

Facilitator's Guide to:

- Counting More Antibodies
- HSE-Style Questions Aligned to CUNY HSE Curriculum Framework, Section 4, Mathematics, Unit 3 – Rate of Change/Starting Amount

Throughout Unit 3, students are challenged to discover relationships between rate of change, starting amount, function rules, solutions to functions and contextualized problems. One of the most comprehensive activities putting rate of change and starting amount in the context of a real world situation is **Counting Antibodies**. In this activity students are presented with a situation in which scientists tested two different medicines to determine how effective they are in producing antibodies in samples of blood.

One of the strengths of **Counting Antibodies** as an activity is that it provides students with a scaffolded approach to putting rate of change and starting amount in the context of the situation. However, when students take an HSE exam they may be asked to put rate of change and starting amount in context with far less support in a relatively short amount of time. The activity **Counting More Antibodies** is designed to begin where **Counting Antibodies** and Unit 3 leave off and transition students to questions that more accurately reflect the length and difficulty that they can expect on the HSE exam.

It is highly recommended that students complete at least **Counting Antibodies** and **The Many Faces of Function Rules and Tables** before they work on **Counting More Antibodies** and continue to the example HSE-style Questions for Unit 3.

Counting More Antibodies

1. Hand out **Counting More Antibodies** and introduce this task as a continuation of the work that students did on **Counting Antibodies**. Make it known that the intention here is to move from this task to questions that look and feel like questions they will see on the HSE exam.

The launch for **Counting More Antibodies** has students review some of the key ideas from the **Counting Antibodies** “experiments”. Students are presented with another set of medicines, C and D and tables of values with accompanying data. Students are then presented with a scaled down, intentionally unlabeled graph. Students are then asked, “Which of the lines must belong to Medicine C? How do you know?”

Give students time to consider the data and write a considered response to the question.

Once they are done, discuss the graph and tables and how students could determine which line belongs to Medicine C and which line belongs to Medicine D. Ask, “What do you notice about these two functions? How are they different? Where are they the same?”

The intention here is to draw students to the idea of starting amount and rate of change, as well as the common solution for both functions.

Ask, “Can we put any numbers on the graph?” And, “Where can we find this information?”

2. Turn to PG. 2, Writing a Linear Function Using Rate of Change and Starting Amount (& Antibodies).

Discuss with students the definition of a linear function and review the definitions for rate of change, and starting amount presented on this handout.

A function that has a constant rate of change is called a **linear function**.

The **starting amount** is the value of the output when the input is zero. This value is sometimes referred to as “b” or “y-intercept”.

The **rate of change** is the constant change in the outputs when the inputs are consecutive. This value is sometimes referred to as m or “slope”.

Then, turn back to the launch page, **Counting More Antibodies**. Have students identify the rate of change and starting amounts for Medicines C and D. Once the rate of change and starting amount have been identified using the tables provided, give students a moment to consider the next questions, “Which of the two medicines would be modeled by the function $a = 50d + 40$? Where do you look for this information?”

Discuss the answers to these questions and explain that the function for Medicine D could also have been written $y = 50x + 40$.

Have students take a moment to then continue and answer the next two questions, “How would we write a similar function for Medicine C?” and “What makes the graphs on the previous page *linear*?”

Explain that x is a variable that is commonly used to represent the input value for a function and y is a variable that is commonly used to represent the output value for a function. However, these values could also be represented by other letters such as a and/or d .

Looking to the graphs from page 1, establish and write on the board, the graph of a linear function is a straight line.

3. Turn to Page 3 and The Problem Concerning Medicine F

Towards the bottom of the page students will find a multiple-choice question. This question is our target, a question that has the look and feel of an HSE-style practice question concerning linear functions in context.

Have students read through question 1.

Then, ask questions like, “what should our table look like?” “What kinds of values belong in each column?”

The point here is to have students create their own table without having it already developed for them.

After setting the stage for the table, have students complete problems 1-5.

4. In summary, stress that problem #4 is modeled after an actual HSE problem. But this problem does not ask for, or provide, or mention a table of values. Creating a table of values is a key strategy for organizing input-output values for a function such as this.

Discuss how the table could help lead us to the correct answer for #4, and then support our answer choice.

HSE-Style Questions Aligned to CUNY HSE Curriculum Framework, Section 4, Mathematics, Unit 3 – Rate of Change/Starting Amount

Having completed Unit 3 and **Counting More Antibodies**, use the questions included in this packet to assess students' readiness on HSE-style questions related to starting amount, rate of change, linear functions, and formal function notation (linear).

