UNIT 4 Using Systems of Equations to Make and Justify Choices

Systems of equations appear frequently on the HSE exam, as well as almost all college-entrance exams. Our students really struggle with this material, especially since it is so often introduced without any real-world context or discussion of its usefulness. However, developing an understanding of how systems of equations work can pay big dividends both in and out of the classroom. The problems in this unit use real-world scenarios to present a scaffolded introduction to the topic. Students will continue to explore the many different views of functions, and they will learn how systems of equations can be a useful tool for drawing conclusions and justifying choices.

SKILLS DEVELOPED

- **Writing systems of equations in two variables.**
- **Using tables and graphs to compare two linear functions.**
- **Understanding the significance of the point on the graph where two lines intersect.**
- **Using tables, graphs, and guess & check to find solutions to systems of linear functions.**
- **Seeing applications of systems of equations in science and social studies contexts.**
- **Using systems of equations to make and justify choices.**
- **interpreting systems of equations as a means of negotiation between competing interests.**

KEY VOCABULARY

equation: a math statement showing that two expressions are equal to one another.

solution to a system of equations: values that satisfy all equations in a system. The solution to a system of linear equations can be represented by a point in the coordinate plane.

system of equations: a collection of two or more equations that we have to consider at once. This unit will focus on systems with only two equations.

Core Problem Overview: Choosing a Cell Phone Plan

The previous core problems have incorporated tables, rules, and graphs to develop understanding of functions. This core problem goes a step further by asking students to reflect on the meaning of the point where two lines on a graph intersect. Before giving out the problem, ask students how they chose their cell phone plans. After students have shared some ideas, tell them we can use functions as a way to make comparisons and decisions and that they'll be looking at a two options for a cell phone plan. *Choosing a Cell Phone Plan* presents students with advertisements from two competing wireless service providers. Each provider charges a base fee—$60 for PEMDAS Wireless and $40 for CCSS Mobile—and a fee for each additional gigabyte of data used. Through a series of scaffolded questions, students will explore how the total cost for each provider grows as data usage increases.

PEMDAS Wireless	
Gigabytes	Total Cost
0	$60
1	$62
2	$64
3	$66
4	$68

CCSS Mobile	
Gigabytes	Total Cost
0	$40
1	$44
2	$48
3	$52
4	$56

Students will first interact with the problem by doing something that should be familiar by this point: completing a table of values. Some students will notice a pattern and complete these tables rather quickly, though others might need some support in order to understand how the price structure works for each provider. At this point, they are asked to make a choice about which provider they would choose based on the information in their tables. Most students will choose CCSS mobile, because the total cost is lower for every one of the inputs in the tables. Some might notice, though, that the cost of CCSS Mobile grows more quickly than it does for PEMDAS Wireless. The next two questions ask students to clarify what the starting point and rate of change are for each provider—both of these are intended to help students get closer to creating a rule that they could use to find the total cost for each company.

The rule that students create will help them to complete another table. This table uses bigger input values and will be the one students use to

construct a graph of the two functions. After students have graphed the lines for both providers, they will answer some questions intended to draw out their thinking about the shapes of the graphs and the point at which they intersect. The question at the end of this activity—about which provider has the best offer—is similar to one that they answered at the beginning, but we're looking for students to show a more nuanced understanding this time around.

TEACHING THE CORE PROBLEM

Depending on how much time your class has spent completing tables and working on functions, it might be worthwhile to have a conversation about the two advertisements before they start working. This will give students the opportunity to hear what their classmates think and may also help them to see something that they didn't see at first. To start, tell the class that they have two minutes to look over the ads, but not to write anything down yet. When time is up, ask them what they noticed. Some possible questions to get the discussion started are:

- *Which of these providers do you think would be the least expensive? Why?*
- *Think about your own phone bill and data usage. Do you use a lot of data, or a little?*
- *Can someone explain how your phone bill would work if you went with PEMDAS Wireless? What about CCSS Mobile?*

Now that students have had the opportunity to talk and get interested in the situation, ask them to work independently. As they're working, make note of student answers to the second question—"Which provider would you choose? Why?"—and have them explain their reasoning. Most students will say that they would choose CCSS Mobile, but they should explain why they would make that choice. If they say, "because it's cheaper," ask them if they think it will always be cheaper. Or ask, *What if a person used a lot of data, like 12 gigabytes? Do you think it would still be cheaper?*

