STOP For these two problems or tasks, see if you can think of more than one path to the solution. Try to think of an approach that is near the bottom of the "ramp" (less sophisticated) and another that is closer to the middle or the top of the "ramp." Do this now before reading further.

The area problem can be solved with materials that directly attack the meaning of the problem. The cover of the book can be completely covered with tiles and then counted one at a time. Moving slightly up the ramp, a child may cover the book with tiles but count only the length of the row and the number of rows, multiplying to get the total. Another child may place tiles only along the edges of the book and multiply. Yet another child may use a ruler to measure the book edges, noting that the tiles are 1 inch on each side.

For the pizza task a direct approach is also possible. Plastic circular fraction pieces (or a drawing) can be used to represent $2\frac{1}{3}$ pizzas, and $\frac{1}{4}$ pieces can be placed on top of these until no more will fit. Another child may know that four fourths make a whole; therefore, two of the pizzas will feed eight friends. Children may or may not know how many fourths they can get from the $\frac{1}{3}$ piece and will have to tackle that part accordingly. A guess and check approach is possible, starting with perhaps six children, then seven, and so on until the pizza is gone. A few children may have learned a computational method for dividing $2\frac{1}{3}$ by $\frac{1}{4}$.

Having thought about these possible entry points, the teacher will be better prepared to suggest a hint that is appropriate for students who are "stuck" but who are likely different in what they bring to the task.

NCTM Standards The Equity Principle challenges teachers to believe that every student brings something of value to the tasks that they pose to their classes. The Teaching Principle calls for teachers to select tasks that "can be solved in more than one way, such as using an arithmetic counting approach, drawing a geometric diagram and enumerating possibilities, or using algebraic equations [so that tasks are] accessible to students with varied prior knowledge and experience" (p. 19).

A Three-Part Lesson Format

U.S. teachers typically spend a small portion of a lesson explaining or reviewing an idea and then go into "production mode," where students wade through a list of exercises. Lessons set up in this explain-then-practice pattern condition students to focus on procedures so that they can get through the exercises. Teachers find themselves going from desk to desk reteaching and explaining to individuals. This is in significant contrast to a lesson built around a single problem, an approach that is quite typical in Asian classrooms (see Figure 4.2). Much more learning occurs and much more assessment information is available when a class works on a single problem and engages in discourse about the validity of the solution.

The emphasis on understanding is evident in the steps typical of Japanese eighth-grade mathematics lessons:

- Teacher poses a complex thought-provoking problem.
- Students struggle with the problem.
- Various students present ideas or solutions to the class.
- Class discusses the various solution methods.
- The teacher summarizes the class's conclusions.
- Students practice similar problems.

In contrast, the emphasis on skill acquisition is evident in the steps common to most U.S. and German math lessons:

- Teacher instructs students in a concept or skill.
- Teacher solves example problems with class.
- Students practice on their own while the teacher assists individual students.

FIGURE 4.2 Comparison of the steps typical of eighth-grade mathematics lessons in Japan, the United States, and Germany.

Source: Unpublished tabulations from the Third International Mathematics and Science Study, Videotape Classroom Study, University of California, Los Angeles, 1996. Used with permission.

Before, During, and After

Teaching through problem solving does not mean simply providing a problem or task, sitting back, and waiting for magic to happen. The teacher is responsible for making the atmosphere and the lesson work. To this end, think of a lesson as consisting of three main parts: *before*, *during*, and *after*. Each portion carries specific agendas and requires specific teacher actions to make the lesson effective (see Figure 4.3).

- *Before.* The agenda in this part of the lesson is to get students mentally prepared to work on the problem and thinking about the kinds of ideas that will help them the most. You want to be sure they understand the task. You want to be sure they understand their responsibilities. At the end of this portion of the lesson, there should be no questions regarding the task or what is to be done. Students have already begun to think about the relevant ideas and are ready to go to work.
- *During.* The first agenda item here is to *let go!* Give students a chance to work without your constant guidance. Give them the opportunity to use *their* ideas and not simply follow directions. Have faith in their abilities. The second point is to *listen.* Find out how different children or groups are thinking, what ideas they are using, and how they are approaching the problem.

