Essential Functions Practice
for Students of HSE-math

Essential Functions Practice for Students of HSE math

What is a function?

> **Function**: A mathematical rule that assigns one, and only one, output for every input.

A function takes an input, and then does something to the input to produce an output. This process is often thought to be like that of a machine. Let's look at "function machine A" below.

Put 2 in	Put 5 in
↓	↓
Function Machine A	**Function Machine A**
Rule: add 1	**Rule: add 1**
↓	↓
3 comes out	6 comes out
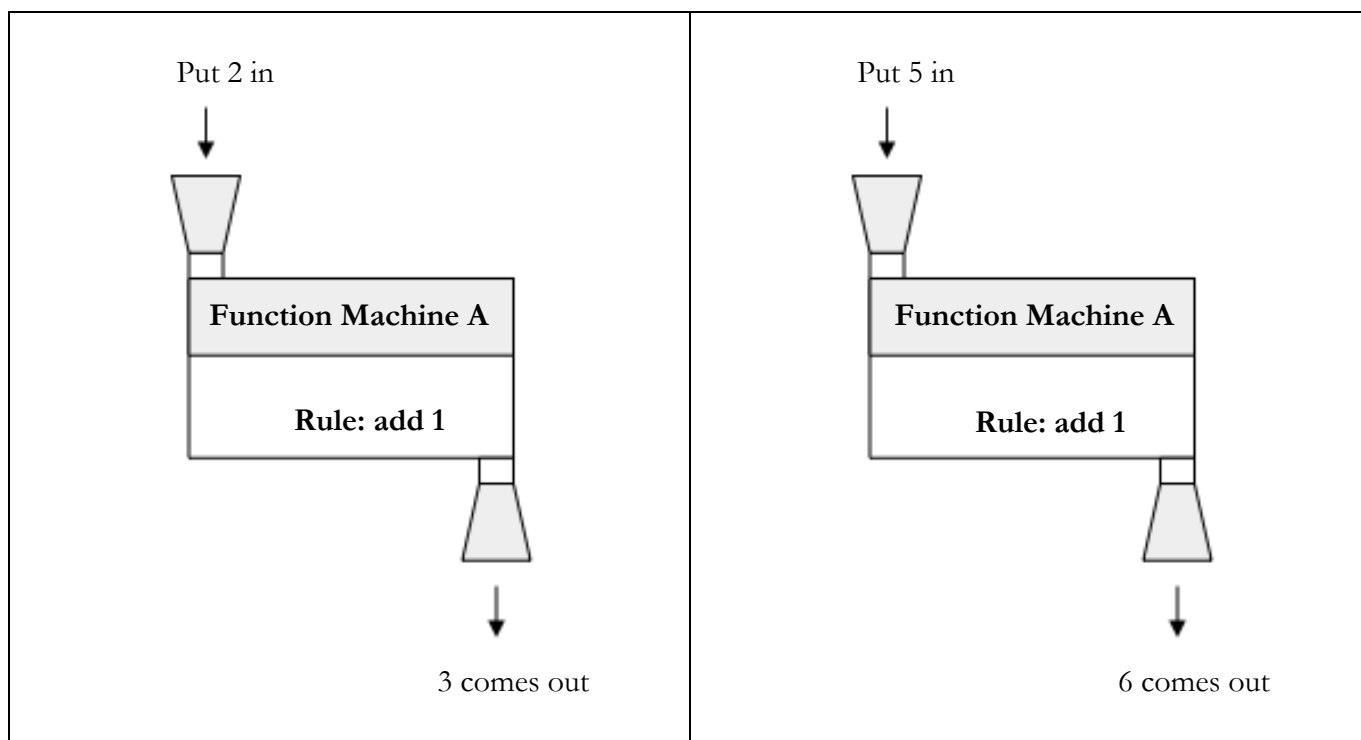
Using Function Machine A, and the rule add 1, 3 is the only output that we can get when 2 is the input. Thus, this rule <u>assigns only one output</u>, 3, to the input 2. Likewise, using Function Machine A, and the rule add 1, 6 is <u>the one and only output</u> that we can get when 5 is the input.

Below you will find practice functions questions like those you will see on the HSE exam. There are many more types of questions you will see on the exam, but these represent some of the most common topics on the test.