The next two questions ask about "starting amount" and "rate of change." These topics are introduced in Unit 3, and so if you haven't done these yet, you might choose to remove these two questions from the activity. It's a good idea to make sure that students give more than a numerical answer for these questions. That is, if a student just writes "60" for PEMDAS's starting amount, ask them to write a little about what that means. You're looking for something like, *"I would have to pay $60 even if I don't use any data, and then it gets more expensive from there."* The same goes for their answer about the rate of change. You want students to talk explicitly about how, for example, CCSS

Mobile costs $4 more for each additional gigabyte of data used. The rate of change is included in the advertisement, but it's a good idea to encourage students to put this into their own words. It will help them when they take on the next task: creating a rule for each provider.

You may or may not have dug into variables and function notation with your class yet. If you have, you can encourage students to write their rules in function notation. So, the function for PEMDAS Wireless would look like $f(x) = 2x + 60$, and the function for CCSS Mobile would look like $f(x) = 4x + 40$. If you haven't discussed function notation or variables, ask your students to write out the process they could use to calculate the total cost for the two providers. The rule for CCSS Mobile might read something like, "Multiply the number of gigabytes by 4, and then add 40." Your students will be able to complete the activity—and gain insight into how systems of equations work—whether they use function notation or not. Try to meet your students where they are and support the method that they are using.

For the last part of the activity, students will use their rules to complete a larger table, and then they will graph each of the points from the table. Your students might need some support with the graphing aspect of this problem, depending on how much graphing work they have done up to this point. If students are struggling, help them to remember another similar graph that they have done in class. You might ask, *"When have we seen a graph like this before?"* Then help them to remember the *Commission Problem* (or one of the supplemental problems) from Unit 2. It's a good idea to check in with each student while they are graphing, to make sure they're plotting points correctly and that they recognize that each company's graph should form a line. After students have finished their graphs and answered the last two questions, move on to the processing part of the activity. This is where you have the opportunity to help your students develop a real understanding of why systems of equations are useful.

PROCESSING THE PROBLEM

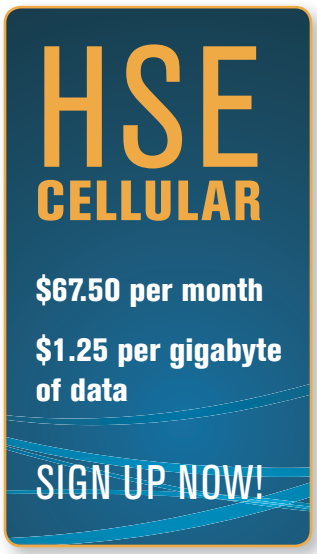
Because this problem is so scaffolded, there is not a lot of room for students to take significantly different approaches in solving it. This is helpful, though, in keeping the whole-class discussion focused on the meaning of the point where the two lines meet. To get the discussion started, ask some general questions about the graphs:

- *What do you notice about these two graphs?*
- *How are the graphs similar? How are they different?*
- *Which company's graph has a steeper line? Why is it steeper?*

Once you have allowed students to discuss their thoughts about the two graphs, ask them to focus on the point where the two graphs meet. Here, you might ask for volunteers to share their answer to question number 8 from the activity. Your goal is for students to see that if you were to use 10 gigabytes of data, both companies would have the same monthly cost. As a few volunteers talk about this, ask clarifying questions to make sure that all students are following along.

- *Can someone restate what she just said?*
- *What questions do you have about this point of intersection?*
- *How would we write this point as a solution?*

When the class has come to an agreement about this solution, write it on the board. To help students develop stronger connections between the point of intersection and the rules that they developed earlier, ask for volunteers to read their rules. You should record these on the board and ask the class if they agree with the rule for each provider. Next, ask students to plug in 10 gigabytes as the input for each, and they will see that the outputs are the same. Explain to them that this is the big idea in systems of equations: finding an input that will produce the same output for both equations. Point out that the solution that fits both functions can be found in the table, or by plugging inputs into the rules, or by looking for the point of intersection on the graph.



Now, you should feel free to open the discussion up to other aspects of the graph, and you can begin by asking students to share their answers to the last question from the handout. By looking at the graph and the table, students should talk about how PEMDAS Wireless is a better deal if you plan to use more than 10 gigabytes of data; otherwise, CCSS Mobile would be the best choice. You could also ask students to write their own word problems about the graph, and then have the class solve them together. If your students are looking for an additional challenge, create another advertisement and ask if it will intersect at the same point as the other two lines. You might try the one on the right.

