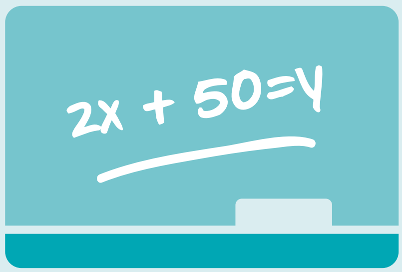
THE CUNY HSE CURRICULUM FRAMEWORK

SECTION 4

Math: Problem-Solving in Functions and Algebra



Contents

The CUNY HSE Curriculum Framework
2015

Principal Authors

Kate Brandt *Social Studies: Integrating Reading and Writing Curriculum Framework*

Rebecca Leece *Science: Matter and Energy Curriculum Framework*

Mark Trushkowsky *Math: Problem Solving in Functions and Algebra Curriculum Framework*

Eric Appleton *Math and Science: Contributing Author and Curriculum Framework Production Coordinator*

Associate Author

Tyler Holzer, *Math: Problem Solving in Functions and Algebra Curriculum Framework*

Special Acknowledgement

Steve Hinds, for his visionary contributions to CUNY HSE math instruction

Significant Contributions

Erna Golden, Shirley Miller

Acknowledgements

Ellen Baxt, Gary Dine, Arnitta McKinley, Joan Stern, Ramon Tercero, Kevin Winkler

Project Director: Leslee Oppenheim

Design and Layout: Renée Skuba | Graphic Design

Video Production: Kieran O'Hare, Zachary Timm

The authors would like to thank their students in the Spring 2015 CUNY HSE Demonstration Class who inspired their teaching and writing.

Sabrina Abreu
Adam Alicea
Janet Alicea
Netea Banks
Eileen Berrios
Asbury Brown
Tiffany Carrian
Fabio Castro
Iquis Dickerson
Sandra Eisenberg
Julien Fils
Renee Gulliver
Brad Lee
Ron Lee
Edith Leon
Lashana Linton
Angel Osorio
Roxanne Perez
Miyako Smith
Natasha Williams

The City University of New York, Office of Academic Affairs

John Mogulescu
Senior University Dean for Academic Affairs and Dean of the School of Professional Studies

Suri Duitch
University Dean for Continuing Education and Workforce Development

Leslee Oppenheim
University Director of Language and Literacy Programs

For information about the CUNY Adult Literacy/HSE Program—its services, educational curricula and instructional materials, or to inquire about training options, please contact CUNY's HSE Professional Development Team at:

http://literacy.cuny.edu

This project was made possible through WIA Incentive Grant funding from the U.S. Department of Labor, with support from the New York State Department of Labor, Division of Employment and Workforce Solutions, in collaboration with the New York State Education Department, Office of Adult Career and Continuing Education Services.

THE CUNY HSE CURRICULUM FRAMEWORK

SECTION 4

Math: Problem-Solving in Functions and Algebra

Overview

Defining Math and Mindset

Imagine traveling alone in a country where you don't speak the language. Maybe you learn a few words like "please" and "thank you" (and become more polite than you are in your native language). Maybe you memorize a whole sentence. You're not sure what it means, but you know if you say it at the right time, people smile. But if you say it at the wrong time, people seem annoyed or disappointed. You can't for the life of you figure out what is the right or wrong time to say it. Slowly, the isolation takes over and leaves you rigid and anxious. It is an anxiety that comes from hearing everyone around you speaking a language you don't know, that is different from the one inside your mind. You can feel like you've lost your identity, like you have no sense of agency. You can't express who you are and it can feel like you are not really there.

Unfortunately, when it comes to math, this experience (and resulting mindset) is how many adult students feel when they enter our classrooms. At the root of this alienation is the way students define math and mathematical learning.

HOW DO ADULT EDUCATION STUDENTS FEEL ABOUT MATHEMATICS?

- Math is a club where only certain people—"math people"—are allowed, and I am not one of them.
- Math is a set of procedures that make no sense that you just have to memorize.
- Math is something you learn by sitting passively and watching someone do a procedure on the board.
- Math is about getting the answer.
- All you need to do is learn all the formulas and you will understand math.
- Doing math means repeating that same procedure to answer 15-25 identical problems.