I. Identifying Functions

Example 1

> Which set of ordered pairs represents a function?
>
> A. {(2, 6), (3, -2), (2, 7), (-1, 5), (0, 4)}
>
> B. {(-2, 6), (-3, 2), (-2, 7), (1, -5), (0, 4)}
>
> C. {(-2, 6), (3, -2), (2, 7), (1, -5), (0, -4)}
>
> D. {(-2, -6), (-3, -2), (-2, -7), (-1, -5), (0, -4)}

Solving Example 1:

We see the words *ordered pairs*. This refers to an input paired with an output, written like this (*input, output*). So, if we look at the ordered pair (2, 6), this means that the number 2 is our input, and 6 is the output. In mathematics, it is common to refer to the (*input, output*) as (x, y), where x represents input values and y represents output values.

Let's look at choice A, {(2, 6), (3, -2), (2, 7), (-1, 5), (0, 4)}. The brackets { } are used to organize a collection, or a *set*, of (input, output) *ordered pairs*. To answer the question, does this set of ordered pairs represent a function, we need to remember that a function pairs <u>exactly one output for each input</u>. So, the set found in choice A fails the test because we have (2, 6) and (2, 7) in the set. If 2 is the input and 6 is the output, as in (2, 6), then we can't also have (2, 7). Essentially, we can look at the first number in each ordered pair. If we have the same number more than once with different outputs, then we don't have a function. Here we have 2 listed as an input twice (2, 6), (2, 7), so we don't have a function.

Let's look at choice B, {(-2, 6), (-3, 2), (-2, 7), (1, -5), (0, 4)}. This set also fails the test for a function. Here we have -2 repeat as in an input since we have the ordered pairs (-2, 6), and (-2, 7) in the same set.

Choice C is the correct answer. Here, all of the inputs are different, and thus, each input is paired with only one output.

Choice D also fails the test for a function. Here, -2 is repeated as an input since we have (-2, -6) and (-2, -7) in the same set.

Testing TIPS!

- Don't worry about trying to find the function, or rule, that fits the set of ordered pairs found in a question like Example 1. We only need to look to see that no input repeats.

- An output can repeat. For example, this set does belong to a function: {(4, 5), (2, 3), (6, 5)}. Yes, we have 5 as an output twice, but this is okay. We are only concerned with whether or not the input repeats.

You Try – Identifying Functions

1. Which of these sets of ordered pairs represents a function?

   A. {(6, -1), (4, 2), (5, -1), (6, 1)}
   B. {(2, -1), (5, -1), (3, -1), (7, -1)}
   C. {(2, -8), (5, -3), (4, 3), (5, 2)}
   D. {(8, -1), (8, 1), (8, 3), (8, 9)}

2. Which of these sets of ordered pairs does NOT represent a function?

   A. {(6, -1), (4, 2), (5, -1), (9, 1)}
   B. {(2, -8), (3, -3), (4, 3), (5, 2)}
   C. {$\left(\frac{1}{2}, 8\right)$, (5, -1), (3, -8), (0.5, -1)}
   D. {(2, -1), (8, 1), (3, 3), (19, 9)}

3. Given the relation G = {(-4, 5), (8, 2), (a, 7), (1, 3)}. Which replacement for *a* makes this relation a function?

   A. 1
   B. 2
   C. -4
   D. 8

II.   Analyzing Domain and Range

Example 2

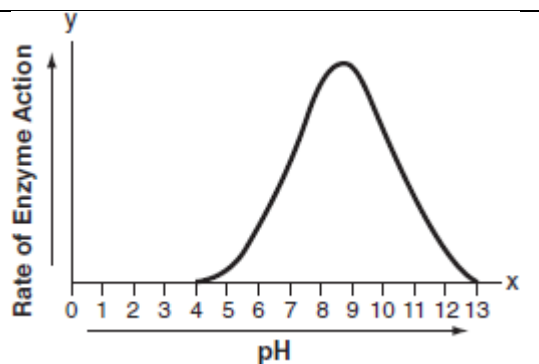The effect of pH on the action of a certain enzyme is shown on the accompanying graph.

What is the domain of this function?

   A. $4 \leq x \leq 13$
   B. $4 \leq y \leq 13$

C. $y \geq 0$

D. $x \geq 0$

Solving Example 2:

Let's first define *domain*, and *range*.

> **Domain – all of the input values for a function.**
> **Range – all of the output values for a function.**

In example 2 above, you will notice the use of x and y. Remember, in mathematics, it is common to use the variables (x, y) to represent inputs and outputs. Also, remember that when we use a coordinate plane to graph two variables, the horizontal axis is the x-axis, and the vertical axis is the y-axis.

