$

Divide numerator and denominator by 5 to get an equivalent decimal.

Decimal: 0.8

Multiply the new numerator and denominator by 100 to get an equivalent percent.

Percent: 80%

Use ratio tables to convert these ratios into decimals and percentages.

original	decimal	percent
	$\div 8$	$\times 100$
3		
8	1	100
	$\div 8$	$\times 100$

Decimal:

Percent:

39)

7		
5	1	100

$\div 5$ $\times 100$
 $\div 5$ $\times 100$

Decimal:

Percent:

40)

10		
25	1	100

Decimal:

Percent:

41)

5		
4		

Decimal:

Percent:

42)

13		
4		

Decimal:

Percent:

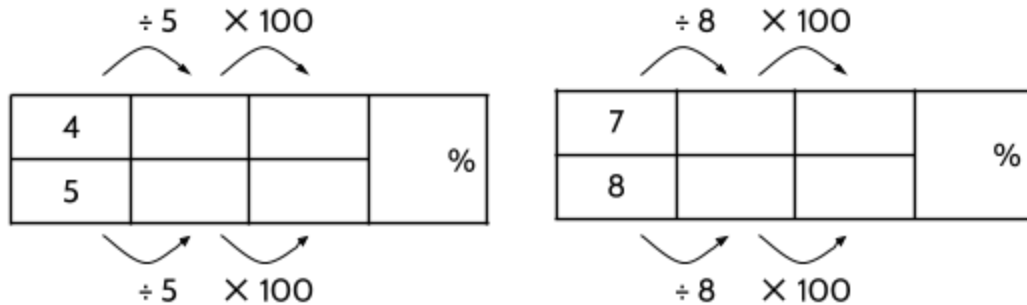
43)

$$\begin{array}{c}
 \div 20 \qquad \qquad \times 100 \\
 \frac{19}{20} = \frac{\quad}{1} = \frac{\quad}{100} = \quad \% \\
 \div 20 \qquad \qquad \times 100
 \end{array}$$

44) $\frac{7}{16} = \frac{\quad}{1} = \frac{\quad}{100} = \quad \%$

Once you convert fractions into a decimal or percent, you can see which is greater.

45) Which is greater, $\frac{4}{5}$ or $\frac{7}{8}$?



46) Which is greater, $\frac{9}{5}$ or $\frac{13}{8}$?

9			
5			%

13			
8			%

47) Which is greater, $\frac{9}{16}$ or $\frac{11}{20}$?

$$\frac{9}{16} = \frac{\quad}{1} = \frac{\quad}{100} = \quad \%$$

$$\frac{11}{20} = \frac{\quad}{1} = \frac{\quad}{100} = \quad \%$$

48) Which is greater, $\frac{7}{25}$ or $\frac{5}{16}$?

49) Which is greater, 30% or $\frac{2}{5}$? (You don't have to use all of the boxes.)

30					
100					

2					
5					

50) Which is greater, 0.48 or $\frac{9}{20}$?

48					
100					

9					
20					

51) Which is greater, 63 out of 300 or 22%?

52) Which is greater, 145% or $\frac{8}{5}$?

Using Rates to Solve for a Missing Value

Find the missing value that keeps the two ratios equivalent, then write an explanation.

53)

oranges	8	
dollars	2	6

As a sentence: If you can buy 8 oranges for \$2.00, then you can buy 24 oranges for \$6.00.

54)

oranges	8	1
dollars	2	

As a sentence:

55)

apples	4	
dollars	2	10

As a sentence:

56)

apples	4	10
dollars	2	

As a sentence:

57)

gallons of gas	1	
dollars	3	18

As a sentence:

58)

gallons of gas	1	4.5
dollars	3	

As a sentence:

59)

U.S. dollars	1	10
japanese yen	100	

As a sentence:

60)

U. S. dollars	1	
japanese yen	100	25

As a sentence:

61)

cups	16	
gallon	1	1.5

As a sentence:

62)

cups	16	40
gallon	1	

As a sentence:

63)

kilometers	1.6	
miles	1	3

As a sentence:

64)

kilometers	1.6	10
miles	1	

As a sentence:

Find the missing value that keeps the two ratios equivalent, then write an explanation.

65) $\frac{2}{6} = \frac{x}{24}$

What is the value of x ? Why?

The value of x is 8. Since 6×4 equals 24, then 2×4 would equal 8. Also, 2×3 equals 6, so 8×3 would equal 24.

66) $\frac{5}{20} = \frac{y}{100}$

What is the value of y ? Why?

67) $\frac{4}{6} = \frac{10}{z}$

What is the value of z ? Why?

68) $\frac{1}{a} = \frac{7}{28}$

What is the value of a ? Why?

69) $\frac{b}{2} = \frac{48}{8}$

What is the value of b ? Why?

70) $\frac{6}{c} = \frac{1.5}{4.5}$

What is the value of c ? Why?

71) $\frac{0.25}{1} = \frac{d}{100}$

What is the value of d ? Why?

72) $\frac{40}{100} = \frac{e}{20}$

What is the value of e ? Why?

A Typical Day

How Jakima spends a typical day

working	working	eating	studying	sleeping	sleeping
working	working	cooking	studying	sleeping	sleeping
working	working	taking care of family	studying	sleeping	sleeping
working	working	taking care of family	using phone	using phone	sleeping

73) Mark the following statements true or false.

Statements	T or F
Jakima typically works 8 hours per day.	T
Jakima works more hours than she sleeps.	
$\frac{1}{3}$ of Jakima's day is spent at work.	
Jakima studies $\frac{3}{8}$ of a typical day.	
For Jakima, the ratio of working to using her phone is 4:1.	
Jakima uses approximately 8% of a typical day taking care of her family.	

74) Complete the following diagram to show 24 hours in one of your typical days.

How do you spend a typical day?

Write mathematical statements about how you spend a typical day. Use what you know about ratios, fractions, and percents.

Ratios Written in Different Ways

Once you start looking for statistics written as ratios, you will see them everywhere. Here are some examples from recent news reports:

- More than 1 in 4 high school students are vaping (using e-cigarettes). About 1 in every 10 middle school students currently vape. (Dec. 5, 2019, U.S. News and World Report)
- About $\frac{3}{5}$ of the New York City high school graduating class of 2018 enrolled in higher education, such as college or a vocational program. (Nov. 21, 2019, Chalkbeat.org)
- U.S. customers gave video streaming services a score of 75 out of 100, compared with 60 out of 100 for cable television. (May 21, 2019, American Customer Service Index)
- Playing for the Houston Rockets basketball team, James Harden has made 87% of his free throws this season. (Dec. 5, 2019, SportingNews.com)
- With a batting average of .335, Tim Anderson of the Chicago White Sox had the best hitting percentage in major league baseball in 2019. (Dec. 8, 2019, ESPN.com)

Each of these values are written in different ways, but they are all ratios:

1 in 4 1 in every 10 $\frac{3}{5}$ 75 out of 100 87% 0.335

- 75) Use a newspaper, advertisements, television, or the Internet to find three examples of statistics written as ratios.
- 76) Write one of the ratios in a couple other ways. For example, if it is a percent, change it to a fraction. Or if it is a fraction, change it to an equivalent fraction.

Using Ratios - Answer Key

- 1) Answers will vary.
- 2) In image A, there is a ratio of 7 apples to 12 squares. This can also be written as 7:12 or $\frac{7 \text{ apples}}{12 \text{ squares}}$. This ratio is equivalent to $\frac{14}{24}$ and $\frac{28}{48}$.
- 3) In image B, there is a ratio of 13 apples to 24 squares. This can also be written as 13:24 or $\frac{13 \text{ apples}}{24 \text{ squares}}$. This ratio is equivalent to $\frac{26}{48}$ and $\frac{6.5}{12}$.
- 4) Image A is more crowded with apples. There are 7 apples for every 12 squares in Image A and 6.5 apples for every 12 squares in Image B. Similarly, there are 14 apples for every 24 squares in Image A and 13 apples for every 24 squares in Image B.
- 5) In image C, the ratio of $\frac{8 \text{ oranges}}{15 \text{ squares}}$ is equivalent to $\frac{16}{30}$, $\frac{24}{45}$, and $\frac{32}{60}$.
- 6) In image D, the ratio of $\frac{12 \text{ oranges}}{20 \text{ squares}}$ is equivalent to $\frac{24}{40}$, $\frac{36}{60}$, and $\frac{48}{80}$.
- 7) Image D has a greater number of oranges per area. It has a ratio of 36 oranges for every 60 squares. Image C has a ratio of 32 oranges for every 60 squares.
- 8) In order to answer this question, it is useful to set up a ratio table for each image and create some equivalent ratios. If you find two ratios that have the same bottom number, you can compare the size of the two ratios. There are many ways to do this. The numbers below are just examples. You may have done it a different way.

Image E:

Bananas	8	4	2	1	50	200
Squares	16	8	4	2	100	400

Image F:

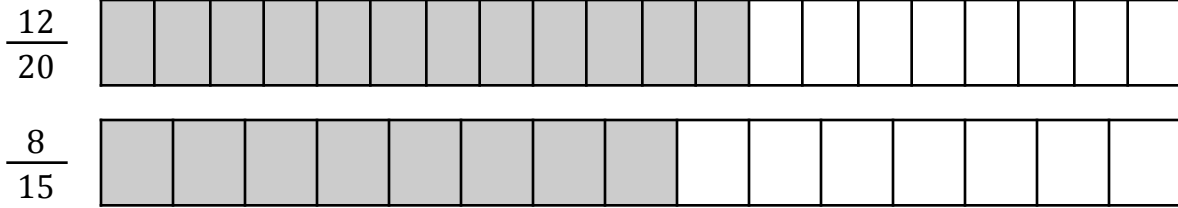
Bananas	10	2	20	30	40	4
Squares	25	5	50	75	100	10

Image E has a higher density of bananas because it has a ratio of 50 bananas to 100 squares while Image F has a ratio of 40 bananas to 100 squares.

- 9) $\frac{3}{4}$ is greater, because $\frac{3}{4}$ of a dollar is 3 quarters or \$0.75. $\frac{2}{4}$ of a dollar is 2 quarters or \$0.50. Another explanation uses fractions. If the denominator is the same, then you can look at the numerator to see which is greater. $\frac{3}{4}$ is greater than $\frac{2}{4}$ because 3 is greater than 2.
- 10) The explanation is after the question. If you already know the answer, that's fantastic, but it's okay if you don't.
- 11) The explanation is on the next page of the text.
- 12) There are different things you might notice. Here are a few:
- $\frac{1}{2}$ is out of a total of 2 pieces. 1 is shaded.
 - $\frac{2}{5}$ is out of a total of 5 pieces. 2 are shaded.
 - Both fractions are less than 1, since the whole rectangle represents 1 whole.
 - The fraction $\frac{2}{5}$ is less than half the length of the whole rectangle.
 - $\frac{1}{2}$ is greater than $\frac{2}{5}$. The shaded area for $\frac{1}{2}$ is longer.
- 13) There are different things you might notice. Here are a few:
- Both $\frac{6}{4}$ and $\frac{8}{5}$ are greater than 1.
 - The fraction $\frac{6}{4}$ is equal to $1\frac{2}{4}$. The fraction $\frac{8}{5}$ is equal to $1\frac{3}{5}$.
 - $\frac{6}{4}$ is out of 2 totals of 4 pieces. 6 are shaded.
 - $\frac{8}{5}$ is out of 2 totals of 5 pieces. 8 are shaded.
 - $\frac{8}{5}$ is greater than $\frac{6}{4}$. The shaded area for $\frac{8}{5}$ is longer.

It may seem weird to have 6 out of 4, or 8 out of 5. If the total is 4, how can 6 be a part? A real-world example might help. Imagine two cakes. Each one is cut into 4 pieces. If you give out 6 pieces of cake to kids at a birthday party, you have given out 6 fourths of cake. At another party, you cut two cakes each into 5 pieces, then give out 8 total pieces of cake. You gave out 8 fifths of cake. You gave out $\frac{6}{4}$ at the first party and $\frac{8}{5}$ of cake at the second party.