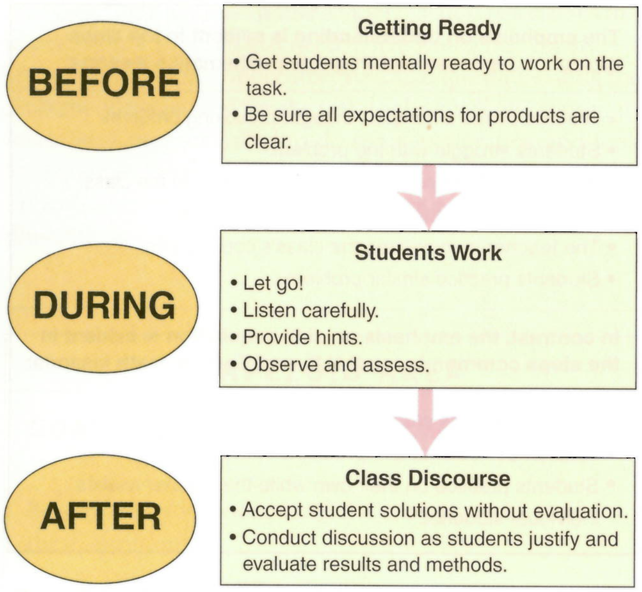


FIGURE 4.3 Teaching through problem solving suggests a simple three-part structure for lessons.

- *After.* In this portion of the lesson, you want to engage the class in productive discourse and help students begin to work as a community of learners. *Do not evaluate.* Students must learn to both contribute to and participate in these discussions. They must listen to others and help decide which approaches and solutions make the most sense and why. Thinking must not stop when the problem is solved. Now is the time to encourage reflection on solutions, methods, and extensions.

If you allot time for each before, during, and after segment, it is quite easy to devote a full period to one seemingly simple problem. As long as the problematic feature of the task is the mathematics you want students to learn, a lot of good learning will result from engaging students in only one problem. The same three-part structure can be applied to small tasks, resulting in a 10- to 20-minute minilesson (common in kindergarten). There are also times when the during and after portions extend into the next day or even longer.

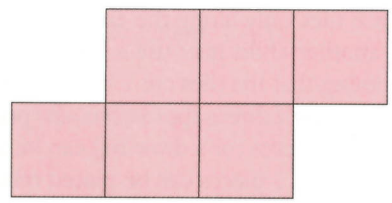
Teacher Actions in the Before Phase

The kinds of things you do in the before phase of a lesson will vary with the task. Some tasks you can begin with immediately. For example, if your students are used to solving story problems and know they are expected to use words, pictures, and numbers to explain their solutions in writing, all that may be required is to read through the problem with them and be sure all understand it. The actual presentation of the task or problem may occur at the beginning or at the end of your "before actions." However, you will first engage students in some form of activity directly related to the problem in order to get them prepared. Consider the following "before actions."

Begin with a Simple Version of the Task

Suppose that you are interested in developing some ideas about area and perimeter in the fourth or fifth grade. This is the task you plan to present (Lappan & Even, 1989).

Assume that the edge of a square is 1 unit. Add squares to this shape so that it has a perimeter of 18.



Instead of beginning your lesson with this problem, you might consider one of the following simpler tasks:

- Draw a 3-by-5 rectangle of squares on the board, and let students tell things they know about the shape. (It's a rectangle. It has squares. There are 15 squares. There are three rows of five.) If no one mentions the words *area* and *perimeter,* you could write those two words on the board and ask if those words can be used in talking about this figure.
- Provide students with some square tiles or grid paper. "I want everyone to make a shape that has a perimeter of 12 units. After you make your shape, find out what its area is." After a short time, have several students share their shapes.

Each of these "warm-ups" gets the vocabulary you will need for your exploration out in the open. The second activity suggests the tiles as a possible model that students may choose to use. It has the added benefit of hinting that there are different figures with the same perimeter.

Dad says it is 503 miles to the beach. When we stopped for gas, we had gone 267 miles. How much farther do we have to drive?

This problem is designed to help children develop an add-on method of subtraction. Before presenting this problem, have students supply the missing part of 100 after you supply one part. Try numbers like 80 or 30 at first; then try

47 or 62. After presenting the actual task, you might ask students if the answer to the problem is more or less than 300.