When it comes to a function, we can say that the x-axis measures the input values, and the y-axis measures the output values.
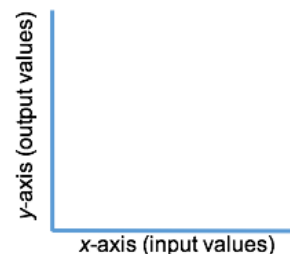

To answer Example 2, we need to define the domain, all of the input values. Looking at the x-axis, we see that the pH values represented by the graph go from 4 to 13 and cover all the values in between. Choice B looks like it could be the correct answer, but be careful, we are not concerned with the y-values when defining the domain of a function. **The correct answer is Choice A**, all of the pH values that are greater than or equal to 4, and also less than or equal to 13.

You Try – Analyzing Domain and Range

4. A meteorologist drew the accompanying graph to show the changes in relative humidity during a 24-hour period in New York City.

   What is the range of this set of data?

   A. $30 \leq y \leq 80$

   B. $30 \leq x \leq 80$

   C. $0 \leq y \leq 24$

   D. $0 \leq x \leq 24$
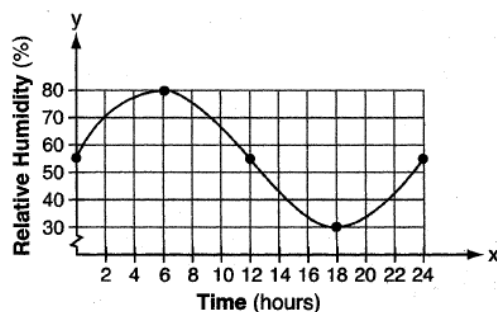
5. What is the range of the function $y = x + 3$ when the domain is $\{-2, 0, 4\}$?

A. {-2, 0, 4}

B. {-5, 3, 7}

C. {-5, -3, 1}

D. {1, 3, 7}

6. It takes Christina 2 minutes to read one page of her 250 page book. Which of the following represents the appropriate domain and range for a function relating the total time she has spent reading the book, x, and the number of pages she has read, y?

A. $0 \leq x \leq 500$
    $0 \leq y \leq 250$

B. $0 \leq x \leq 125$
    $0 \leq y \leq 250$

C. $0 \leq x \leq 250$
    $0 \leq y \leq 500$

D. $0 \leq x \leq 250$
    $0 \leq y \leq 250$

II. Function Notation

Example 3

If $f(x) = 4x^2 - 5x + 6$, what is $f(-3)$?
A. -45
B. -15
C. 27
D. 57

Solving Example 3:

The equation in Example 3 is written in a specific, and possibly unfamiliar, way called *function notation*. It is common in algebra that a quantity outside of parentheses next to a quantity inside parentheses means to multiply. That is <u>not</u> the case in function notation. Here, the left side of the equation, $f(x)$, is read "f of x". Put simply, x is our input and $f(x)$ is the output we get when we use the function called "f".

In Example 3, $f(x) = 4x^2 - 5x + 6$. We are asked, what is $f(-3)$? So, we use -3 as our input using the function "f" to find the output. The first step is to <u>substitute</u> -3 for all values of x.

Step 1: Substitute -3 for all values of x.

$$f(-3) = 4(-3)^2 - 5(-3) + 6$$

Step 2: Evaluate the right side of the equation using the order of operations.

$$f(-3) = 4(9) - 5(-3) + 6$$
$$f(-3) = 36 - (-15) + 6$$
$$f(-3) = 36 + 15 + 6$$
$$f(-3) = 57 \qquad \textbf{The correct answer is choice D.}$$

Testing TIPS!

- Remember the order of operations. If, $f(x) = 3x^2$, then $f(4) = 3 \cdot 4^2 = 3 \cdot 16 = 48$. It does not mean $(3 \cdot 4)^2 = 12^2 = 144$.
- Be careful, -3^4 should be entered as $(-3)^4$ using the parentheses on your calculator. This way you can be sure that the calculator is doing what you want; $-3 \cdot -3 \cdot -3 \cdot -3 = 81$.

You Try – Function Notation

7. If $h(x) = 2x^2 - 6x + 1$, what is $h(-2)$?

   A. -3
   B. 5
   C. 21
   D. 29

8. If $g(a) = 2a^3 - 6a^2 + 1$, what is $g(-4)$?

   A. -607
   B. -223
   C. 33
   D. 417

9. If $f(x) = 3x^4 + 5$, what is $f(-2)$?

   A. -1291
   B. -43
   C. 53
   D. 1301

10. A company uses the function $S(x) = 0.4x + 2$ to calculate shipping costs.

- $S(x)$ is the total cost (in dollars) to ship an envelope.
- x is the weight of the envelope in ounces.