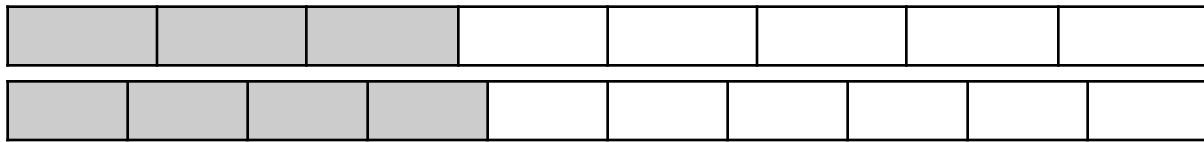
14)



You may notice:

- $\frac{12}{20}$ is greater than $\frac{8}{15}$. The shaded area for $\frac{12}{20}$ is longer.
- $\frac{12}{20}$ is out of a total of 20 pieces. 12 are shaded.
- $\frac{8}{15}$ is out of a total of 15 pieces. 8 are shaded.

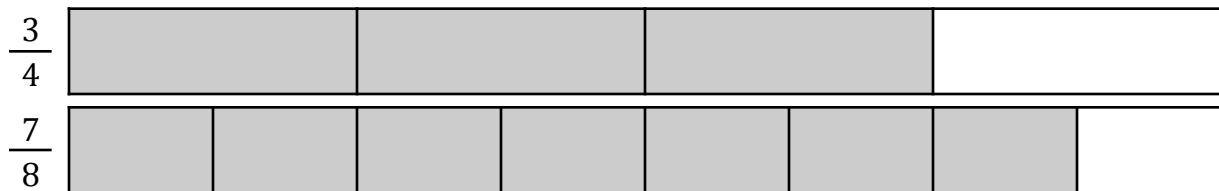
15)



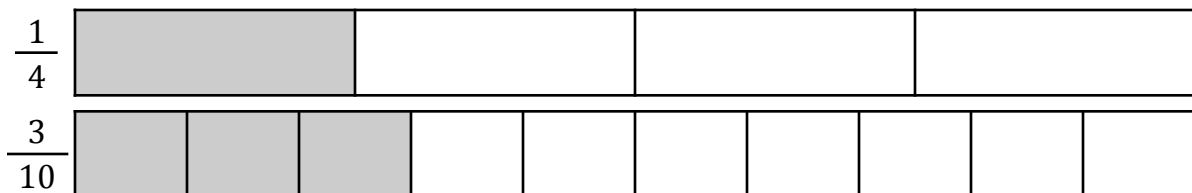
You may notice:

- $\frac{4}{10}$ is greater than $\frac{3}{8}$. The shaded area for $\frac{4}{10}$ is longer.
- $\frac{3}{8}$ is out of a total of 8 pieces. 3 are shaded.
- $\frac{4}{10}$ is out of a total of 10 pieces. 4 are shaded.

16) $\frac{7}{8}$ is greater than $\frac{3}{4}$.



17) Using money: $\frac{1}{4}$ is like 1 quarter out of 4 quarters (1 dollar) or \$0.25. $\frac{3}{10}$ is like 3 dimes out of 10 dimes (1 dollar) or \$0.30.



- 18)
- | | | |
|---|----|-----|
| 3 | 15 | 75 |
| 4 | 20 | 100 |
- | | | |
|----|----|-----|
| 7 | 14 | 70 |
| 10 | 20 | 100 |
- $\frac{3}{4}$ is greater than $\frac{7}{10}$ because 15 for every 20 is more than 14 for every 20.
 - We can also see that $\frac{3}{4}$ is equivalent to $\frac{75}{100}$ and $\frac{7}{10}$ is equivalent to $\frac{70}{100}$. 75 is greater than 70, so $\frac{3}{4}$ is greater than $\frac{7}{10}$.

- 19) $\frac{2}{5}$ is greater than $\frac{1}{4}$ because $\frac{8}{20}$ is greater than $\frac{5}{20}$ and $\frac{40}{100}$ is greater than $\frac{25}{100}$.

2	4	8	40
5	10	20	100

1	5	25	blank
4	20	100	blank

- 20) $\frac{6}{8}$ is greater than $\frac{9}{15}$ because $\frac{75}{100}$ is greater than $\frac{60}{100}$.

9	3	6	60
15	5	10	100

6	3	15	75
8	4	20	100

21) $\frac{6}{8}$

22) $\frac{8}{5}$

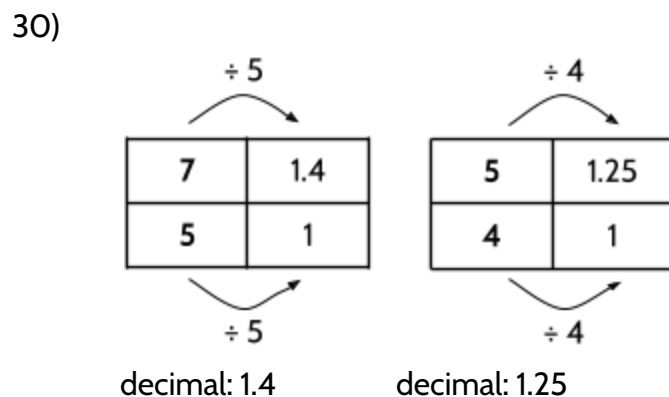
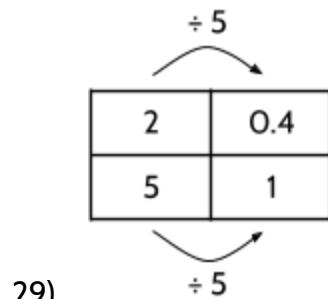
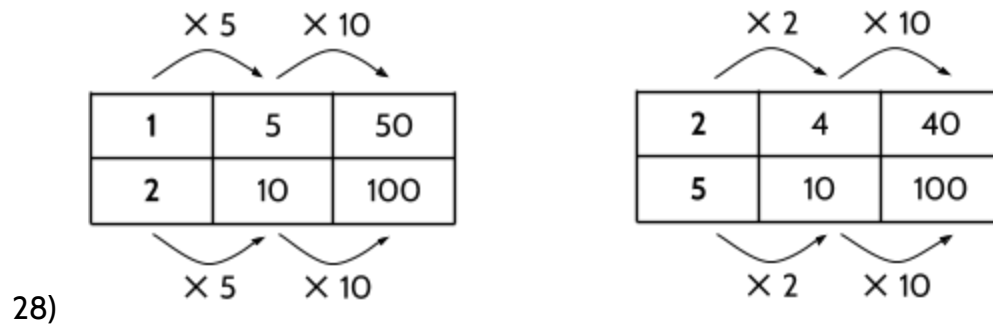
23) $\frac{14}{20}$

24) $\frac{8}{3}$

25) \$80.00 for every 5 hours of work is more money. This is equivalent to \$16.00 per hour. \$45.00 for every 3 hours of work is the same as \$15.00 per hour.

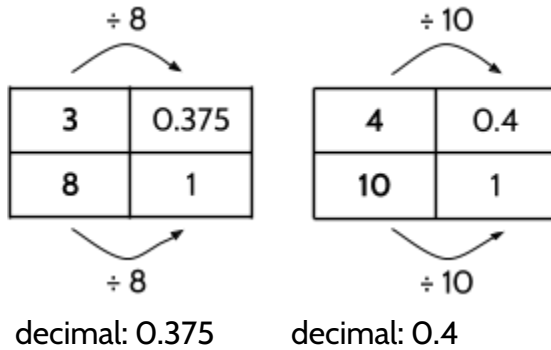
26) 4 oranges for \$3.00 is a better deal. This is equivalent to \$0.75 per orange. 5 oranges for \$4.00 is equivalent to \$0.80 per orange.

27) 90 miles in 1.5 hours is faster. This is equivalent to 60 miles per hour. 200 miles in 4 hours is equivalent to 50 miles per hour.



$\frac{7}{5}$ is greater because the decimal 1.4 is greater than the decimal 1.25.

31)

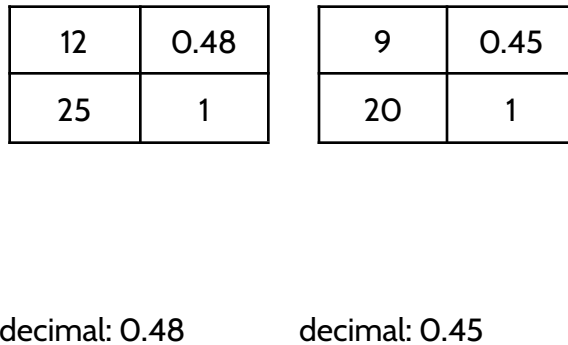


$\frac{3}{8}$ is equal to 0.375.

$\frac{4}{10}$ is equal to 0.4.

$\frac{4}{10}$ is greater because the decimal 0.4 is greater than the decimal 0.375.

32)

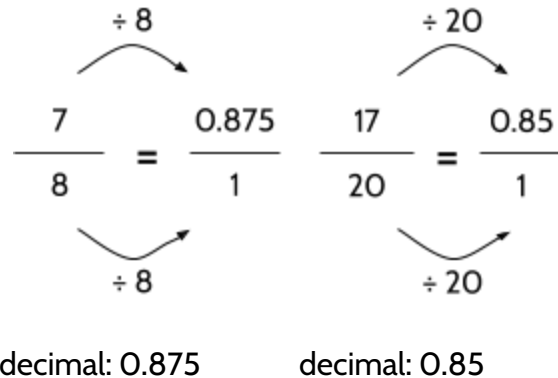


$\frac{12}{25}$ is equal to 0.48.

$\frac{9}{20}$ is equal to 0.45.

$\frac{12}{25}$ is greater because the decimal 0.48 is greater than the decimal 0.45.

33)



$\frac{7}{8}$ is equal to 0.875.

$\frac{17}{20}$ is equal to 0.85.

$\frac{7}{8}$ is greater because 0.875 is greater than 0.85.

34)

$$\frac{7}{16} = \frac{\quad}{1}$$

$$\frac{11}{25} = \frac{\quad}{1}$$

decimal: 0.4375 decimal: 0.44

$\frac{7}{16}$ is equal to 0.4375.

$\frac{11}{25}$ is equal to 0.44.

$\frac{11}{25}$ is greater because 0.44 is greater than 0.4375.

35)

$$\frac{6}{5} = \frac{\quad}{1} \quad \frac{23}{20} = \frac{\quad}{1}$$

decimal: 1.2 decimal: 1.15

$\frac{6}{5}$ is equal to 1.2.

$\frac{23}{20}$ is equal to 1.15.

$\frac{6}{5}$ is greater because 1.2 is greater than 1.15.

36) $\frac{4}{5} = 0.8$ and $\frac{6}{8} = 0.75$. $\frac{4}{5}$ is greater.

original	decimal	percent
	$\div 5$	$\times 100$
4	0.8	80
5	1	100
	$\div 5$	$\times 100$

37)

original	decimal	percent
	$\div 8$	$\times 100$
3	0.375	37.5
8	1	100
	$\div 8$	$\times 100$

38)

Decimal: 0.375, Percent: 37.5%

39) Decimal: 1.4, Percent: 140%

40) Decimal: 0.4, Percent: 40%

41) Decimal: 1.25, Percent: 125%

42) Decimal: 3.25, Percent: 325%

$$43) \frac{19}{20} = \frac{0.95}{1} = \frac{95}{100} = 95\%$$

$$44) \frac{7}{16} = \frac{0.4375}{1} = \frac{43.75}{100} = 43.75\%$$

45) $\frac{4}{5}$ is equivalent to 0.8 or 80%. $\frac{7}{8}$ is equivalent to 0.875 or 87.5%, which is more.

46) $\frac{9}{5}$ is equivalent to 1.8 or 180%. $\frac{13}{8}$ is equivalent to 1.625 or 162.5%, which is less.

47) $\frac{9}{16}$ is equivalent to 0.5625 or 56.25%. $\frac{11}{20}$ is equivalent to 0.55 or 55%, which is less.