Brainstorm

Often you can present a problem and have students suggest solutions or strategies. Their suggestions will not solve the problem for others because students must still work out the solution and an explanation. The following problem for middle school students is designed to address the ideas of ratio and proportion as well as data analysis. Since this is not a straightforward task, brainstorming will likely produce a variety of approaches, resulting in more profitable solutions by more students.

Enrollment data for the school provide information about the students and their families as shown here:

	School	*Class*
Siblings		
None	36	5
One	89	4
Two	134	17
More than two	93	3
Race		
African American	49	11
Asian American	12	0
White	219	15
Travel-to-school method		
Walk	157	10
Bus	182	19
Other	13	0

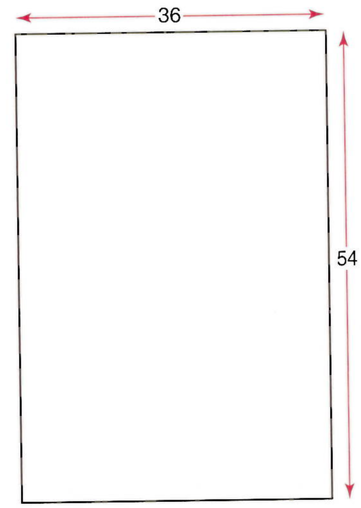
If someone asked you how typical your class was of the rest of the school, how would you answer? Write an explanation of your answer. Include one or more charts or graphs that you think would support your conclusion.

Estimate or Use Mental Computation

When the task is aimed at the development of a computational procedure, a useful before action is to have students actually do the computation mentally or suggest a ballpark answer. Have students independently think of an estimate or compute mentally, and then list these "pre-answers" on the board. You may even have students explain their reasoning. Again, this process will not spoil the problem for the class.

This technique would be useful with the earlier problem concerning how many miles to go to the beach. The following task is another example in which preliminary estimates or mental computations would be useful.

How many small squares (ones or units) will fit in a rectangle that is 54 units long and 36 units wide? Use base-ten pieces to help you with your solution.



Make a plan for figuring out the total number of pieces without doing too much counting. Explain how your plan would work on a rectangle that is 27 units by 42 units.

Prior to estimation or mental computation for this problem, several simpler problems will also help. For example, rectangles such as 30 by 8 or 40 by 60 could be explored, or these may have been problems from earlier days.

Be Sure the Task Is Understood

This action is not optional! You must always be sure that students understand the problem before setting them to work. Remember that their perspective is different from yours.

Consider the task of mastering the multiplication facts. The most difficult facts can each be connected or related to an easier fact already learned.

Use a fact you already know to help you solve each of these facts: 4 × 6, 6 × 8, 7 × 6, 3 × 8.

For this task, it is essential that students understand the idea of using a helping fact. They have most likely used helping facts in addition. You can build on this by asking, "When you were learning addition facts, how could knowing 6 + 6 help you figure out 6 + 7?" You may also need to help students understand what is meant by a fact they know—one they have mastered and know without counting.

When using a word problem, it is useful in the lower grades to ask a series of direct questions that can be answered just by looking at the problem.

The local candy store purchased candy in cartons holding 12 boxes per carton. The price paid for one carton was $42.50. Each box contained 8 candy bars that the store planned to sell individually. What was the candy store's cost for each candy bar?