Using the given function, how much would it cost to ship an envelope that weighs 6-ounces?

IV. Interpreting Linear Functions (High Emphasis)

Example 4

The results of an experiment testing the effectiveness of a medication in raising the number of antibodies in a sample of blood are shown in the graph at right.

Which of the following functions correctly models the relationship between d, the days that have passed in the experiment, and a, the number of antibodies in the sample of blood?

    A. $d = 70a + 50$

    B. $d = 50a + 70$

    C. $a = 70d + 50$

    D. $a = 50d + 70$
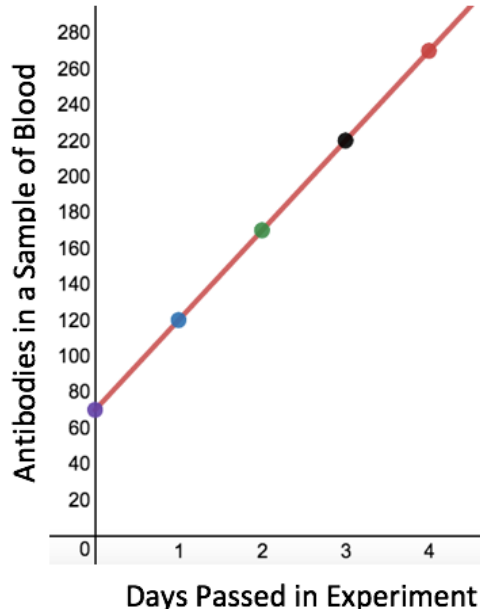
Solving Example 4:

Functions are commonly shown in three ways: 1. As an equation; 2. As a graph; and 3. As a table of values. Example 4 is essentially asking for you to match the graph of a function with the appropriate equation.

Looking at the graph of the function, we see that this function forms a straight line when graphed. A function of this form is called a *linear function*. Perhaps you notice that the word linear has the word <u>line</u> in it. This is not a coincidence.

A linear function can always be written in the form:

    output value = (rate of change)(input value) + starting amount

This is also known as:

    $y = mx + b$

One way to determine the rate of change and starting amount is to look at a table of values for the function. Since we aren't given a table of values here, we can construct one using the graph. Here we see that the *x-axis* displays the days passed in the experiment, or d. These are our input values. The *y*-axis displays the antibodies in a sample of blood, or a. These are our output values.

Days Passed in Experiment d	Antibodies in a Sample of Blood a
0	70
1	120
2	170
3	220
4	270

The *starting amount* is the output value when the input value is 0. Here, when 0 days have passed, there are 70 antibodies in a sample of blood. So, the starting amount is 70.

The *rate of change* is the change in the output values when the input values change by 1. In the table, we see that the number of antibodies in the sample of blood increase by 50 when the days passed in the experiment increase by 1. The output values go from 70 to 120 to 170 to 220 to 270. Each time, the output values increase by 50. So, we say that the *rate of change* is 50.

We now know that the starting amount is 70, and the rate of change is 50. Using our standard form for a linear equation, we can now see that **the correct answer is Choice D. $a = 50d + 70$.**

This choice gives us the correct starting amount and the correct rate of change. Be careful, though, to not be fooled by Choice B. $d = 50a + 70$. This choice also gives us the correct starting amount and the correct rate of change. However, this choice is incorrect, because the input value is d, and the output value is a. Remember, the form of a linear equation we introduced was output value = (rate of change)(input value) + starting amount.

Testing Tips

- Rate of change is also known as **slope**, or m. And starting amount is also known as **y-intercept**, or b. So, the functions $f(x) = 2x + 5$, and $g(x) = 2x + 3$ have the same rate of change, or the same slope, but different starting amounts, or different y-intercepts.

You Try - Interpreting Linear Functions

11. A company uses the function $S(x) = 0.4x + 2$ to calculate shipping costs.
    - $S(x)$ is the total cost (in dollars) to ship an envelope.
    - x is the weight of the envelope in ounces.

   What do the values 0.4 and 2 represent in the function?