48) $\frac{7}{25}$ is equivalent to 0.28 or 28%. $\frac{5}{16}$ is equivalent to 0.3125 or 31.25%, which is more.

49) $\frac{2}{5}$ is greater.

50) 0.48 is greater.

51) 22% is greater.

52) $\frac{8}{5}$ is greater.

53)

oranges	8	24
dollars	2	6

54)

oranges	8	1
dollars	2	0.25

Sentences will vary.

55)

apples	4	20
dollars	2	10

Sentences will vary.

56)

apples	4	10
dollars	2	5

Sentences will vary.

57)

gallons of gas	1	6
dollars	3	18

Sentences will vary.

58)

gallons of gas	1	4.5
dollars	3	13.5

Sentences will vary.

59)

U.S. dollars	1	10
Japanese yen	100	1000

Sentences will vary.

60)

U.S. dollars	1	0.25
Japanese yen	100	25

Sentences will vary.

61)

cups	16	24
gallons	1	1.5

Sentences will vary.

62)

cups	16	40
gallons	1	2.5

Sentences will vary.

63)

kilometers	1.6	4.8
miles	1	3

Sentences will vary.

64)

kilometers	1.6	10
miles	1	6.25

Sentences will vary.

65) The value of x is 8. Since 6×4 equals 24, then 2×4 would equal 8. Also, 2×3 equals 6, so 8×3 would equal 24.

66) $y = 25$. What is your explanation?

67) $z = 15$. What is your explanation?

68) $a = 4$. What is your explanation?

69) $b = 12$. What is your explanation?

70) $c = 18$. What is your explanation?

71) $d = 25$. What is your explanation?

72) $e = 8$. What is your explanation?

73)

Statements	T or F
Jakima typically works 8 hours per day.	T
Jakima works more hours than she sleeps.	T
$\frac{1}{3}$ of Jakima's day is spent at work.	T
Jakima studies $\frac{3}{8}$ of a typical day.	F
For Jakima, the ratio of working to using her phone is 4:1.	T
Jakima uses approximately 8% of a typical day taking care of her family.	T

74) Answers will vary.

75) We like to hear what statistics you found. You can email your statistics to **info@collectedny.org**.

76) Answers will vary.

Introduction to Probability

Probability is different from math you may have studied before.

Every day we talk about things that might happen or they might not happen. We can't say for sure if they will occur, especially when we talk about things in the future. When we are talking about these situations we talk about chance and ask, "What are the chances that this will happen?"



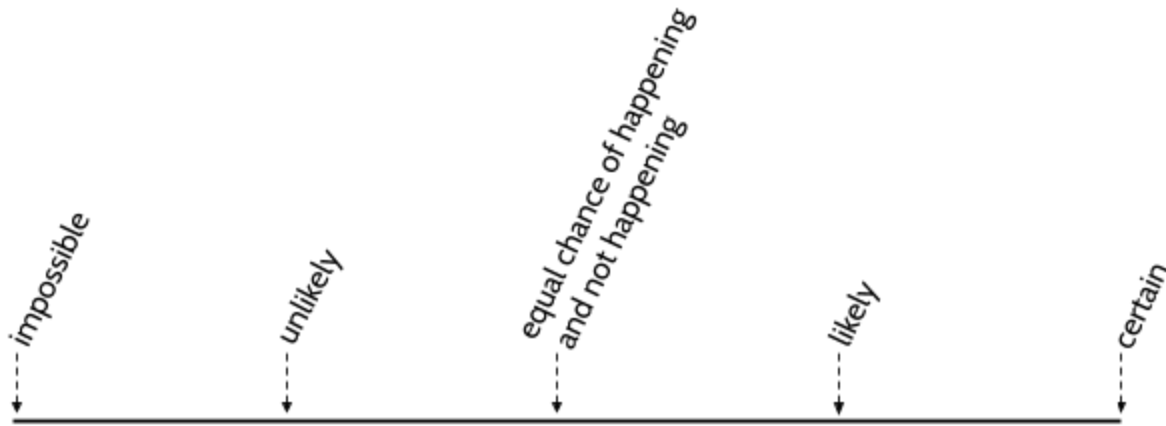
Probability is the mathematics of chance. We use probability to help us make decisions. If you think your chances of getting the flu are high, you might get a flu shot. If you are not sure if your seeds will sprout, you might plant extra seeds. If you believe it will rain, you might decide to carry an umbrella. If you think one surgery might be more successful than another, that is the one you will probably choose.

Probability is the math we can use to measure and analyze *chance*. Chance is the possibility of something happening.

When we talk about chance, there are some helpful words we can use:

- **Impossible** - It will never happen.
- **Unlikely** - It probably will not happen.
- **Likely** - It probably will happen.
- **Certain** - It will definitely happen.

A *probability line* helps us see the possibilities. Impossible events are on one end of the line. Events that are certain to happen are on the other end.



Imagine that we drop a glass bottle.

If we drop a glass bottle, we can be certain that it will fall.

If we drop a glass bottle onto something soft, it is unlikely that it will break.

If we drop a glass bottle on the sidewalk, it is likely that it will break.

If we drop a glass bottle, it is impossible that it will turn into a bird and fly away.

We can use the word “event” to describe “things that happen.”

Where would you place the following *events* on the probability line? Shade in the box that best fits your answer.

- 1) For you to beat South African Olympic gold medalist Caster Semenya in a race.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

- 2) You will eat cereal for breakfast one morning this week.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

- 3) A Democrat will win the next presidential election.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

- 4) The sun will rise tomorrow morning.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

- 5) You will hear your favorite song on the radio.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

- 6) The human population of the world will continue to grow.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

- 7) It will rain at least once every month in your city or town.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

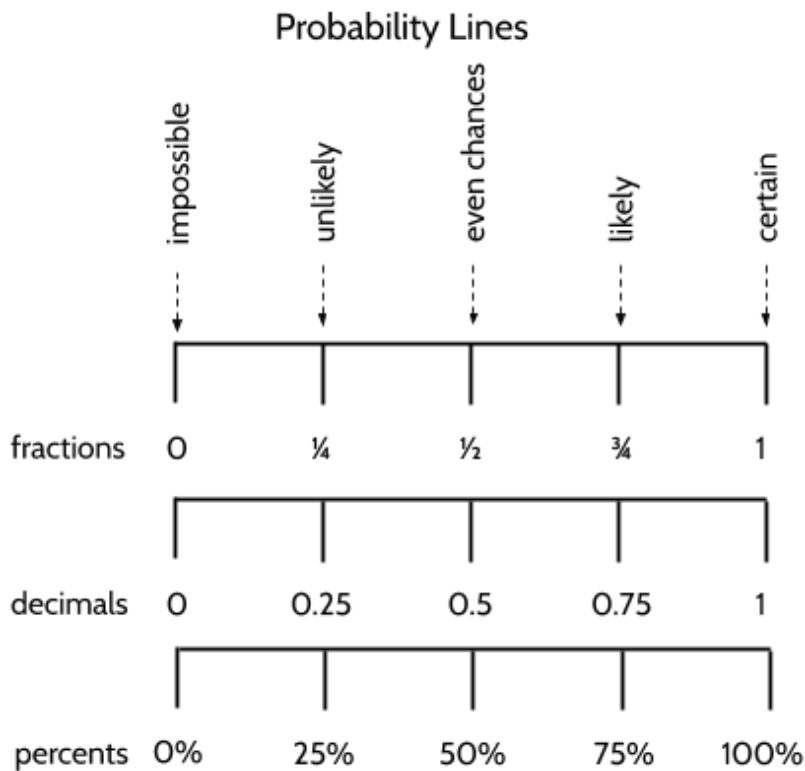
- 8) A sunflower seed planted today will bloom tomorrow.

Impossible	Unlikely	Likely	Certain
------------	----------	--------	---------

- 9) Write one event that is certain to happen.
- 10) Write one event that is impossible - something that definitely will not happen.
- 11) Write one event that is likely to happen.
- 12) Write one event that is unlikely to happen.

In everyday conversation, we use words like *impossible*, *unlikely*, *likely*, and *certain* to talk about what might happen in the future, but mathematicians use numbers to compare the likelihood of different events.

Consider the diagram below.



- 13) What do you notice?

Mathematicians use numbers to communicate and calculate probabilities like *impossible*, *even chances*, and *certain*. If an event is impossible (the sun will rise in the West), the probability is 0. If an event will always happen (the sun will come up tomorrow), the probability is 1. If there are even chances of an event happening or not happening, the probability is $\frac{1}{2}$. A probability of 1 is the same as 100%, in the same way that a probability of $\frac{1}{2}$ is the same as 50%.

When we measure the chance that something will happen or will not happen, we use the numbers between 0 and 1. It is hard to think of events that will never happen or will always happen. Events with probabilities in between 0 and 1 are more interesting and useful to think about. Will the stock market go up? Will I win the lottery? Will it rain tomorrow? What are my chances of catching the flu? Probabilities between 0% and 100% cover most things that happen in the world, since most events in the world are somewhere between impossible and certain.

Almost anything you can think of has a probability between 0 and 1. The probability of flipping a coin and getting "heads" is $\frac{1}{2}$. The probability of winning Powerball is 0.000000003. The chance that it will rain in New York City tomorrow (as I write this) is 90%. Each of these numbers ($\frac{1}{2}$, 0.000000003, and 90%) are between 0 and 1. By the way, you might have noticed that we can write each of these probabilities as fractions, decimals, or percents.

14) Describe an example of a probability in your life.

Introduction to Probability - Answer Key

For the first 8 questions, the chances of some of the events occurring depends on you.

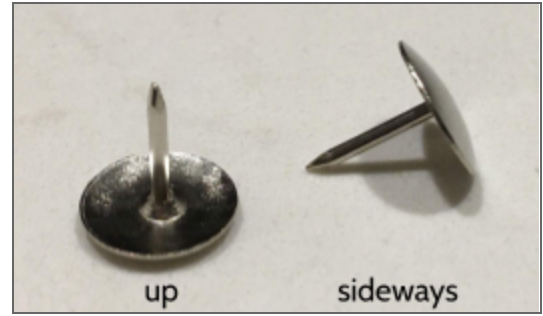
- 1) Impossible or Unlikely
- 2) Likely
- 3) This depends on the election and your opinion or what will happen.
- 4) Certain
- 5) Depends on how popular your favorite song is.
- 6) Likely
- 7) Likely
- 8) Impossible
- 9) Example: It is certain that you will make mistakes while learning.
- 10) Example: It is impossible for humans to hold their breath for 24 hours.
- 11) Example: It is likely that you will finish this packet.
- 12) Example: It is unlikely that you will win the lottery.
- 13) You may notice that the fraction $\frac{1}{4}$, the decimal 0.25, and the percent 25% all mean “unlikely” in probability. Other equivalent ratios like $\frac{3}{4}$, 0.75, and 75% also have a similar meaning in probability. You should also notice that 1 means 100%, the probability that something will definitely happen.
- 14) Answers will vary.

Experimental Probability

To better understand chance, we will investigate probability by looking at a few experiments. In an experiment, we observe what happens and record data. We analyze the data after the experiment to understand *experimental probability*.

Tossing Tacks

If you drop a tack, it might land on its side or it might land with the point sticking up. These are the two possible *outcomes*.



- 1) If you dropped 100 tacks, how many do you predict would land with the point up?

If we were flipping coins, we might predict that half of the flips would result in “heads” and half would result in “tails,” but that is because one side of the coin is the same as the other side of the coin. The probability of tossing tack is harder to predict. In order to understand the probability, we will need to collect data.

We can use an experiment to learn about the probability of tossing tacks. We will toss 10 tacks at a time and see what happens.

- 2) Here are the results of the first 10 tosses. Record the results in the table.



First 10 tosses		
up	sideways	total
		10

- 3) Based on these tosses, how many tacks do you think would land “up” if we toss 20?

Note: There are vocabulary words used in probability that may be new to you. There may also be words you have seen before, but are used in different ways. For example, in the language of probability, each time we toss the tack is a *trial*. There are two possible *outcomes*: landing up or landing sideways. These two different outcomes are called *events*. For now, we are interested in the “up” tack event. How many times something happens is called *frequency*. The frequency of “up” tacks in the first 10 trials was 3. There have been 10 total trials so far (since there were 10 tossed tacks).