Sections:

- Being good at math means being fast.
- Understanding math means getting all the answers correct.
- Math is something that doesn't have to make sense—if you know the keywords, they will tell you what to do.
- Math has little to do with "me" (except in terms of money).
- Math has nothing to do with communication, creativity or intuition.
- There is one right way to solve a math problem—so if you don't know it, you might as well put down your pencil and wait for the teacher to show you.

Not all of our students feel this way about math, but many of them do. And there is a strong correlation between students who believe these statements and students who have real problems and unproductive struggle with math.

How is it that students from all across the U.S. and from countries all over the world have the same misconceptions about what it means to do, learn and understand math? It has to do with the way almost all of us were taught math. Not many of us had teachers who told us those things explicitly (though I have heard some stories). But still the message got across through the way we were taught. Even those of us who were successful in our own math education are not necessarily immune.

It is telling how many adult education math teachers say they are learning more math as teachers than they ever did in school. That statement really says a lot about how adults (and people in general) learn math. As teachers, we not only have to work on math problems, but we need to find engaging ways to talk about mathematical concepts. We need to find different strategies and ways to represent and explain things when students aren't understanding. We need to try to understand each student's reasoning, both when it is correct and when it is not. Each of those aspects of teaching are activities at the heart of what it means to do and study mathematics.

> **Mathematics is an "interconnected body of ideas and reasoning processes… [learned through] collaborative activity in which learners are challenged and arrive at understanding through discussion."** (Swan, 2005)

There is a lot we can do through the choices we make as teachers. We can give students the genuine experiences they need to reshape the harmful preconceived notions of what math is and who they can be in a math classroom. For example, we can present problems before showing students how to solve them. By putting the problem before the explanation, we put student thinking and reasoning at the heart of our lessons. We convey to students that math can come from them, that they can develop strategies for working on problems they've never seen before.

One classroom routine that can get students to question the idea that math is just a set of procedures to be memorized is an activity called, *"Sometimes, Always, Never."* The basic idea is you give students a mathematical statement and ask them to categorize it as always being true, sometimes being true or never being true. If it is true, students have to give an example, and if it is not true (or not always true), they need to provide a counterexample. For example, you might ask students to consider, "Difference means subtraction." This statement is sometimes true, but not always. A counterexample might be, "There is a 35 year age difference between Paulina and her son. If Paulina's son is 8, how old is Paulina?" Students who think "difference" always means subtraction will subtract 8 from 35 and say that Paulina must be 27. *"Sometimes, Always, Never"* is an important activity to do multiple times over the course of a semester. The more you do it, the more you will see your students bring the question to other aspects of your class—when learning a new content, when considering a classmate's strategy, they'll begin to question the boundaries and exceptions of mathematical statements. For more on this idea, look for the article "13 Rules that Expire" cited in the math resources section.

STUDENT MISTAKES ARE EXPECTED, RESPECTED AND INSPECTED

Students tend to think of mistakes in math as failures and as proof that they "are just not good at math." For teachers, when student mistakes or misunderstandings come out, it can be tempting to move past them as quickly as possible. Teachers often keep calling on different students until the correct answer comes out. As an alternative, what if we asked questions designed to draw out those common mistakes and misconceptions? What if we treated student mistakes like the discoveries they are and framed them as opportunities for everyone to learn something? We need to hear from students who don't know the answer, as much as we need to hear from those who do. A mistake that is not addressed is usually a mistake that will be repeated. A student mistake can also serve as a "canary in a mineshaft"—if one student is capable of making it, others might as well, even if they didn't this time.

For more on how to use student mistakes, check out the following reviews on CollectEdNY.org:

- http://www.collectedny.org/2015/03/my-favorite-no-a-great-way-to-celebrate-student-mistakes-in-math/
- http://www.collectedny.org/2015/04/mistakes-in-math-expected-respected-and-inspected/

A FINAL WORD ON WHAT IT MEANS TO DO MATH

The Common Core Mathematics Standards are composed of two parts of equal importance. The content standards address what math content students should learn at different grade levels. The 8 Standards for Mathematical Practice (MP), are a redefinition and a guide for what it means to do math and what it means to be a mathematically proficient student. The 8 MPs are what students should be doing in math classes at every level, with whatever content they are learning. Phil Daro (2015), one of the writers of the CC math standards has said the MPs are a way to define the content of students mathematical character. Too many of our students say, "I am not a math person." They say that because to them, being a math person means already knowing the math content. The 8 Standards of Mathematical Practice are a way of redefining what it means to be a "math person"—it's not how much math content you know, but more about your attitudes and what you do to make sense of and learn math. The 8 MPs define the way mathematical knowledge comes together and gets used, as well as productive attitudes and habits of mind in the study of math. The math lessons and activities in this curriculum framework offer ample opportunities for students to explore all 8 Math Practices.