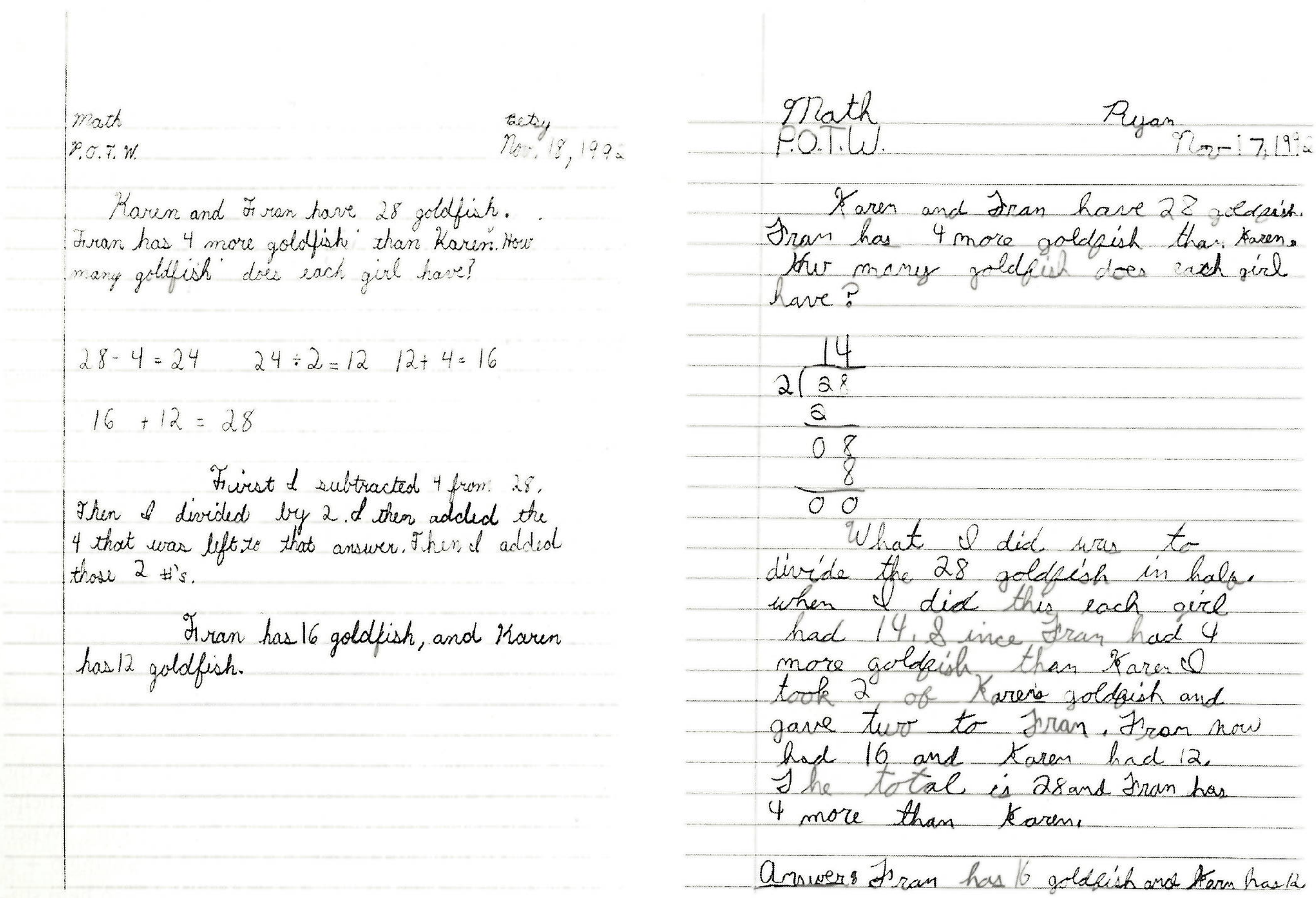
"What did the candy store do? What is in a carton? What is in a box? What is the price of one carton? What does that mean when it says 'each box'?" The last question here is to identify vocabulary that may be misunderstood. It is also useful to be sure students can explain to you what the problem is asking. Rereading a problem does little good, but having students restate the problem in their own words forces them to think about what the problem is asking.

Establish Expectations

As with the previous action, this one is essential. Students need to be clearly told what is expected of them beyond an answer. It is always a good idea to have students write out an explanation for their solution. There is a significant difference between "Show how you got your answer" and "Explain why you think your answer is correct." With the former direction, students may simply record their steps ("First we did . . . , and then we . . . ") or present their work as self-evident. You can't expect students to explain their mental processes unless you ask for an explanation. In Figure 4.4 the work of two students illustrates the contrast.

When students are working in groups, only one written explanation should come from the group. All in the group must understand what is written, all should put their names on it, and any one in the group should be ready to make an oral presentation.

It is never too early to begin written explanations, even in kindergarten. There the writing may be in the form of drawings and numbers, but this early form of written com-


Math
P.O.T.W.
Betsy
Nov. 18, 1992

Karen and Fran have 28 goldfish. Fran has 4 more goldfish than Karen. How many goldfish does each girl have?