We can calculate the *experimental probability* of an event by making a part-whole ratio with the data after the experiment.

$$\frac{\text{frequency of event}}{\text{total number of trials}} = \frac{3 \text{ "up" tacks}}{10 \text{ tossed tacks}}$$

The part-whole ratio above is called *relative frequency*, because it is the frequency of the event *relative* to the total number of trials. *Experimental probability* has this name because it is a probability that is based on doing experiments.

Let's use the experimental probability of $\frac{3}{10}$ to make some equivalent ratios.

- 4) Complete the table.

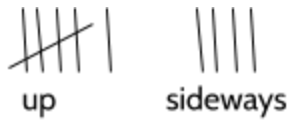
Trials 1-10	“up” tacks	3				
	total trials	10	20	30	50	5

- 5) Equivalent ratios can help us make predictions about what would happen if we toss different numbers of tacks. Based on these 10 trials, can you predict how many times a tack would land “up” out of 100 tosses?

- 6) Imagine we threw a mystery number of tacks. If the experimental probability was $\frac{3}{10}$ and there were 24 “up” tacks after throwing all tacks, how many trials were there?

“up” tacks	3	24
total trials	10	

Now, we will continue to toss tacks and then analyze the number of times they land “up.” Here are the results of trials 11 through 20.



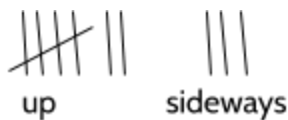
7) Complete the table.

		original ratio	“out of 50”	“out of 100”	“out of 1”
Trials 11-20	“up” tacks				
	total trials	10	50	100	1

In the table above, you had to calculate the number of “up” tosses out of 1. That may seem strange, since the “up” number is less than 1. However, the ratio $\frac{6 \text{ up}}{10 \text{ total}}$ is equivalent to the decimal 0.6. Probability is often recorded as a decimal less than 1.

8) Based on trials 11 through 20, how many times do you predict that a tack would land “up” out of 100 tosses?

Here are the results of trials 21 through 30.



9) Complete the table.

		original ratio	“out of 50”	“out of 100”	“out of 1”
Trials 21-30	“up” tacks				
	total trials	10	50	100	1

10) Based on these 10 tosses, how many times would a tack land “up” out of 100 tosses?

- 11) The experimental probability of $\frac{7 \text{ up}}{10 \text{ total}}$ is based on the results of trials 21 through 30. What is the percent equivalent of this ratio?

A. 0.7%

C. 35%

B. 7%

D. 70%

So far, we have tossed 10 tacks three times for a total of 30 trials. Each set of 10 trials had a different experimental probability.

- 12) Complete the table.

	Trials 1-10	Trials 11-20	Trials 21-30
“up” tacks out of 10	3		7
part-whole ratio	$\frac{3}{10}$	$\frac{6}{10}$	
“out of 1” - decimal equivalent		0.6	0.7
“out of 100” - percent equivalent			70

It is normal for the experimental probability to be different each time you do an experiment. But how can we make predictions using experimental probability if it keeps changing? Our predictions for the number of “up” tacks out of 100 are 30, 60, and 70. Which prediction is best? Or is there another prediction that would be better? We will try to answer this together.

In probability, you will need to be able to convert numbers from their original ratios into equivalent ratios of decimals and percents.

- 13) Practice converting $\frac{7}{20}$ into a decimal equivalent and a percent.

	original	decimal	percent
"up" tacks	7		
total trials	20	1	100

$\div 20$ $\times 100$

$\div 20$ $\times 100$

What if we put our tack-tossing trials together and calculate new experimental probabilities? Let's start by looking at the first 20 trials.

- 14) Complete the table for trials 1 through 20.

		total for trials 1-20	original ratio	"out of 1"	"out of 100"
Trials 1-20	"up" tacks	3 + 6	9		
	total trials	10 + 10	20	1	100

- 15) Let's look at the first 30 trials now. Complete the table below for trials 1 through 30.

		total for trials 1-30	original ratio	decimal equivalent	percent
Trials 1-30	"up" tacks	3 + 6 + 7		0.53	
	total trials	10 + 10 + 10	30	1	100

- 16) The experimental probability of $\frac{9 \text{ up}}{20 \text{ total}}$ is based on the results of trials 1 through 20.

What is the decimal equivalent of this ratio?

- A. 0.9
B. 0.45
C. 45
D. 90

- 17) The experimental probability of $\frac{16 \text{ up}}{30 \text{ total}}$ is based on the results of trials 1 through 30.

What is the percent equivalent of this ratio?

- A. 0.53%
B. 16%
C. 53%
D. 104%

Being Counted: Probability & Statistics (Part 1)

Below is the data for 100 tosses of the tacks. Each row shows the total number of “up” tacks and tosses so far.

- 18) Complete the table. Round to the hundredths place. For example, round the decimal 0.575 to 0.58.

	Number of Up Tacks	Total Tosses	Relative Frequency of “Up” Tacks as a Decimal	Relative Frequency of “Up” Tacks as a Percent
Trials 1-10	3	10	0.30	30%
Trials 1-20	9	20	0.45	45%
Trials 1-30	16	30	0.53	53%
Trials 1-40	23	40	0.58	
Trials 1-50	31	50		
Trials 1-60	35	60		
Trials 1-70	41	70		
Trials 1-80	46	80		
Trials 1-90	51	90		
Trials 1-100	58	100		

- 19) What do you notice when you look at the data above?

You may have noticed that the experimental probability changes less as we have more and more trials. Once we had more than 50 trials, the relative frequency of “up” tacks is between 56% and 58%. When you do more trials in a probability experiment, the experimental probability changes less and less.

If we have a small number of trials, like we did at the beginning, the experimental probability can change a lot. In our experiment above, there were only 10 trials in each group. When we did more trials, the experimental probability started changing less.

- 20) We tossed even more tacks to see what would happen. Analyze the data for larger numbers.

Number of Up Tacks	Total Trials	Experiment Ratio	Relative Frequency of “Up” Tacks as a Decimal	Relative Frequency of “Up” Tacks as a Percent
121	200	$\frac{121}{200}$	0.605	
307	500			
611	1,000			

- 21) Imagine that you are tossing tacks in a new experiment. Based on the data above, what is the best prediction for the number of “up” tacks out 100 tosses?

Shooting Free Throws

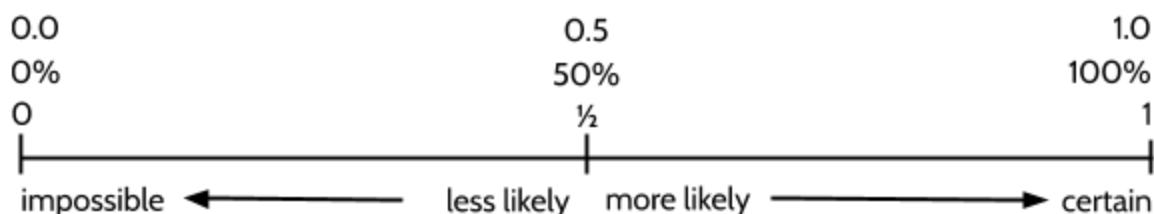
When we think about experimental probability, sometimes the experiment has already been done for us. Steph Curry is a player for the Golden State Warriors basketball team. As of February 2020, he had the best free throw⁷ percentage of anyone in the history of the National Basketball Association (NBA). Over his 11-year career, Curry had scored 2552 free throws out of 2819 free throw attempts.



22) Curry's experimental probability of making free throws was:

$$\frac{2552 \text{ free throws made}}{2819 \text{ free throw attempts}} = 0.______ \text{ or } _____\%$$

23) How likely is it that Curry will make a free throw basket? Make a mark on the probability line below to show the likelihood that he will make a basket.



The experimental probability calculated above tells us about how many free throws we can expect Steph Curry to make. For every 100 free throws he takes, we can expect that he will make about 90 shots.

But probability can't predict exactly what will happen each time he shoots. If Curry comes up to shoot a free throw, we don't know if he will make this particular basket. Probability gives us information about the likelihood that an event will happen, but it can't take the mystery out of life. Will Curry make the shot? Probably. Do we know for sure? No way.

⁷ In basketball, free throws (also known as a foul shot) are when a player is allowed to shoot a basket from behind the free throw line. Free throws are usually given after a foul by the opposing team.

The Colors of M&Ms

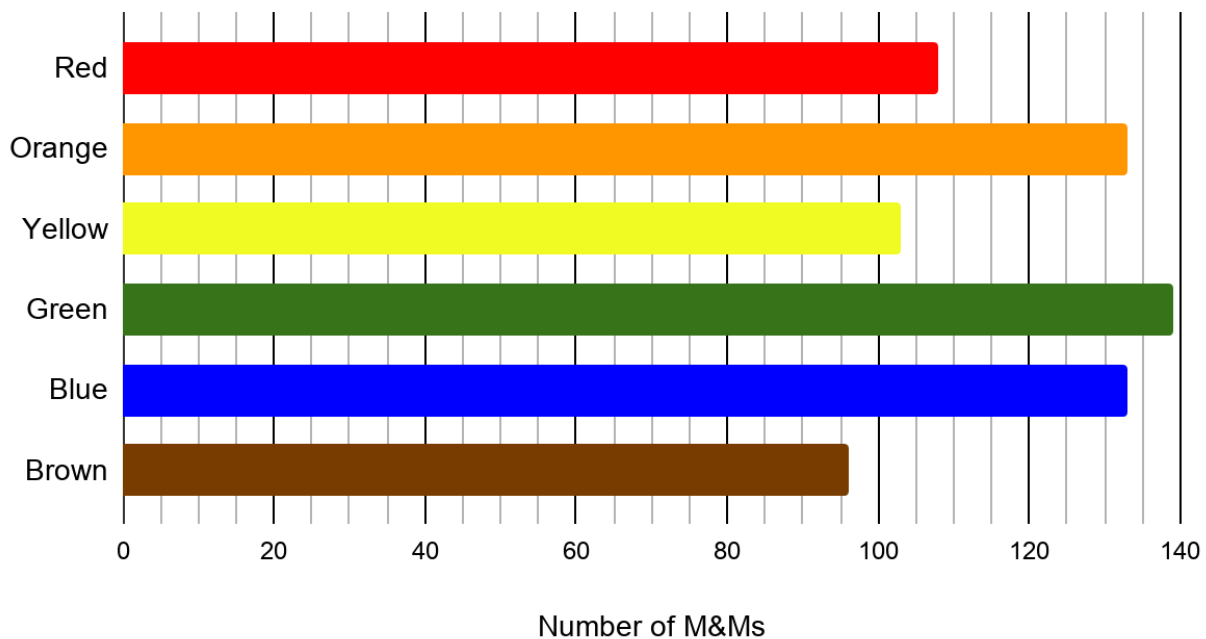
Have you looked at a bag of M&Ms and wondered how many there are of each color? And if every bag has the same number of each color?

Rick Wicklin is a computer programmer who became interested in the *distribution*⁸ of M&Ms. In 2017, he worked for a company that had free bowls of M&Ms available in the break room. He collected one scoop each week and counted the colors. Eventually, he collected 712 M&Ms.⁹



When Rick counted the number of each color, he found the following color distribution.

M&Ms color distribution in Rick Wicklin's experiment



- 24) Which color M&M had the highest frequency in Rick's experiment? About how many M&Ms of this color were there?

⁸ The *distribution* is the number or percentage of each category in a set of data.

⁹ Wicklin, The distribution of colors for plain M&M candies,
<https://blogs.sas.com/content/iml/2017/02/20/proportion-of-colors-mandms.html>

After Rick calculated the relative frequency of each color, he contacted the company that makes M&Ms and asked them for the color distribution of M&Ms overall. The company wrote back and told him that M&Ms are produced at two factories in the United States, one in Ohio and one in New Jersey. They said that the color distributions are different for each factory.