FOOTNOTE: Elements of the Overview as well as Units 1, 2, and 3 draw on and adapt pieces of curricula written by Steve Hinds, especially his lesson set titled, *Functions Rule*.

Underlying Teaching and Learning Principles

The following research-based teaching principles are at the center of the work we do at the CUNY Adult Literacy and High School Equivalency Program. These principles informed all of the work in this document, from the content selection down to the design of each problem.

CORE PRINCIPLE #1

Implement a Content-Based Approach

> **"The model of the student as an empty vessel to be filled with knowledge provided by the teacher must be replaced. Instead, the teacher must actively inquire into students' thinking, creating classroom tasks and conditions which student thinking can be revealed. Students' initial conceptions then provide the foundation on which more formal understanding of the subject matter is built."**
>
> – from *How People Learn: Brain, Mind, Experience and School* (2004)

TEACH DEPTH OVER BREADTH

We need to teach with Focus and teach less, so students can learn more.

One of the biggest challenges we face in adult education math instruction is the amount (and difficulty) of material we feel pressured to teach and that students are expected to learn. All teachers want what is best for their students and sometimes it feels like we should try to cover as much as possible, to give them a taste of everything they might see on their HSE exam.

Now more than ever, it is important to remember both the short-term and the long-term learning goals we have for our students. Both the Common Core and the research back up what our experience tells us. When students receive instruction that is "a mile wide and an inch deep," they often lack the ability to apply what they know to new situations, or remember what they've already "learned." Many of us have asked ourselves questions like, "My students knew how to do this two weeks ago, what happened?" or "Why don't my students see that they can use what we learned to solve that problem to solve this one?" The answer to both questions is the same: *Without a deep enough understanding, students won't transfer what they know to new situations or to retain what they do in class.* It may feel strange to spend more time on fewer topics, but if we try to cover everything, we end up having to re-teach things anyway, often right in the middle of a lesson on a completely different topic.

"You can't make a plant grow by pulling on it, you only make it rootless."
—Student

We should accept that we cannot teach everything and then make some choices about how we will focus our resources, especially our time. We should not let any assessments work us up into such a panic that we lose sight of some of our greatest strengths—our practice of starting from where students are and our serious respect for their learning processes. As a student once told me, "You can't make a plant grow by pulling on it, you only make it rootless."

The math problems students will see on their HSE exams will be far more complex than what we find in workbooks and what was on the old G.E.D. Just knowing formulas will not be enough. Experience with routine problems that can be answered simply by using a memorized set of procedures or steps will not be enough. At the heart of teaching with focus is time. We need to give students time. Time to work on and struggle with complex problems. Time to present different solution methods. Time to discuss, appreciate and analyze each other's methods. Time to debate and to write in math class. Time to revise their work. Time to reflect on what they are learning, what they understand, how they understand it and what questions they still have. Students feel great urgency to find shortcuts and move quickly to the test. It is our responsibility to slow the classroom down so that people learn the content.

There are several examples of the "teach less, learn more" philosophy working. Here is one:

> The TIMSS (Trends in International Mathematics and Science Study) is one of those international tests you often hear about, on which students in the US do not perform particularly well. Because it is an international test, it is not written to favor one country's curriculum over another. In fact, the curriculum in Singapore only covers 45% of what is on the TIMSS test. The U.S. curriculum covers 80% of what is on the TIMSS test. And yet students in Singapore were number 1 in the world on the TIMSS, while the

U.S .was in the middle of the pack. In this case, U.S. students may have covered more topics, but they did not develop a deep enough understanding to transfer it to different problems. The students from Singapore were not hurt by the fact that the learned less than half of the content on the TIMSS assessments. The student from the US may well have been hurt by curricula that tried to cover too much too quickly.

In the spirit of narrowing how much content we cover, this curriculum framework targets functions and algebraic reasoning. Concentrating on these topics allowed us to create a greater coherence between the topics and to go deeply into some high yield topics that will position students for mathematical success on their HSE exams, college, career and life. Focusing deeply on the major concepts in these two topics will allow students to secure the mathematical foundations, conceptual understanding, procedural skill and fluency, and ability to apply the math they have learned to solve all kinds of problems—inside and outside the math classroom.