28 - 4 = 24 24 ÷ 2 = 12 12 + 4 = 16

16 + 12 = 28

First I subtracted 4 from 28. Then I divided by 2. I then added the 4 that was left to that answer. Then I added those 2 #'s.

Fran has 16 goldfish, and Karen has 12 goldfish.

Math
P.O.T.W.
Ryan
Nov. 17, 1992

Karen and Fran have 28 goldfish. Fran has 4 more goldfish than Karen. How many goldfish does each girl have?

2) 28 = 14

What I did was to divide the 28 goldfish in half. When I did this each girl had 14. Since Fran had 4 more goldfish than Karen I took 2 of Karen's goldfish and gave two to Fran. Fran now had 16 and Karen had 12. The total is 28 and Fran has 4 more than Karen.

Answer: Fran has 16 goldfish and Karen has 12

FIGURE 4.4 **Betsy tells each step in her solution but provides no explanation. In contrast, Ryan's work includes reasons for his steps.**

Kindergarten
How many ways can you show what 5 means?

FIGURE 4.5 A kindergarten student shows her thinking about ways to make 5.

munication is just as important. Figure 4.5 shows one student's solution for ways to make 5.

The written explanations may be in a journal or on a separate sheet of paper. Sometimes a group report can be prepared on a large sheet of newsprint. This is useful when students are going to make a presentation.

There may be occasions when you will not require written work. A very simple yet useful technique is to have students share their ideas with a partner and then between the two of them select the best approach to be presented. This causes students to defend their ideas to a peer and prepares them to talk to the class.

Students preparing to explain and defend their answers will spend time reflecting on the validity of their results and will often make revisions even before sharing them. They will have a greater interest in the class discussion because they will want to compare their solution with others' solutions. They will have "rehearsed" for the class discussion and be ready to participate. Even when students fail to solve a problem, they should be expected to show the ideas and the work that they have considered.

Teacher Actions in the During Phase

Once you are comfortable that students are ready to work on the task, it is time to let go. You must demonstrate confidence and respect for your students' abilities. Set them to work with the expectation that they will solve the problem. You *must* let go!

Students should get in the habit of working in groups so that there is no lost time in moving from the full-class discussion to the small groups. Groups of three or at most four work well, but pairs of students, especially at the K–2 level, are often best. Your role now shifts to that of facilitator and active listener.

Listen Actively

This is one of two opportunities you will get in the lesson (the other is in the discussion portion of the lesson) to find out what your students know, how they think, and how they are approaching the task you have given them. You might sit down with a group and simply listen for a while, have the students explain what they are doing, or take notes. If you want further information, try saying, "Tell me what you are doing," or "I see you have started to multiply these numbers. Can you tell me why you are doing that?" You want to convey a genuine interest in what students are doing and thinking. This is *not* the time to evaluate or to tell students how to solve the problem.

Students may initially be timid when you sit down to join them, or they may change their minds as soon as you ask a question, taking it as a signal that they are wrong. However, soon they will accept that you are simply there to find out what they are doing. When they see that the more you learn about their reasoning and ideas the better you are able to help them, they will no longer be anxious when you are listening.

In Chapter 5, you will read about techniques for recording the information you find out during this time.

Provide Hints and Suggestions

How much help to give students is always going to be an issue. Should you let them stumble down the wrong path? How much direction should you provide? Do you correct errors you see?

If a group is searching for a place to begin, a hint may be appropriate. You might suggest that the students try using a particular manipulative or draw a picture if that seems appropriate.

In Fern's Furniture Store, Fern has priced all of her furniture at 20 percent over wholesale. In preparation for a sale, she tells her staff to cut all prices by 10 percent. Will Fern be making 10 percent profit, less than 10 percent profit, or more than 10 percent profit? Explain your answer.

For this problem, consider the following hints:

- Try drawing a picture or a diagram of something that shows what 10 percent off means.
- Try drawing a picture or a diagram that shows what 20 percent more means.
- Maybe you could pick a price of something and see what happens.
- Let's try a simpler problem. Suppose that you had 8 blocks and got 25 percent more. Then you lost 25 percent of the new collection.