    A. The cost to ship a 2-ounce envelope is $0.40.

    B. The cost to ship a 0.4-ounce envelope is $2.00.

    C. The cost to ship an envelope is $2.00 plus $0.40 per ounce.

    D. The cost to ship an envelope is $0.40 plus $2.00 per ounce.

12. The gas tank in a car holds a total of 16 gallons of gas. The car travels 75 miles on 4 gallons of gas. If the gas tank is full at the beginning of a trip, which graph represents the rate of change in the amount of gas in the tank?
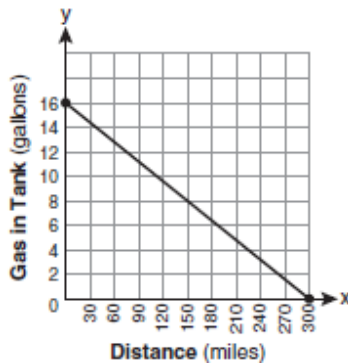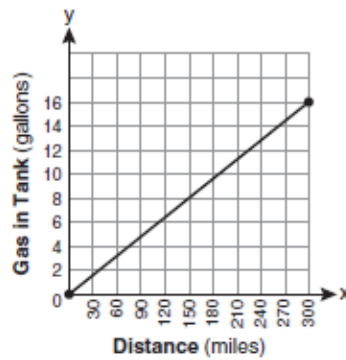
A.
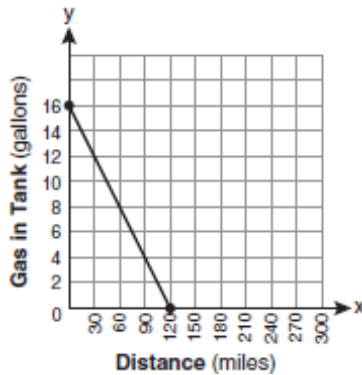
B.
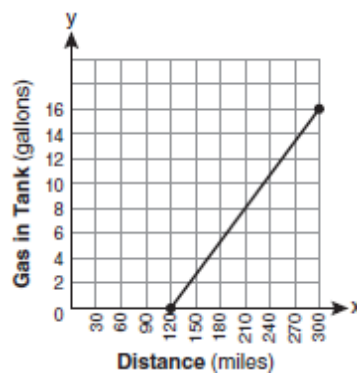
C.

D.

13. Wacky Willie pays his employees $100 a day plus $40 for every TV they sell. Which function below models the relationship of t, the number of TV's sold, and p, the pay an employee makes that day?

A. $t(p) = 140p$

B. $p(t) = 100t + 40$

C. $t(p) = 40p + 100$

D. $p(t) = 40t + 100$

Essential Functions Practice for Students of HSE-math
Answer Key

Solutions to: You Try – Identifying Functions

#	√	Explanation
1	B	In the set {(2, -1), (5, -1), (3, -1), (7, -1)} each input is different. Because each input is different, then every input given is paired with only one output. This set represents a function.
2	C	The question asks for a set of ordered pairs that **does not** represent a function. Therefore, we are looking for a set that has at least one input that does repeat and is matched with a different output. At first glance, it appears that all of the inputs in the set $\{(\frac{1}{2}, 8), (5, -1), (3, -8), (0.5, -1)\}$ are different. However, the set contains both $\frac{1}{2}$ and 0.5 as inputs. Those values are equivalent, and essentially the same number, or input. Therefore this set fails the test for a function.
3	B	To have a function, we need to have all of the inputs be different. Since the set already contains -4, 8, and 1 as inputs, then we cannot repeat any of those numbers. Choice B. 2 is the only number we do not have as an input yet, so this is the correct answer.

Solutions to: You Try – Analyzing Domain and Range

#	√	Explanation
4	A	The range is defined as all the outputs in the set of data. As on all coordinate planes, we look to the y-axis to measure the output values. Looking at the plane, and the y-axis, we see that the graph reaches a minimum value of 30, and a maximum value of 80. The graph crosses all points between 30 and 80. Therefore, $30 \leq y \leq 80$. This is read y is greater than, or equal to, 30 and less than, or equal to, 80.
5	D	The domain {-2, 0, 4} gives us the three x-values we have as inputs for the function $y = x + 3$. To find y, we must separately substitute each of those values for x in the function. $y = -2 + 3 = 1$ $y = 0 + 3 = 3$ $y = 4 + 3 = 7$ Using those inputs, we get outputs of {1, 3, 7}. This is the range for the function.
6	A	If it takes Christina 2 minutes to read one page, then it will take 500 minutes for Christina to read 250 pages. Since we know that x represents the total time she has spent reading the book, then the domain would be $0 \leq x \leq 500$. And since we know that y represents the number of pages she has read, and the book is 250 pages long, then the range would be $0 \leq y \leq 250$. The correct answer is A.