The questions included here are certainly not exhaustive of all types of questions that could possibly be asked within the domain of linear functions. You may want to continue with more formal discussions of slope and y-intercept, however the problems posed by Unit 3 and in this packet are reflective of the types of questions that have been presented in past HSE exams.

Problems #7 and #8 do ask students to identify and compare the slope from a table of values and the slope found in four answer choices that are linear equations. These questions purposefully follow question #6 which asks a similar question, but in the context of an *antibodies medication*. The idea of rate of change and slope from question #6 to #7 is virtually interchangeable, and requires little to no additional skills. However, this topic may require a more formal introduction. To that end, the definition for rate of change found on **Counting More Antibodies** does reference the fact that *rate of change* can also be referred to as *slope*. Question #8 has the potential to be markedly more difficult than other rate of change questions for some students. Throughout Unit 3 there have been fewer examples where rate of change is displayed as a fraction. This is another area that may deserve more attention before you give this assessment.

Counting More Antibodies: Building Functions Using Rate of Change and Starting Amount

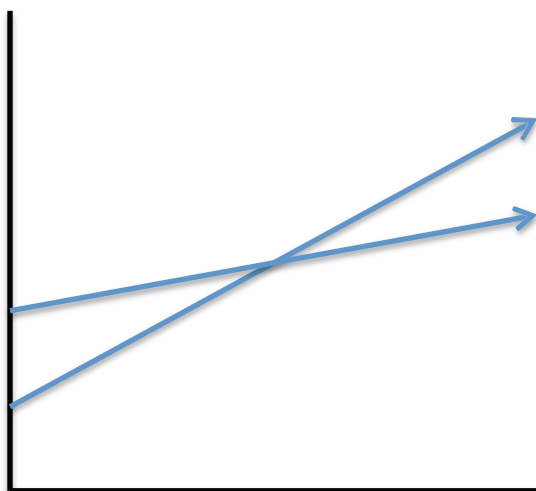
Researchers conducted tests to determine how effective two medicines are at increasing the number of antibodies in the blood of patients.

The data charting the effectiveness of the two medicines is found below.

Data for Medicine C	
Days passed	Antibodies in the sample of blood
d	a
0	80
1	120
2	160
3	200
4	240
5	280
6	320

Data for Medicine D	
Days passed	Antibodies in the sample of blood
d	a
0	40
1	90
2	140
3	190
4	240
5	290
6	340

After conducting their experiments the researchers sketched the graph below, but they made a major error. They forgot to label their graph.



Which of the lines must belong to Medicine C? How do you know?

Writing a Linear Function Using Rate of Change and Starting Amount (& Antibodies)

Linear Function - A function that has a constant rate of change is called a **linear function**.

One way to write this type of function is like this:

$$\text{output value} = (\text{rate of change})(\text{input value}) + \text{starting amount}$$

For example:

$$a = 50d + 40$$

Where we have defined the following:

The **starting amount** is the value of the output when the input is zero.

- This value is sometimes referred to as “b” or “y-intercept”.

The **rate of change** is the constant change in the outputs when the inputs are consecutive.

- This value is sometimes referred to as m or “slope”.

Looking back to Medicines C and D from the previous page, which of the two medicines would be modeled by the function $a = 50d + 40$? Where do you look for this information?

How would we write a similar function for Medicine C?”

What makes the graphs on the previous page “linear”?

The Problem Concerning Medicine F

A scientist tested “Medicine F” in order to determine how effective it is in producing antibodies. After conducting the test, the scientist developed the linear equation

$y = 35x + 200$ to model the medicine's ability to increase antibodies.

y is the number of antibodies in the sample of blood

x is the number of days that have passed since the patient took the medication

1. Create and complete a table that shows the relationship between the number of days passed and the number of antibodies in the sample of blood as a result of Medicine F.
2. What is the starting amount for the function and what does the starting amount represent here?
3. What happens after each day that passes?
4. Which statement describes the medicine's effectiveness at producing antibodies?
 - A. The patient's blood sample starts with 0 antibodies and increases to 235 antibodies after one day.
 - B. The patient's blood sample starts with 200 antibodies and increases by 35% each day that passes.
 - C. The patient's blood sample starts with 200 antibodies and increases by 35 each day that passes.
 - D. The patient's blood sample starts with 35 antibodies and increases by 200 each day that passes.
5. How can we use the table you filled in to support your answer to the question above?