Encourage Testing of Ideas

Students will look to you for approval of their results or ideas. Avoid being the source of "truth" or of right and wrong. When asked if a result or method is correct ask, "How can you decide?" or "Why do you think that might be right?" or "I see what you have done. How can you check that somehow?" Even if not asked for an opinion, asking, "How can we tell if that makes sense?" reminds students that answers without reasons are not acceptable.

Suggest Extensions or Generalizations

Lots of good problems are simple on the surface. It is the extensions that are excellent. The area and perimeter task is a case in point. Many students will quickly come up with one or two solutions. "I see you found one way to do this. Are there any other solutions? Are any of the solutions different or more interesting than others? Which of the shapes you found with a perimeter of 18 is the largest and which the smallest? Does the perimeter always change when you add another tile?"

What can you find out about that? This general question is at the very heart of mathematics as a science of pattern and order. It asks students to look for something interesting, to generalize. For example, in the area and perimeter problem, squares can be added to a shape in three situations: If a new square touches the old shape on only one side, the perimeter increases by 2. If it touches on two sides, there is no increase. If it fits into a "U" and touches on three sides, the perimeter actually decreases by 2.

Questions that begin "What if you tried . . . ?" or "Would that same idea work for . . . ?" are also ways to suggest different extensions. For example, "Suppose you tried to find all the shapes possible with a perimeter of 18. What could you find out about the areas?"

Find a Second Method

The value of students' solving a problem in more than one way cannot be overestimated. It shifts the value system in the classroom from answers to processes and thinking. It is a good way for students to make new and different connections.

For example, consider this sixth-grade problem.

The dress was originally priced at $90. If the sale price is 25 percent off, how much will it cost on sale?

This is an example of a straightforward problem with a single answer. Many students will solve it by multiplying by 0.25 and subtracting the result from $90. The suggestion to find another way may be all that is necessary. Others may require specific directions: "How would you do it with fractions instead of decimals?" "Draw me a diagram that explains what you did." "How could this be done in just one step?" "Think of a way that you could do this mentally."

Second graders will frequently solve the next problem by counting or using addition.

Maxine had saved up $9. The next day, she received her allowance. Now she has $12. How much allowance did she get?

"How would you do that on a calculator?" and "Can you write an equation that tells what you did?" are ways of encouraging children to connect 9 plus ? = 12 with 12 − 9.

Solving a problem in a second way can help students who have made an error find their own mistake. Perhaps a student has used a symbolic method of her own invention and has made an error in reasoning. Suggesting only that she try to do the problem with physical models ("so that I can understand what you did") may be all that is necessary to have her make a self-correction.

Teacher Actions in the After Phase

Be certain to plan ample time for this portion of the lesson and then be certain to *save* the time. It is not necessary for every student to have finished. Often this is when the best learning will take place. Twenty minutes or more is not at all unreasonable for a good class discussion and sharing of ideas. This is not a time to check answers but for the class to share ideas. Over time, you will develop your class into a community of learners who together are involved in making sense of mathematics. This atmosphere will not develop easily or quickly. You must teach your students about your expectations for this time and how to interact with their peers.

Engage the Class in Discussion

You may want simply to list answers from all of the groups and put them on the board without comment. Following that, you can return to one or more students to get explanations for their solutions or to explain their processes. At other times, you may choose to call on groups or individuals that you noticed earlier doing something interesting or something of special value. This should not be restricted to correct solutions.

When there are different answers, the full class should be involved in the discourse concerning which answers are correct. Allow those responsible for the answers to defend them, and then open the discussion to the class. "Who has an idea about this? George, I noticed that you got a different answer than Tomeka. What do you think of her explanation?" Resist with every fiber in you the temptation to judge the correctness of an answer. Even when the class fails to come to a conclusion, rather than provide your solution, suggest that perhaps you will need to return to this later. Later will be when students have gained some additional insights to help them.

One of your functions is to make sure that all students participate, that all listen, and that all understand what is being said. Moving along too quickly cheats students who are not quite able to follow the faster explanations of the best children in the room. Encourage students to ask questions. "Pete, did you understand how they did that? Do you want to ask Mary a question?"