And finally, if you have done the *Commission Problem* already, this would be a good time to ask students to think about how the two problems are similar and how they are different. Help your students to see connections between the work they did on the *Commission Problem* and the work they did on this one. You could ask questions like:

- *What do you remember about working on the Eric and Nancy problem? Did it help you with this one?*
- *Which one was more challenging? Why?*
- *Try to remember the graph of Eric's income and Nancy's income. Did it look like this graph? In what ways was it similar?*

- *What mathematical tools did you have this time around that you didn't have when you worked on the Eric and Nancy problem?*

SUPPLEMENTAL PROBLEMS

- **The Price of Math Books**
  This problem is very similar to the core problem in terms of its structure and mathematical content, but it also helps students to see how systems of equations have real-world application and it builds their background knowledge about a fundamental concept in economics: supply and demand. The numbers used in this problem are small, so that students can plot supply and demand on a standard one-quadrant graph. After students have done this activity, you could talk about how supply and demand actually operates on a much larger scale. They can think of these numbers as being a small sample that is used to represent a bigger picture, and you could lead this into a discussion of proportional reasoning. One idea might be to tell students that this represents the supply for just one store, and then ask them to calculate the publisher's expected revenue for 1,500 stores.

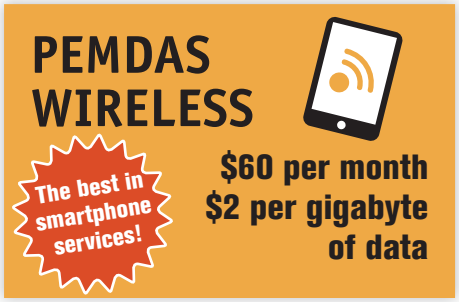
- **Picking Apples**
  This problem is also similar to the core problem, but it incorporates decimals. It also emphasizes the idea that systems of equations can be used to make choices. This time, however, students will not have the benefit of a graph. They will need to rely on a table or guess and check, as well as their knowledge of starting amount and rate of change, to find the point at which both orchards cost the same.

- **Another Commission Problem**
  This problem revisits the *Commission Problem* from Unit 2. Eric and Nancy have both gotten raises from their employers, and they want to figure out how many fish tanks they would each need to sell so that they could bring in the same amount of money in a given month. The difference here is that Nancy has created functions—written in function notation—to calculate their respective incomes. Students will need to know function notation in order to solve this problem. The problem could also be used as an introduction to function notation. Just ask students to read through the first part of the problem, and then go back to talk about how function notation works and what it means.

Core Problem

Choosing a Cell Phone Plan

Bernard is trying to choose a data plan for his smartphone. He narrows his decision down to two providers: PEMDAS Wireless and CCSS Mobile. When he searches online, he sees an advertisement for each of the providers.



1. Using the information from the advertisements, complete the tables for both providers.

PEMDAS Wireless	
Gigabytes	Total Cost
0	
1	
2	
3	
4	

CCSS Mobile	
Gigabytes	Total Cost
0	
1	
2	
3	
4	

2. Based on the data in the table, which provider would you choose? Why?

3. What is the starting amount for PEMDAS Wireless? What is the starting amount for CCSS mobile? What do these numbers mean in the context of the question?

PEMDAS Wireless:

CCSS Mobile:

4. What is the rate of change for PEMDAS Wireless? What is the rate of change for CCSS mobile? What do these numbers mean in the context of the question?

PEMDAS Wireless:

CCSS Mobile:

5. For each provider, determine a rule that could be used to calculate the total cost for any number of gigabytes used. You should write two different rules.

PEMDAS Wireless:

CCSS Mobile:

6. Using your function rules for each provider, complete the tables below. Make sure to calculate carefully! You will be using these tables to graph the functions.