Ohio factory

	Relative Frequency
Red	13.1%
Orange	20.5%
Yellow	13.5%
Green	19.8%
Blue	20.7%
Brown	12.4%

New Jersey factory

	Relative Frequency
Red	12.5%
Orange	25.0%
Yellow	12.5%
Green	12.5%
Blue	25.0%
Brown	12.5%

- 25) Look back at the graph on the previous page. Fill in the approximate number of candies of each color, then calculate the missing percents.

	Red	Orange	Yellow	Green	Blue	Brown
Frequency	108		103	139		
Total Number of M&Ms	712	712	712	712	712	712
Relative Frequency			14%			13%

- 26) Rick decided that the M&Ms must have come from one of the factories. Which one do you think it was? Explain your answer.

Flipping Coins

When you flip a coin, it will either come up “heads” or “tails.” These are the two possible outcomes of flipping a coin.



Heads

Tails

- 27) Have you ever flipped a coin to help you make a decision? Explain what happened.

- 28) Why do you think people flip coins to make decisions?

The chances of getting “heads” is the same as the chances of getting “tails”. The two outcomes are equally likely. So, what happens if you start flipping coins? Do you always get half “heads” and half “tails”?

- 29) Flip a coin 10 times. Record the outcome of each flip in the table on the right. After each flip, put a checkmark ✓ in the heads or tails column.

- 30) Did half of your flips come up “heads”?

	heads	tails
1st Flip		
2nd Flip		
3rd Flip		
4th Flip		
5th Flip		
6th Flip		
7th Flip		
8th Flip		
9th Flip		
10th Flip		
Totals		

John Kerrich (1903-1985) was an English mathematician who visited Denmark during World War II in 1940 and was stuck there when the Nazis invaded. He ended up in an internment camp during the war. While he was there, he conducted a series of probability experiments to keep himself busy. In one of his experiments, he flipped a coin 10,000 times! He kept track of the results and published his experiment after the war.¹⁰

31) In the first 25 flips, Kerrich got 13 heads. Which of these show the relative frequency of “heads” for these trials? Choose all that apply.

A. $\frac{13}{25}$

C. 0.52

B. 52%

D. 0.520

The decimal 0.520 may look a little funny, but it is one correct way to write 13 out of 25. It is accurate to the thousandths (3rd decimal) place.

32) Here is some sample data from Kerrich’s 10,000 flip experiment. Complete the table.

Number of Trials	Number of Heads	Relative Frequency as a Decimal
10	4	0.400
50	25	
100	44	
500	255	
1,000	502	
5,000	2533	
10,000	5067	

33) What do you notice?

¹⁰ Thank you to Steve Hinds for telling us this story. Visit https://en.wikipedia.org/wiki/John_Edmund_Kerrich for more information.

Experimental Probability - Answer Key

- 1) There is no right answer at this point. It might be impossible to know just by looking at a tack. We have to do some experiments to find out.
- 2) 3 up, 7 sideways
- 3) We might guess that 6 tacks will land “up.”

4)

Trials 1-10	“up” tacks	3	6	9	15	1.5
	total trials	10	20	30	50	5

- 5) Based on the ratio so far, there would be 30 “up” tacks.

6)

“up” tacks	3	24
total trials	10	80

7)

		original ratio	“out of 50”	“out of 100”	“out of 1”
Trials 11-20	“up” tacks	6	30	60	0.6
	total trials	10	50	100	1

- 8) 60

9)

		original ratio	“out of 50”	“out of 100”	“out of 1”
Trials 21-30	“up” tacks	7	35	70	0.7
	total trials	10	50	100	1

10) 70

11) D

12)

	Trials 1-10	Trials 11-20	Trials 21-30
"up" tacks out of 10	3	6	7
part-whole ratio	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{7}{10}$
"out of 1" - decimal equivalent	0.3	0.6	0.7
"out of 100" - percent equivalent	30	60	70

	original	decimal	percent
	$\div 20$	$\times 100$	
"up" tacks	7	0.35	35
total trials	20	1	100
	$\div 20$	$\times 100$	

13)

14)

		total for trials 1-20	original ratio	"out of 1"	"out of 100"
Trials 1-20	"up" tacks	3 + 6	9	0.45	45
	total trials	10 + 10	20	1	100

15)

		total for trials 1-30	original ratio	decimal equivalent	percent
Trials 1-30	"up" tacks	3 + 6 + 7	16	0.53	53
	total trials	10 + 10 + 10	30	1	100

16) B

17) C

18)

	Number of Up Tacks	Total Tosses	Relative Frequency of "Up" Tacks as a Decimal	Relative Frequency of "Up" Tacks as a Percent
Trials 1-10	3	10	0.30	30%
Trials 1-20	9	20	0.45	45%
Trials 1-30	16	30	0.53	53%
Trials 1-40	23	40	0.58	58%
Trials 1-50	31	50	0.62	62%
Trials 1-60	35	60	0.58	58%
Trials 1-70	41	70	0.59	59%
Trials 1-80	46	80	0.58	58%
Trials 1-90	51	90	0.57	57%
Trials 1-100	58	100	0.58	58%

19) One thing you might notice is that the relative frequency of "up" tacks stays between 57% and 59% after 60 tosses. The experimental probability changes more earlier in the experiment. After more trials, the experimental probability doesn't change as much.

20)

Number of Up Tacks	Total Trials	Experiment Ratio	Relative Frequency of "Up" Tacks as a Decimal	Relative Frequency of "Up" Tacks as a Percent
121	200	$\frac{121}{200}$	0.605	60.5%
307	500	$\frac{307}{500}$	0.614	61.4%
611	1,000	$\frac{611}{1000}$	0.611	61.1%

21) 61 tacks seems like the best guess, since that was the relative frequency of “up” tacks out of 1,000 tosses, the most we have thrown. The higher the number of trials, the more confident you can be in the experimental probability.

22) $\frac{2552 \text{ free throws made}}{2819 \text{ free throw attempts}} = 0.905... \text{ or } 90.5\%$

23) Your mark should be about $\frac{1}{2}$ inch to the left of 100%.

24) Green. 139.

25)

	Red	Orange	Yellow	Green	Blue	Brown
Frequency	108	133	103	139	133	96
Total Number of M&Ms	712	712	712	712	712	712
Relative Frequency	15%	19%	14%	20%	19%	13%

26) The relative frequencies from Rick’s sample of 712 M&Ms are more similar to the Ohio factory’s relative frequency than to the color distribution at the New Jersey factory. For example, orange in Rick’s sample is 19%. In Ohio, orange is 20.5% and in New Jersey, it is 25%. The color distribution in Rick’s sample is closer to Ohio in every color except brown, but the percentages for brown are nearly identical at the two factories.

27) Answers will vary.

28) Probably because there is an equal chance of “heads” or “tails” so it’s a fair way to make a random decision. And coins are common, so people usually have one.

29) Answers will vary.

30) Our guess is that you didn’t get exactly 5 “heads.” That will happen sometimes, of course, but most of the time you won’t get exactly half. However, if you keep flipping coins, you will usually get something close to half.

31) A, B, C, D. All of the above.

32)

Number of Flips	Number of Heads	Relative Frequency
10	4	0.400
50	25	0.500
100	44	0.440
500	255	0.510
1,000	502	0.502
5,000	2533	0.507
10,000	5067	0.507

33) You may notice that the relative frequency changed more when there were fewer flips. Later on, after there are thousands of flips, the relative frequency changed less. The more trials in a probability experiment, the less the relative frequency will change. This is why it is important to do a large number of trials to find out the experimental probability.

You may also notice that the experimental probability was never exactly 50%, even when there were 10,000 trials.

Theoretical Probability

Rolling Dice

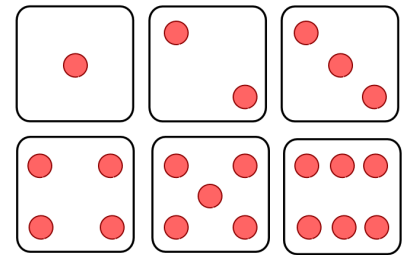
Sometimes we can figure out the exact probability of an event without conducting any experiments and without collecting any data. For these events, we can calculate the chances by analyzing the event. If we know all the possible outcomes, we can calculate the probability by using a part-whole ratio similar to the one you used to calculate relative frequency.

$$\text{Probability of an event} = \frac{\text{The number of outcomes that result in that event}}{\text{The total number of all possible outcomes}}$$

For example, consider a six-sided die.



When we roll a six-sided die, we know all of the possible outcomes. With every roll, we will get a 1, 2, 3, 4, 5, or a 6.



If we roll a single die, what is the probability of rolling a 5? We could create an experiment and roll the die 100 times and use our results to calculate a relative frequency.

However, we don't have to do this because we know the total number of possible outcomes and they are each equally likely. There are 6 possible outcomes and one of those outcomes is rolling a 5. The probability is 1 outcome out of 6 possible outcomes, or $\frac{1}{6}$.

$$\begin{array}{ccc} \text{Probability of an event} = \frac{\text{The number of outcomes that result in that event}}{\text{The total number of all possible outcomes}} & \longrightarrow & \text{Probability of rolling a 5 on a six-sided die} = \frac{1}{6} \end{array}$$

1) What is the probability of rolling a die and getting a 1?

2) Complete the table.

Rolling a...	1	2	3	4	5	6
Probability	$\frac{1}{6}$				$\frac{1}{6}$	

You probably noticed that when rolling a die, the probability of rolling any number is $\frac{1}{6}$. All of the possible outcomes are **equally likely**. That means you are just as likely to roll a 6 as you are to roll a 3.

3) What do you get when you add up all of the probabilities?

When you roll a die, you are guaranteed to get 1, 2, 3, 4, 5, or 6. It is certain that you will get one of those numbers. So the sum of all 6 probabilities is 1.

For any situation, if you add the probability of something happening to the probability of it not happening, the sum will be 1.

Another example of this is flipping a coin. When you flip a coin, you have a $\frac{1}{2}$ probability of the coin landing on heads, and a $\frac{1}{2}$ probability of the coin not landing on heads. $\frac{1}{2} + \frac{1}{2} = 1$

Calculate the probability of the following events, when rolling a single six-sided die.

4) What is the probability of not rolling a 4?

5) What is the probability of rolling an odd number?

- 6) What is the probability of getting a number greater than 2?
- 7) When rolling a normal single die, what are the chances of rolling a 7?
- 8) If a six-sided die is rolled once, which of the following is the least likely to occur?
- A. Rolling an odd number
 - B. Rolling an even number
 - C. Rolling a number less than 6
 - D. Rolling a number greater than 4
- 9) If the probability that it will rain next Wednesday is $\frac{5}{6}$, what is the probability that it will *not* rain on Wednesday?
- A. 0
 - B. $\frac{1}{6}$
 - C. $\frac{5}{6}$
 - D. 1
- 10) If a six-sided die is rolled once, which event has a probability of 0?
- A. rolling an even number less than 5
 - B. rolling an even number larger than 5
 - C. rolling an odd number less than 5
 - D. rolling an odd number larger than 5

Sample Space

We often need to know the total number of possible outcomes when we calculate the probability of an event. Mathematicians refer to the total number of possible outcomes as the *sample space*. When we roll a single die, it is easy to calculate the sample space. We can count the number of sides and see there is a sample space of 6 possible outcomes. What about situations where there are more possibilities in the sample space?

In this section, you will practice a few strategies for counting all of the possibilities in a sample space. Try this problem.

Coin Combinations

- 11) I have 25 cents in my pocket. How many different possible combinations of coins are there?

Record all the possibilities.

			
Penny	Nickel	Dime	Quarter
.01	.05	.10	.25

Being Counted: Probability & Statistics (Part 1)

There are 13 possible ways to make 25 cents. Did you find them all?

When finding the sample space of possible outcomes, we need to identify all of the possibilities. To stay organized, we can record the possibilities in a chart. This helps us keep track of all the possibilities and identify any that we may have missed.

In the chart below, each row represents one way to make 25 cents. Two rows have been completed as an example. Can you find the other 11 possibilities in our sample space?

12) Complete the chart.