Use Praise Cautiously

Be an attentive listener to both good and not so good ideas. Support and praise effort and risk taking, but *expect* students to do good work. Praise offered for correct solutions or excitement over interesting ideas suggests that the students did something unusual or unexpected. This can be negative feedback for those who do not get praise. Comments such as "Good job!" and "Super work!" roll off the tongue easily. We use these statements to help children feel good about themselves. There is evidence to suggest that we should be cautious when using expressions of praise, especially with respect to student products and solutions (Schwartz, 1996). Praise not only supports students' feelings but also evaluates. "Good job!" says, "Yes, you did that correctly." "Nice work" can establish an expectation for others that products must be neat or beautiful in order to have value. This can make students who do excellent mathematical science hide their efforts if they appear sloppy or poorly presented.

In place of praise that is judgmental, Schwartz (1996) suggests comments of interest and extension: "I wonder what would happen if you tried . . . " or "Please tell me how you figured that out." Notice that these phrases express interest and value the child's thinking. They also can and should be used regardless of the validity of the responses.

Engage *All* Class Members

Teachers are finding a wider range of academic diversity in their classrooms. NCTM in its *Standards* documents is very clear in expressing the belief that all children can learn the mathematics of the regular curriculum. This view is supported by a number of prominent mathematics educators who have worked extensively with at-risk populations (Campbell, 1996; NCTM, 1989, 1991; Silver & Stein, 1996; Trafton & Claus, 1994).

But the needs and abilities of children are different, and it requires skill and practice to conduct a large group discussion that is balanced and includes all children. Rowan and Bourne (1994) offer some excellent suggestions based on their work in an urban, multiracial, multiethnic, low-socioeconomic school district. They emphasize that the most important factor is to be clear about the purpose of group discussion—that is, to share and explore the variety of strategies, ideas, and solutions generated by the class and to learn to communicate these ideas in a rich mathematical discourse. Every class has a handful of students who are always ready to respond. Other children learn to be passive or do not participate. So rule number one is that the discussion is more important than hearing an answer.

A second suggestion is to begin discussions by calling first on the children who tend to be shy or lack the ability to express themselves well. Rowan and Bourne (1994) note that the more obvious ideas are generally given at the outset of a discussion. When asked to participate early and given sufficient time to formulate their thoughts, these reticent children can more easily participate and thus be valued.

Make it a habit to ask for explanations to accompany *all* answers. Soon the request for an explanation will not signal an incorrect response, as children initially believe. Many incorrect answers are the result of small errors in the context of otherwise excellent thinking. Likewise, many correct answers may not represent the insightful thinking you might have assumed. A child who has given an incorrect answer is very likely to see the error and correct it during the explanation. Try to support children's thinking without evaluating responses. "Does someone have a different idea or want to comment on what Daniel just said?" All children should hear the same teacher reactions that only the "smart kids" used to hear.

Many times a student will get stuck in the middle of an explanation. The silence can be difficult, and there is a temptation to call on someone else to "help out" or to suggest that the child get assistance from a classmate. Though well intentioned, the message this sends to the child is that he is not capable on his own. Children must learn that time will be given and that their classmates trust and believe in one another. This attitude conveys support and confidence and is usually all that is necessary to get a quality response.

There will, of course, be times when a response is simply not forthcoming. When this occurs, you might suggest taking some time to get thoughts together or to work out the idea with some materials. Promise to return to the child later, and then be *certain* to return to hear what was figured out.

Identify Rules, Hypotheses, and Future Problems

When you are satisfied with the discussion around the answer and the solution, summarize the main points of the discussion, and make sure that all students understand what has been agreed on. When a problem involves creating a procedure, the class may decide on one or more methods of solution, and these may be written on the board. For example, suppose that the task involved finding clever methods of adding when one of the addends is a 9 (e.g., 5 + 9 or 9 + 3). Some students may have devised a method of taking 1 off of the smaller number and putting it with the 9. This might be listed as Maggie's rule: *Shift 1 from the little number to make 10. 5 + 9 becomes 4 + 10.* Other students may have added 10 to the smaller number and subtracted 1. Yolanda's rule might be written: *Add 10 instead of 9 and then take 1 off. 5 + 9 is 5 + 10 − 1.* When students invent procedures for computation or measurement formulas, you may want to help them find a good way to write them, but do not change the students' basic ideas. No student should be required to use the ideas of others, but all students must try to understand them.