PEMDAS Wireless	
Gigabytes	Total Cost
0	
2	
4	
6	
8	
10	
12	

CCSS Mobile	
Gigabytes	Total Cost
0	
2	
4	
6	
8	
10	
12	

7 For each provider, plot the points from the table and connect them. Be sure to label each graph!

8 What is the significance of the point where the two lines intersect?

Supplemental Problem 1

The Price of Math Books

Consumers and businesses are constantly engaged in a tug of war. As consumers, we want to spend as little money as possible, and businesses want to make as much money as possible. It is a balance. Consumer power is the power to not buy—in most cases, a business can't just charge whatever it wants for something. If they charge too much, no one will buy it. A business's power is in its power to choose what to produce or sell. If consumers are not willing to pay enough for them to make a profit, they won't make or sell that product.

Imagine that everyone in your class wants to buy a math book to study for the HSE exam. If the book was priced at $200.00, would anyone in your class want to buy it? Probably not. What if the book was priced at $4.00? In this case, almost everyone would be willing to buy a copy. This example shows that when the price of an item is very high, then few people want to buy it.

When the price of the item goes down, more and more people are willing to buy it. So, we can say that the demand for the math book is higher when it is priced at $4.00 per copy. The demand for the math book is low when it's priced at $200.00 a copy.

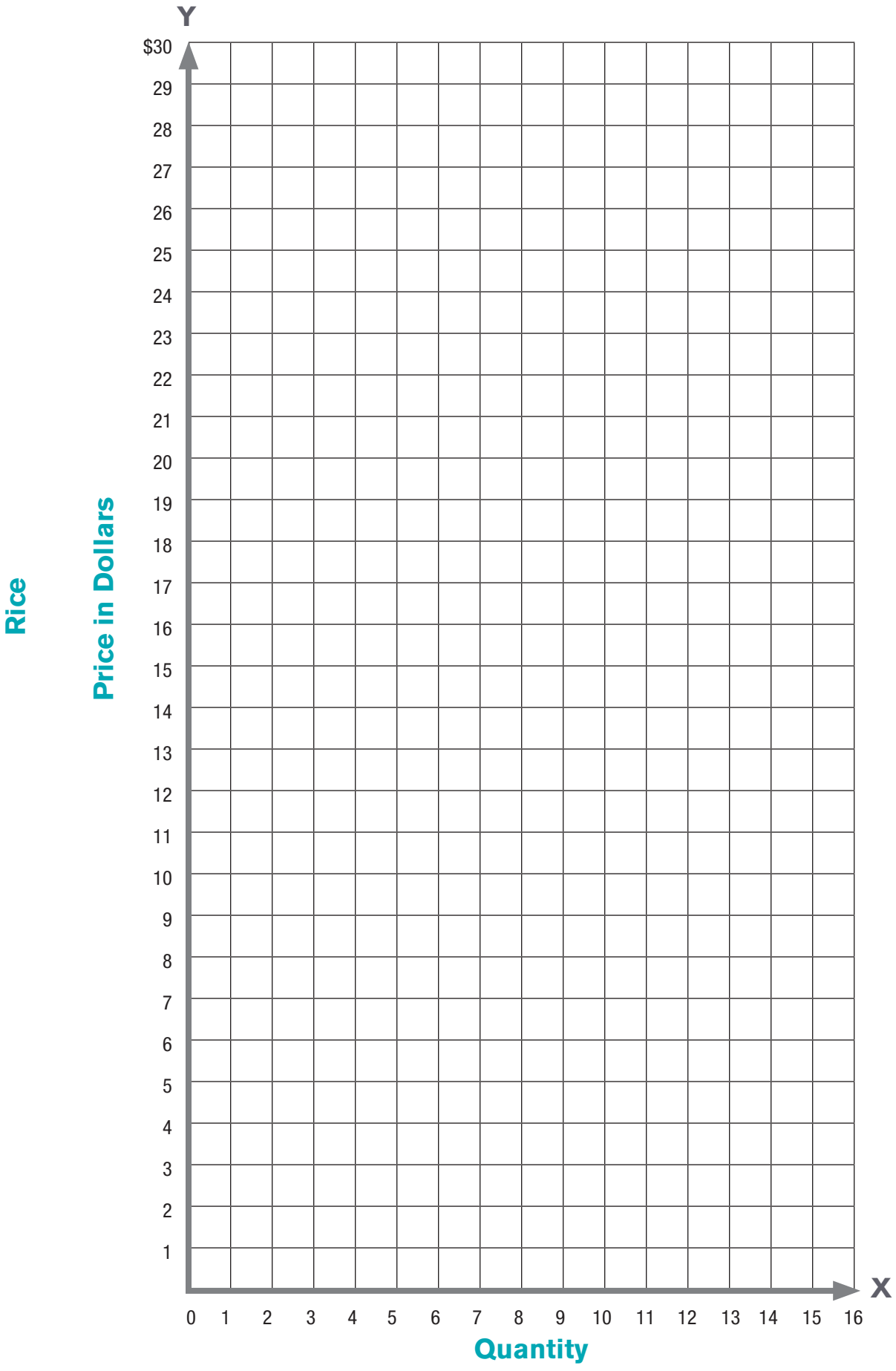
The table to the left gives an example of how many people would want to buy a math book at each different price.