Quarters	Dimes	Nickels	Pennies
1	0	0	0
0	2	1	0

Once we know the sample space, we can figure out the probability of having the different combinations.

If I have 25 cents in my pocket...

- 13) Three of the possible ways of making 25 cents include exactly 5 pennies. What is the probability that I have exactly 5 pennies in my pocket?
- 14) What is the probability that I have an odd number of coins in my pocket?
- A. $\frac{1}{13}$ C. $\frac{9}{13}$
- B. $\frac{1}{25}$ D. $\frac{9}{25}$
- 15) What is the probability that I have fewer than 10 coins in my pocket?
- 16) What is the probability that I have exactly 4 coins in my pocket?
- 17) Which possibility has a $\frac{6}{13}$ probability of being true?
- A. Having at least one quarter C. Having at least one nickel
- B. Having at least one dime D. Having at least one penny

Dice Battles

Two students decided to play a few games with dice. To play, they roll two dice and then add the two dice together to get the sum.

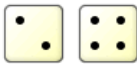











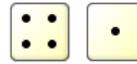
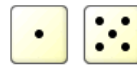








Rules of Game One

If the sum is 3, 5, 7, 9, or 11, Player A gets a point.

If the sum is 2, 4, 6, 8, 10, or 12, Player B gets a point.

- 18) Look carefully at the rules of the game. Which player do you think will win? Why?
- 19) The two students play a game with 20 rolls of the dice. Complete the table with their rolls below. Total the scores at the end.

Roll	Sum of two dice	Player A	Player B
	6		✓
	5	✓	
	10		✓
			
			
			
			
			
			
			

Roll	Sum of two dice	Player A	Player B
			
			
			
			
			
			
			
			
			
			
Total			

To play a dice game yourself, you can use <http://www.random.org/dice/> to get random dice rolls.

Player A got 12 points and Player B got 8 points. Player A won the game even though they scored only with five possible sums (3, 5, 7, 9, 11) compared with Player B who scored with six possible sums (2, 4, 6, 8, 10).



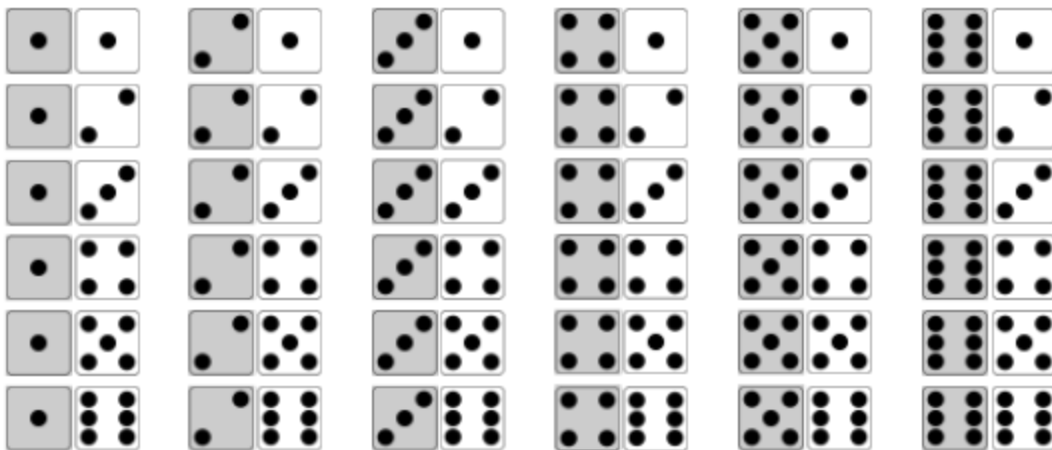
20) Were you surprised by who won? Why or why not?

Earlier, you calculated probabilities for rolling a single die. When rolling a single six-sided die, there is a sample space of six possible outcomes. A single die can come up with 1, 2, 3, 4, 5, or 6 dots. To calculate the probabilities for rolling two dice, we need to find out how many possible outcomes are in the sample space.

21) How many possible outcomes do you think are in the sample space when you roll a pair of dice?

My best guess is that there are _____ possible outcomes when you roll two dice.

Look carefully at the image below.



22) What do you notice?

The image above shows all the possible outcomes when rolling two six-sided dice.

23) How many possible outcomes are there when rolling two six-sided dice?

- A. 2
- B. 6
- C. 12
- D. 36

24) How many of these outcomes include at least one die with 2 dots?

- A. 2
- B. 11
- C. 18
- D. 36

Once you know the answer to the last two questions, you can figure out the theoretical probability of rolling at least one 2 with two six-sided dice.

$$\begin{array}{lcl} \text{Probability} & & \text{Probability of} \\ \text{of an event} & = & \text{rolling at least} \\ & \frac{\text{The number of outcomes}}{\text{The total number of all}} & \text{one 2 with two} \\ & \text{possible outcomes} & \text{six-sided dice} \\ & \longrightarrow & = \frac{11}{36} \end{array}$$

25) What is the probability of at least one of the numbers being a 5?

- A. $\frac{1}{36}$
- B. $\frac{6}{36}$
- C. $\frac{11}{36}$
- D. $\frac{12}{36}$

26) What is the probability that both of dice show the same number? Example:



- A. $\frac{1}{36}$
- B. $\frac{6}{36}$
- C. $\frac{11}{36}$
- D. $\frac{12}{36}$

27) What is the probability of rolling two dice that add up to 9? Example:



- A. $\frac{4}{36}$
- B. $\frac{9}{36}$
- C. $\frac{11}{36}$
- D. $\frac{18}{36}$

There are different ways to analyze the sums of two dice. We will use an addition table first. An addition table is like a multiplication table, except that the numbers are added instead of multiplied.

28) Complete the addition table by adding two numbers. Some examples are done for you.

Addition Table

		first die					
second die	+	1	2	3	4	5	6
	1	2	3				
	2	3					
	3			6			
	4					9	
	5		7				
	6						

What do you notice?

29) How many different ways are there to roll a sum of 10?

30) What is the probability of rolling a sum of 10?

31) Which is more likely, the dice adding up to 3 or the dice adding up to 8?

Explain your answer.

32) If you roll the two dice, what sum do you have the highest probability of rolling?

Explain your answer.

You can also use a table to show the possible ways of rolling different sums on two dice.

33) Fill in the missing possible ways of making each sum, then enter the number of ways below.

Table of Combinations

Sum	2	3	4	5	6	7	8	9	10	11	12
Possible Ways	(1, 1)	(1, 2) (2, 1)	(1, 3) (2, 2) (3, 1)								
Number of Ways	1	2									

What do you notice?

Think about the dice battle game from a few pages back. Each player rolls two dice and scores points according to these rules.

Rules of Game One

If the sum is 3, 5, 7, 9, or 11, Player A gets a point.

If the sum is 2, 4, 6, 8, 10, or 12, Player B gets a point.

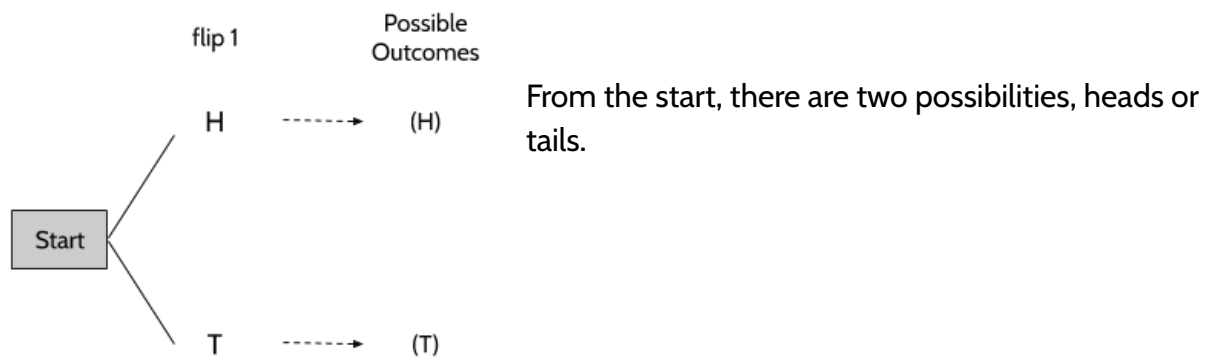
34) Is this a fair game? Why or why not?

Tree Diagrams

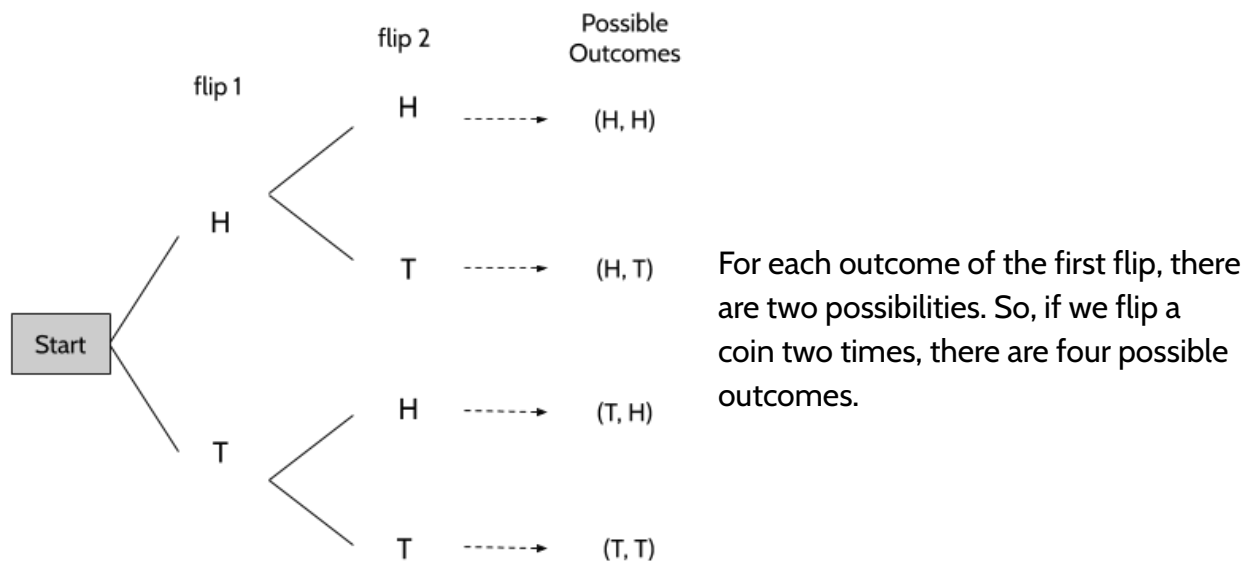
Another tool that can help us determine the possible outcomes (or sample space) is a tree diagram. If we flip a coin, there are only two possible outcomes, heads or tails. We have a $\frac{1}{2}$ probability of it landing on heads, and a $\frac{1}{2}$ probability of it landing on tails.

But what if we flip a coin twice? How many different possibilities are there? A tree diagram can help us answer questions like these.

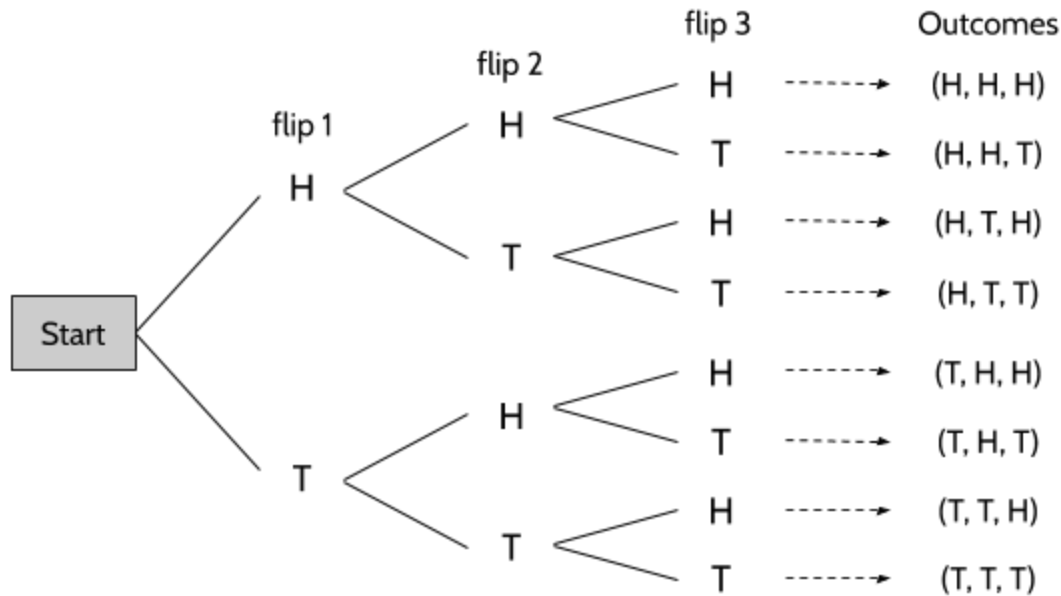
We start with the first flip:



Now for the second flip.