Often someone will make a generalization or an observation that he or she strongly believes in but cannot completely justify. These ideas should always be listened to with interest, even if they are incorrect. Untested ideas can be written up on the board as "Andrew's Hypothesis." Explain the meaning of *hypothesis* as an idea that may or may not be true. Testing the hypothesis may become the problem for another day, or the hypothesis may simply be kept on the board until additional evidence comes up that either supports or disproves it. For example, when comparing fractions, suppose that a group makes this generalization and you write it on the board: *When deciding which fraction is larger, the fraction in which the bottom number is closer to the top number is the larger fraction. Example:* $\frac{4}{7}$ *is not as big as* $\frac{7}{8}$ *because 7 is only 1 from 8 but 4 is 3 away from 7.* This is not an unusual conclusion, but it is not correct in all instances. A problem for a subsequent day would be to decide if the hypothesis is always right or to find fractions for which it is not right (counterexamples).

Even when students have not suggested hypotheses, discussions will often turn up interesting questions that can profitably be used for tasks to help clarify an emerging idea.

Designing and Selecting Effective Tasks

A task is effective when it helps students learn the ideas you want them to learn. It must be the mathematics in the task that makes it problematic for the students so that it is the mathematical ideas that are their primary concern. Therefore, the first and most important consideration for selecting any task for your class must be the mathematics. That said, where do you look for tasks?

Your Textbook

Most teachers find their textbook to be the main guide to their day-to-day curriculum. However, when teachers let the text determine the next lesson, they assume that children learned from each page what was intended. Avoid the "myth of coverage": If we covered it, they must have learned it. Good teachers use their text as a resource and as a basic guide to their curriculum. In the face of the current pressures from state-level standards, the state curriculum guide is also an important consideration.

Using Traditional Textbooks

Traditional textbooks are designed to be teacher directed, a contrast to the approach you have been reading about. But they should not be discarded. Much thought went into the content and the pedagogical ideas. Your book can still be used as a prime resource if you think about translating units and lessons to a problem-oriented approach.

Adopt a *unit perspective*. Avoid the idea that every lesson and idea in the unit requires attention. Examine a chapter or unit from beginning to end, and identify the two to four *big ideas*, the essential mathematics in the chapter. (Big ideas are listed at the start of each chapter in Section 2 of this book. These may be helpful as a reference.) Temporarily ignore the smaller subideas that often take up a full lesson.

With the big ideas of the unit in mind, you can now do two things: (1) adapt the best or most important lessons in the chapter to a problem-solving format and (2) create or find tasks in the text's teacher notes and other resources that address the big ideas. The combination will almost certainly provide you with an ample supply of tasks.

Adapting a Traditional Textbook Lesson

Figure 4.6 shows a page from a first-grade textbook. The lesson addresses an important idea: the connection of addition and subtraction. The approach on this page is fine: A bar of cubes in two colors is used to suggest an addition and a subtraction equation, thus connecting these concepts. However, as is typical of traditional K–2 texts, there are blanks to be filled in with very specific correct answers. Student attention turns almost immediately to how to get the right numbers in the blanks. Imagine for a moment how you might help students complete this page correctly. It is easy to slip into a how-to mode that focuses more on the blanks than on addition and subtraction. Let's convert this lesson to a problem-oriented task. How can children be challenged to wrestle with this idea? You might want to try this before reading on.

One thought is to provide a bar of cubes of one color and have students break the bar into two parts. The students' task is to write addition and subtraction equations that tell what they did with the bar. They should draw a picture to show how they broke the bar.

Another idea is to create a scenario where there are two amounts, such as toy cars, on two different shelves. *In the toy store, there were 11 cars, 4 on the top shelf and 7 on the next shelf.* Have the students create two story problems about the 11 cars, one that is an addition story and another that is a subtraction story.

In the first modification, the page was translated directly to a task. In the second, a similar task was designed that did not look the same but addressed exactly the same mathematics. In each case, the students will do only one or two ex-