Solutions to: You Try – Function Notation

#	√	Explanation
7	C	To find $h(-2)$, we substitute -2 in for all values of x in $h(x) = 2x^2 - 6x + 1$. $h(-2) = 2(-2)^2 - 6(-2) + 1$ $h(-2) = 2(4) + 12 + 1$ $h(-2) = 8 + 12 + 1$ $h(-2) = 21$
8	B	To find $g(-4)$, we substitute -4 in for all values of a in $g(a) = 2a^3 - 6a^2 + 1$. $g(-4) = 2(-4)^3 - 6(-4)^2 + 1$ $g(-4) = 2(-64) - 6(16) + 1$ $g(-4) = -128 - 96 + 1$ $g(-4) = -223$
9	C	To find $f(-2)$, we substitute -2 in for all values of x in $f(x) = 3x^4 + 5$. $f(-2) = 3(-2)^4 + 5$ $f(-2) = 3(16) + 5$ $f(-2) = 48 + 5$ $f(-2) = 53$
10	$4.40	To find the cost to sip a 6-ounce envelope using the function $S(x) = 0.4x + 2$, we replace x, the weight of the envelope, with 6. $S(6) = 0.4(6) + 2$ $S(6) = 2.4 + 2 = 4.4 = \$4.40$ Write and bubble either 4.4 or 4.40 in the grid.

Solutions to: You Try – Interpreting Linear Functions

#	√	Explanation
11	C	In this function, x is defined as the weight of the envelope in ounces. For example, if we wanted to use the function $S(x) = 0.4x + 2$ to find the weight of a 3-ounce envelope, we would replace x with a 3. To see how the cost changes as the weight of an envelope changes, we can look at a table of values. <table><tr><td>Weight in Ounces x</td><td>Cost in Dollars $S(x) = 0.4x + 2$</td></tr><tr><td>0</td><td>$S(0) = 0.4(0) + 2 = 0 + 2 = 2 = \$2.00$</td></tr><tr><td>1</td><td>$S(1) = 0.4(1) + 2 = 0.4 + 2 = 2.4 = \$2.40$</td></tr><tr><td>2</td><td>$S(2) = 0.4(2) + 2 = 0.8 + 2 = 2.8 = \$2.80$</td></tr><tr><td>3</td><td>$S(3) = 0.4(3) + 2 = 1.2 + 2 = 3.2 = \$3.20$</td></tr></table> Notice we started with a weight of 0. This is important, because the starting amount is defined as the output we get when the input is 0. We see that the price of shipping an envelope goes from \$2.00 to \$2.40 to \$2.80 to \$3.20 as we increase the weight by 1 ounce. Therefore, the rate of change is \$0.40 for each

		ounce. So, the cost to ship an envelope is $2.00 plus $0.40 per ounce. The correct answer is C.
12	C	As a car travels (Distance), the amount of gas in the tank goes down. Based on this, we know that the answer must be either choice A or C. These two graphs show that as the distance, or number of miles (*x*-axis), driven increases, the gas in the tank goes down (*y*-axis). We also know that the car travels 75 miles on 4 gallons of gas. To make a table, we start at a distance of 0-miles driven, and a full tank of gas of 16-gallons. For every 75 miles driven, the gas will go down 4 gallons. Our table could look like this:

Distance (miles)	Gas in Tank (gallons)
x	*y*
0	16
75	12
150	8
225	4
300	0

In Choice A., the car only travels a total of 120 miles. In choice C., the car travels a total of 300 miles. This agrees with our table. The correct answer is Choice C. |
| 13 | D | Wacky Willie pays his employees $100 a day plus $40 for every TV they sell. This means that an employee that sells 0 TV's will still earn $100 for that day. So, our *starting amount* is $100. Then, for every TV an employee sells, they will earn an additional $40. Therefore, our *rate of change* is $40. The standard form for a linear equation is *output value = (rate of change)(input value) + starting amount*. Since the pay depends on how many TV's are sold, the input value is *t*, number of TV's sold. That makes *p*, the pay an employee makes that day the output value. Using $100 as our starting amount, $40 as our rate of change, *t* as our input, and *p(t)* as our output, we have a function of $p(t) = 40t + 100$. The correct answer is Choice D. |