We can continue tree diagrams to record additional experiments. If we flip a coin three times, our tree diagram would look something like this:



If you flip a coin three times, what is the probability of...

35) ...getting three tails?

36) ...getting exactly two heads?

37) ...getting tails for the first and last flip?

38) How much do the number of possible outcomes change with each additional flip?
There are 2 possible outcomes with one flip, 4 possible outcomes with two flips, ...

39) How many possible outcomes are there if you flip a coin 4 times?

Explain your answer.

- 40) Draw a tree diagram showing all of the possible outcomes of flipping a coin 4 times.
- 41) Without drawing a tree diagram, how many possible outcomes are there for flipping a coin 10 times?
- 42) Before every soccer game, there is a coin toss. The team that correctly guesses heads or tails gets to choose which goal they want to attack. For the last six games of the Liverpool Field Club, the coin toss has landed on heads. What is the probability that the coin will land on heads for the next game?
- A. $\frac{1}{7}$
- B. $\frac{1}{6}$
- C. $\frac{1}{5}$
- D. $\frac{1}{2}$

- 43) Kate works an early shift. In order to start her shift at 6am, she has to leave her house at 5am. This means she has to get dressed in the dark, because she doesn't want to wake her wife.

In Kate's closet, she has the following clothes:

Shirts	Pants
blue shirt brown shirt black shirt red shirt purple shirt	blue pants brown pants

Since it is too dark to see, Kate has to choose her clothes by feel. She grabs a shirt and a pair of pants, but she can't see the color until she is already dressed. The shirts all feel the same, so she has an equal chance of grabbing any of them. The same is true for her pants.

Make an organized list or a tree diagram to determine all of the possible combinations of shirts and pants.

- 44) Kate also has 3 pairs of socks.

Socks
gray socks blue socks brown socks

How many combinations are possible now?

- 45) What is the probability that Kate gets dressed in an all blue outfit (shirt, pants, and socks)?
- 46) What is the probability that Kate is wearing at least one item of blue clothing (shirt, or pants or socks)?
- 47) What is the probability that Kate is not wearing a piece of red clothing?

-
- 48) An ice cream shop offers customers the choice of a cup or a cone. It offers a choice of three different flavors: chocolate, vanilla, or strawberry. It also offers three different toppings: sprinkles, nuts, or hot fudge. How many different combinations result from choosing a cone or a cup, one kind of ice cream, and one topping?

Probability and Multiple Choice Questions

When you take a standardized test like the high school equivalency exam, what are the chances of getting the right answer by making an educated guess?

Let's consider a multiple choice question where there are 4 answer options.

49) If you have no idea and make a completely random guess, what are your chances of being correct?

- A. 25%
- B. 50%
- C. 75%
- D. 100%

Sometimes when we are working on multiple choice questions, we can eliminate one of the choices.

50) If you are able to eliminate 1 of the 4 choices, what are your chances of being correct?

- A. $\frac{1}{3}$
- B. $\frac{1}{2}$
- C. $\frac{2}{3}$
- D. $\frac{3}{4}$

51) If you are able to eliminate 2 of the 4 choices, which of the following has the same probability of you choosing the correct answer? (Choose all possible answers)

- A. Flipping a coin and getting heads.
- B. Rolling a die and getting a 4.
- C. Rolling a die and getting an odd number.
- D. Rolling a die and getting a 5 or a 6.

Theoretical Probability - Answer Key

1) $\frac{1}{6}$

2)

Rolling a...	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

3) $\frac{6}{6}$ or 1

4) $\frac{5}{6}$

5) $\frac{3}{6}$ or $\frac{1}{2}$

6) $\frac{4}{6}$ or $\frac{2}{3}$

7) 0

8) D. There is a $\frac{2}{6}$ (or $\frac{1}{3}$) chance of rolling a number greater than 4. There are six possible outcomes and 2 of them are greater than 4.

9) B. If there is a $\frac{5}{6}$ chance that it will rain on Wednesday, then there is a $\frac{1}{6}$ chance it will *not* rain. It will either rain on Wednesday or it will not rain. Those are the only two options and one of them will definitely happen. Since it is certain that on Wednesday it will either rain or not rain, the probability is one. If there is a 5 out of 6 chance of rain, then there is a 1 out of 6 chance that it will not rain.

10) D. 7 is the smallest odd number larger than 5 and there are no 7's on a six-sided die.

11) There are 13 possible ways to make 25 cents with quarters, dimes, nickels, and pennies.

12)

Quarters	Dimes	Nickels	Pennies
1	0	0	0
0	2	1	0
0	2	0	5
0	1	3	0
0	1	2	5
0	1	1	10
0	1	0	15
0	0	5	0
0	0	4	5
0	0	3	10
0	0	2	15
0	0	1	20
0	0	0	25

13) $\frac{3}{13}$

14) C

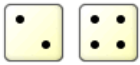
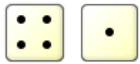








15) $\frac{7}{13}$

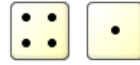
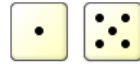








16) $\frac{1}{13}$

17) B

18) Make your best guess.

19)

Roll	Sum of two dice	Player A	Player B
	6		✓
	5	✓	
	10		✓
	7	✓	
	11	✓	
	9	✓	
	3	✓	
	4		✓
	10		✓
	11	✓	

Roll	Sum of two dice	Player A	Player B
	5	✓	
	6		✓
	7	✓	
	10		✓
	5	✓	
	2		✓
	7	✓	
	9	✓	
	10		✓
	5	✓	
Total		12	8

20) Answers will vary.

21) You might have guessed 11 possible outcomes, because there are 11 different possible sums with two dice: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. However, there are actually 36 different ways of combining the rolls of two dice. That means there are 36 different possible events in the sample space.

22) There are many things you might notice:

- The grey dice are arranged in columns of 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, and 6 dots.
- The white dice are arranged in rows of 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, and 6 dots.
- Each combination of a grey die and a white die is unique. For example, there is only 1 combination that has a grey die with 2 dots and a white die with 3 dots.

Being Counted: Probability & Statistics (Part 1)

- There are 36 total combinations.
- The image shows all the possible outcomes when rolling two six-sided dice.

23) D

24) B. The 11 possible rolls that have at least one die with 2 dots are 2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 1-2, 3-2, 4-2, 5-2, and 6-2.

25) C

26) B

27) A. The 4 possible rolls are 3-6, 4-5, 5-4, and 6-3.

28)

		first die					
second die	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

There are many things you might notice:

- The sums are repeated in diagonal lines.
- Some sums happen more than others. For example, there is only one way to get a sum of 2 and there are 4 ways to get a sum of 9.
- There are more 7's than any other number.

29) 3. You can roll a 4-6, 5-5, or 6-4.

30) $\frac{3}{36}$, $\frac{1}{12}$, or 8.3%. Each of these probabilities are equivalent.

31) Rolling a sum of 8 is more likely since there are five ways to roll a sum of 8 and there are only two ways to roll a sum of 2.

32) 7 is the most likely sum, since there are six different ways to roll a sum of 7. No other sum has than many different possibilities.

33)

Sum	2	3	4	5	6	7	8	9	10	11	12
Possible Ways	(1, 1)	(1, 2) (2, 1)	(1, 3) (2, 2) (3, 1)	(1, 4) (2, 3) (3, 2) (4, 1)	(1, 5) (2, 4) (3, 3) (4, 2) (5, 1)	(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)	(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)	(3, 6) (4, 5) (5, 4) (6, 3)	(4, 6) (5, 5) (6, 4)	(5, 6) (6, 5)	(6, 6)
Number of Ways	1	2	3	4	5	6	5	4	3	2	1

There are many things to notice:

- The sums are the same as in the addition table.
- The sums look like a pyramid or triangle.
- 7 has the highest number of ways of being the sum of two dice.

34) It is a fair game. Both players have an equal chance of winning.

Player A	Player B
3 -> 2 possible outcomes 5 -> 4 possible outcomes 7 -> 6 possible outcomes 9 -> 4 possible outcomes 11 -> 2 possible outcomes <hr/> total of 18 possible outcomes out of 36	2 -> 1 possible outcome 4 -> 3 possible outcomes 6 -> 5 possible outcomes 8 -> 5 possible outcomes 10 -> 3 possible outcomes 12 -> 1 possible outcome <hr/> total of 18 possible outcomes out of 36

35) $\frac{1}{8}$

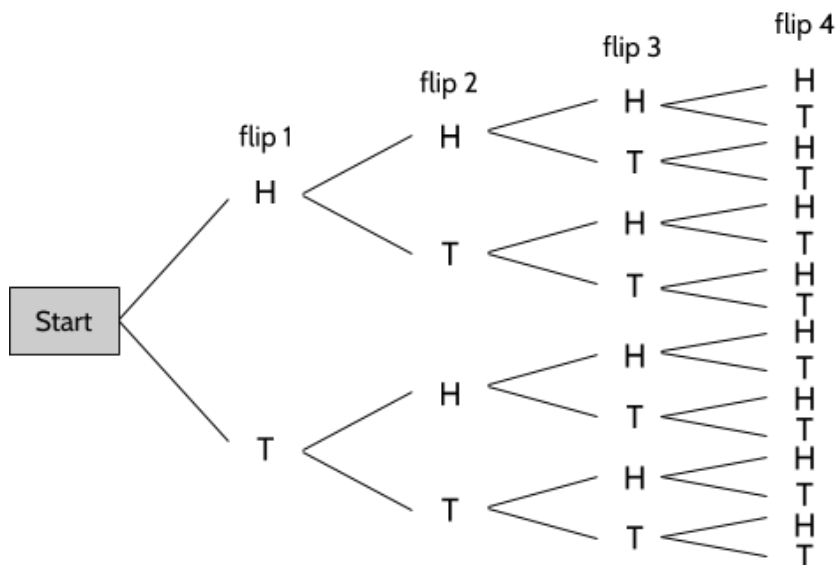
36) $\frac{3}{8}$

37) $\frac{2}{8}$ or $\frac{1}{4}$

38) There are twice the number of outcomes with each additional flip.

39) There are 16 possible outcomes if you flip a coin four times. Three flips results in a sample space of 8 possibilities.

40)



41) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$ possibilities.

42) Choice D. The results of previous coin tosses have no impact on the probability of the coin landing on heads. Every time you flip a coin, you have a 50% chance of getting heads and a 50% chance of getting tails.

- 43) Kate has 10 possible outfits for work.

Here's one way of listing the combinations:

blue shirt & blue pants
brown shirt & blue pants
black shirt & blue pants
red shirt & blue pants
purple shirt & blue pants
blue shirt & brown pants
brown shirt & brown pants
black shirt & brown pants
red shirt & brown pants
purple shirt & brown pants

And here's another way:

blue shirt & blue pants
blue shirt & brown pants
brown shirt & blue pants
brown shirt & brown pants
black shirt & blue pants
black shirt & brown pants
red shirt & blue pants
red shirt & brown pants
purple shirt & blue pants
purple shirt & brown pants

- 44) By adding three pairs of socks, there are 30 possible combinations.