Price of Math Books	Quantity Demanded
$4	12
$10	9
$16	6
$22	3
$28	0

1. What is the relationship between the price of the math books and the number of books that people would want to buy?

2. Notice that in the graph on the next page, the ***x-axis*** represents ***Quantity*** and the ***y-axis*** represents ***Price***. Plot each of the five points in the table. Why is the Quantity Demanded graph decreasing?

3. How many math books would be demanded by the students in the class if the price was $14? What if the price was $8?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Rice
Price in Dollars
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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Quantity

Supply

Price of Math Books	Quantity Supplied
$8	1
$10	3
$12	5
$15	8
$20	13

Now let's think about supply. Supply refers to how many items a company would want to produce.

If you were a publishing company, you would not want to use your resources producing books if you weren't going to be able to charge enough to make a profit, especially since producing the books costs money. But if you knew that you people would be willing to spend $28 or more on books, you would want to produce a lot of them because you could make more money.

This table to the left shows how many books a publisher would want to supply at a bookstore for several different prices.

1. What is the relationship between the price of math books and the quantity that the publisher would want to supply?

2. On the same sheet of graph paper that you used for demand, plot the five points and connect them. How would you describe the shape of the ***Quantity Supplied*** graph?

A graph of supply and demand is useful because it represents a balancing point between what consumers want and what companies are able to produce. In economics, this balancing point is called **equilibrium**.

3. How is the supply graph similar to/different from the graph for demand?

4. Look carefully at your two graphs. The publisher needs to determine how many books to supply and how much they should cost. Based on your graphs of supply and demand, help them make a final decision.

   Quantity to supply:

   Cost:

Supplemental Problem 2

Picking Apples

Anna and Chris want to pick apples. They find two orchards right next to each other; David's orchard and Pam's orchard.

The signs below are at the entrance to the orchards:

1 Anna wants to pick 6 pounds of apples.

a. How much does this cost at David's Orchard? _____
Show your calculations.

b. How much does it cost at Pam's Orchard? _____
Show your calculations.

2 Chris has $30 to spend.

a. How many pounds of apples will he get if he goes to David's Orchard?_____ Explain how you figured it out.

b. If Chris goes to Pam's Orchard, how many pounds of apples will he get?_____ Explain how you figured it out.

3. Which orchard is a better deal? Be prepared to explain your reasons to someone who disagrees with you.

4. One student says that apples are cheaper at David's orchard, and another student says that they are cheaper at Pam's. Can both of these statements be correct? Why?

Adapted from a task created by the Mathematics Assessment Resource Service

Supplemental Problem 3

Another Commission Problem

This version is a sequel to the Commission Problem in Unit 3: Three Views of a Function.

Eric and Nancy both had a successful year selling fish tanks for their respective employers, and so they were both given raises.

Eric: Eric's base salary is still $1400, but now he makes a commission of $100 for each fish tank that he sells.

Nancy: Nancy now gets a base salary of $500 per month, but her commission has stayed the same at $250 per fish tank.

Eric and Nancy still want to make sure that they contribute the same amount to their total monthly income, and Nancy proposes using algebra to figure out how many fish tanks that they would each need to sell. She tells Eric that he can calculate his monthly income by using the formula $f(x) = 100x + 1400$. She says that she can calculate her own salary by using the function $g(x) = 250x + 500$.

1. What does the variable x represent in Nancy's formulas? How do you know?

2. Who makes more if they each sell 4 fish tanks? Show all your work.

3. Who makes more if they each sell 8 fish tanks? Again, show all your work.

4. How many fish tanks would each person need to sell so that they made the same amount of money?

Good instruction spirals backwards and forwards, reinforcing new ideas and building off of old ones.