45) $\frac{1}{30}$

46) $\frac{22}{30}$ or $\frac{11}{15}$

47) $\frac{24}{30}$ or $\frac{4}{5}$

- 48) 18 possible combinations

Chocolate, Sprinkles, Cone	Vanilla, Sprinkles, Cone	Strawberry, Sprinkles, Cone
Chocolate, Sprinkles, Cup	Vanilla, Sprinkles, Cup	Strawberry, Sprinkles, Cup
Chocolate, Nuts, Cone	Vanilla, Nuts, Cone	Strawberry, Nuts, Cone
Chocolate, Nuts, Cup	Vanilla, Nuts, Cup	Strawberry, Nuts, Cup
Chocolate, Fudge, Cone	Vanilla, Fudge, Cone	Strawberry, Fudge, Cone
Chocolate, Fudge, Cup	Vanilla, Fudge, Cup	Strawberry, Fudge, Cup

- 49) A

- 50) A

- 51) A and C

Vocabulary Review

Glossary

You can use this section to look up words used in this math packet.

average (noun): See <MODE>, <MEDIAN>, and <MEAN> below.

cause (verb): to make something happen

causation (noun): When something happens because of something else.

census (noun): a census is an official count involving every member of a population. The US conducts the US Census every 10 years to count all the people living in the United States.

certain (adjective): Something that is certain is something that is sure to happen. We say something that is certain has a probability of 1 or 100%. For example, It is certain that if you drop a drinking glass, it will fall.

chance (adjective): the likelihood that something will happen. For example, “What are the *chances* that it will snow tomorrow?”

coin (noun): In the United States, we have the following coins: cents (pennies), nickels, dimes, quarters, half dollars, and dollar coins. Each of these coins has a *head* and a *tail*, the two sides of the coin. The heads of US coins refers to the side with the faces or heads of former presidents and leaders.

combination (noun): A combination is any of the ways we can combine things when order does not matter. For example, if we have a pair of red pants, a pair of blue pants, a red shirts and a blue shirt, the following 4 combinations are possible: {red shirt, red pants}, {red shirt, blue pants}, {blue shirt, red pants}, {blue shirt, blue pants}

correlation (noun): A relationship or connection between two things. Sometimes, one thing causes the other thing. There is a correlation between the amount of exercise and the number of calories burned. Exercise causes calories to be burned. Sometimes, there is a connection, but one doesn't cause the other. There is a correlation between the amount of ice cream sold and the number of sunglasses sold. Increased ice cream sales doesn't cause people to buy sunglasses, or vice-versa.

data (noun): Facts that can be analyzed or used to gain knowledge or make decisions; information.

denominator (noun): The bottom number in a fraction. Shows how many equal parts the item is divided into.

dice (noun): A die is a solid object with numbers or dots on each face. The most common dice are cubes with dots on them of the numbers 1 - 6. Dice can be used to generate random numbers because there is an equal chance of rolling a 1, 2, 3, 4, 5, or a 6.



die: This is the singular of “dice.” Example: Minerva rolled one die to see which child would go down the slide first.

difference (noun): How much one number differs from another. One way to visualize difference is how far one number is from another on a number line. For example, Paulina is 35 and her son is 11, so there is a 24 year age difference between them.



distribution (noun): The number or percentage of each group in a set of data. For example, the Mars, Inc. company has said that the distribution of M&Ms is 24% blue, 20% orange, 16% green, 14% yellow, 13% red, and 13% brown. These are the percents overall, but the distribution of colors in a bag of M&Ms are probably different.

estimate (verb): to come up a value that is close enough to the actual value

event (noun): An outcome from an experiment.

experiment (noun): Something that can be repeated that has a set of possible results. Flipping a coin or rolling a die can be part of a probability experiment.

fair (noun): Treated equally, without having an advantage or disadvantage. A fair game is a game in which each player has the same chance of winning.

frequency (noun): The number of times that something happens.

likelihood (noun): The probability or the chances of something happening. An event might have a small likelihood (low probability) or a high likelihood (high probability) of happening.

mean (noun): The equal share when numbers are added together and divided by the number of total values.

measures of center (noun): This term refers to the measurements mode, median, and mean, which are all ways of finding the center of a set of data.

measures of spread (noun): This term refers to the measurements such as range, which is the difference between the largest and smallest number in a data set, and shows how spread out the data is.

median (noun): The middle number in a set of numbers when the numbers are placed in order from smallest to largest.

mode (noun): The most common number in a set of numbers.

numerator (noun): The top number in a fraction. It shows how many parts we have.

odds (noun): the ratio of the number of ways an event *can* happen to the number of ways it *can not* happen. For example, the odds of a month starting with the letter J are 3:9 (or 1:3) because there are 3 months that start with a J (January, June, July) and 9 months that do not start with a J.

outcome (noun): A possible result of an experiment. For example, when flipping a coin, a “head” or a “tail” are the two possible outcomes. When rolling a six-sided die, a 1, 2, 3, 4, 5, or a 6 are the possible outcomes. An outcome is sometimes called an event.

percent (noun): Parts per 100. It is a ratio “out of 100.”

population (noun): a group that is being studied.

probability (noun): the mathematics of chance. Probability is the study of the chance that something will happen, over the long term. The probability of an event is expressed as a ratio, fraction, decimal, or percent. The probability of an event is between 0 and 1, where 0 means there is no chance of the event happening and 1 means the event will definitely happen.

experimental probability (noun): The actual result of an experiment. The probability found by experimenting.

theoretical probability (noun): A probability found by using calculations to analyze a situation.

random (adjective): When something happens without being made to happen on purpose.

random sample (noun): a random sample is a smaller selection of a larger population. A sample is random if every member of the population has the same chance of being selected.

range (noun): the difference between the maximum value and the minimum value in a set of numbers. For example, if Troy's paychecks for the past 4 weeks are \$380, \$420, \$450, and \$325 then the range in his paychecks would be the difference between \$450 and \$325.

ratio (noun): A relationship that shows the size of one value in comparison to one or more other values. For example, the ratio of teachers to students in a school might be 1:20, which means that there is one teacher for every 20 students.

relative frequency (noun): A ratio that compares the number of times something happens to the total number of outcomes. Example: The relative frequency of students with children was 3:4. This means that 3 out of 4 students had children. 1 out of 4 students did not. On a survey, the *relative frequency* is the frequency of one response compared with the total number of responses

relative frequency (noun): How often an event happens divided by all outcomes.

sample (noun): a group of something (people, animals, objects, etc) selected from a larger population. For example, a blood sample is not all the blood in your body, but a smaller selection.

representative sample (noun): a sample whose characteristics do a good job of reflecting the larger population from where the sample was selected.

sample space (noun): A set or list of all the possible outcomes for an experiment.

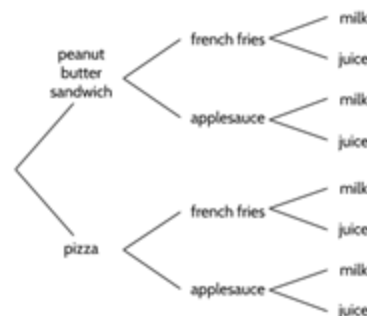
sampling bias (noun): sampling bias is what happens when the sample of a population is not selected randomly. For example, imagine you want to figure out how many hours a week the people of a town spend reading. If you only survey people as they come out of the library, it is unlikely that your sample will represent the general population. Since people who visit the library are more likely to read than the general population, your sample will be biased.

set (noun): A set is a collection of things, which could be numbers or objects. In this packet, the sets are groups of numbers. {3, 5, 8, 10} is a set of 4 numbers.

statistics (noun): The science of collecting, reviewing, and analyzing data.

survey (noun): A process of collecting data about the opinions or experience of a group of people, based on a series of questions.

tree diagram (noun): a diagram using lines that can be used to map out the possible outcomes and the total outcomes in a trial. For example, in the tree diagram to the right all of the possible combinations for a child's lunch order are shown. There are 8 possible different orders shown in this tree diagram. <See *SAMPLE SPACE*>



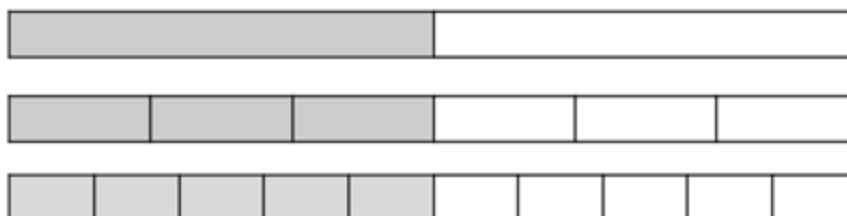
trial (noun): In math, a trial is a test or an experiment. Throwing dice or tossing a coin are examples of a trial.

typical (adjective): The characteristics that you would expect of a member in this group. Averages like mode, median, and mean help us know what would be a typical number in a set.

variability (noun): Differences in the data, how much different data points *vary* from each other. For example, there is a lot of *variability* in the ages of students in an adult education class. Students might be anywhere from 17 years old and up.

variation (noun): Differences in the data. How much different data points *vary* from each other. For example, there is a lot of variation in the ages of students in an adult education class. Students might be anywhere from 17 years old and up.

visual representation (noun): a picture, diagram, or graph that shows data and a relationship between quantities. For example, the visual representation below can be used to show that $\frac{1}{2}$, $\frac{3}{6}$, and $\frac{5}{10}$ are all equivalent fractions.

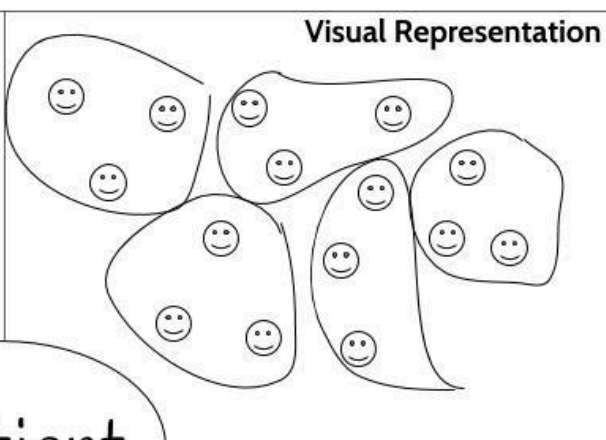


Using Graphic Organizers to Learn Vocabulary

In order to learn math vocabulary, we need practice using words in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.

To start, choose a word from the packet and complete the graphic organizer:

- **What is the definition of the word?** You can look at the vocabulary review on page 111 for help. Write the definition in your own words to really make the word yours.
- **Make a visual representation.** You can make a drawing or diagram that will help you remember what the word means.
- **What are some examples of the word you're studying?** Below you can see that there are examples of *quotients*, which are the answers to division problems.
- **What are some non-examples of this word?** These are things that are **not** the word you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

<p>What is it?</p> <p>A quotient is the result of dividing one number by another. It is the answer to a division question.</p>	<p>Visual Representation</p> 
<p>What are some examples?</p> <p>15 divided by 3 equals 5</p> <p>$66 \div 6 = 11$</p> <p>$63/18 = 3.5$</p> <p>5, 11 and 3.5 are quotients in these calculations.</p> <p>population \div area = density</p>	<p>What are some non-examples?</p> <p>4 times 6 equals 24</p> <p>$18 + 5 = 23$</p> <p>$17 - 2.5 = 14.5$</p> <p>$3.5 \times 18 = 63$</p>

<p>Visual Representation</p>	<p>What are some non-examples?</p>
<p>What is it?</p>	<p>What are some examples?</p>

<p>Visual Representation</p>	<p>What are some non-examples?</p>
<p>What is it?</p>	<p>What are some examples?</p>

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Versions

V. 1.6, 10/4/2021:

- Corrected missing blank for student answer, answer key errors, fixed pagination.

V. 1.7, 7/22/2022

- Corrected typos, answer key errors

V. 1.8, 10/13/2023

- Added Assessment and Placement Recommendations.
- Revised Welcome! letter for consistency with other packets
- Moved vocabulary graphic organizer to the Vocabulary Review section for consistency with other packets

V.1.9, 12/13/2023

- Corrected typos