AIM FOR COHERENCE

Teaching with coherence means that each lesson is not a new event, but builds on the knowledge students bring to each activity/concept/class. It also means making explicit connections between math topics, problems, and solution methods. Limiting the math we teach, focusing on high yield content allows us the time we need to help students develop an understanding of the structure of mathematics as a discipline, which helps with both retention and transference. Aiming for coherence means helping students understand how things fit together—and that they are related in the first place.

CORE PRINCIPLE #2

Provide Scaffolded Instruction

We need to start where our students are, or we run the risk of leaving them behind. Teachers of adult math learners often have to contend with "mixed-level" classrooms—classrooms with a dramatically wider range of student abilities than are generally found in the other subject areas. Sometimes, the mixed-level can even be embodied in a single student. You might have an immigrant student who is very good at calculations, but who struggles with word problems. You might have a student who works in construction and can solve complex problems involving volume and operations with fractions, but who can't read a graph. At the core of our scaffolded math instruction are rich, non-routine problems that can be approached with a wide variety of student methods, from drawing pictures, guess-and-check and writing equations. Teachers prepared with support questions and extension questions can keep an entire class of "mixed-level" students working on the same problem and engaged in the productive struggle at the heart of each student's proximal zone of development (ZPD). ZPD is a concept developed by Lev Vygotsky that looks at three kinds of learner experiences—there are the things that can do without help, there are the things they can't do and there are the things they can do with some guidance. The third category is the ZPD and where learning happens. We offer students problems that have a low entry and a high ceiling. A problem is a "low entry" if it is accessible to wide range of different different students, where every student can begin and find their own level of engagement. It has a "high ceiling" if there are a lot of possibilities for students to get into challenging mathematics, where the problem can be extended for students who need to be challenged further in order to say in their ZPD. The lessons, teacher supports and additional resources to follow are all built around these types of problems.

People learn math best when they can start with intuitive understandings, move into concrete manipulation, then into representational, and finally abstract and communicative levels of understanding and discussing mathematical concepts (Sharma, 1988).

Here's an example of a scaffolded progression:

In exploring how multiplication is about sets or groups, a teacher might start by flashing a photo of a carton of eggs and asking the class how many eggs they saw. Students will easily say 12. Did you count them all? No, I just knew that eggs come in a dozen. This is the intuitive level.

The next step may be to show a baking tin full of 12 muffins in 3 rows of 4 each. If we ask students to tell us how many muffins are there without counting each individual muffin, then ask how they know, they may say that there counted by 3's or counted by 4's or a student may say that there are 3 rows of 4, so they multiplied 3 times 4. This is the concrete level.

The next question may ask students to use colored tiles to create as many rectangles as they can with 12 tiles. Students may show rectangles formed of these dimensions: 1×12, 2×6 and 3×4. This could also be considered the concrete level.

A next question may ask students to use grid paper to draw rectangles with 24 squares (also concrete), but then ask for a number of squares that can't be drawn on the given grid paper. The resulting drawing of, say, a rectangle 60 squares tall and 20 squares wide would become representational, since it would no longer be efficient to count every square.

Finally, a symbolic relationship between the length, width and the area could be discovered: **length × width = area**. This is a representational level of understanding.

Each of the steps from intuitive to concrete to representational to abstract are scaffolds for students' thinking. Some students may need intuitive and concrete ways of thinking more than others in order to understand the concepts, but the opportunity for all students to use these ways of thinking and to communicate with each other about them deepens everyone's knowledge and helps prepare students for abstract thinking.

Our principle of providing scaffolded instruction is explored further in the Lessons, the Teacher Supports and especially in the Adaptation section.

CORE PRINCIPLE #3

Stimulate Active Learning

ENGAGE STUDENTS IN PRODUCTIVE STRUGGLE

Look up "Perseverance" in the dictionary and it will say something like:

> **"To persist in or remain constant to a purpose, idea, or task in the face of obstacles or discouragement."**

In order for students to develop perseverance in mathematical problem-solving, they have to learn how to work through struggle. They have to build up some experiences of feeling stuck, sticking with it and having a breakthrough. They will not be able to do this if we step in too soon or too often.

I like to think of our role as teachers as similar to that of a weight-lifting spotter in a gym. If the spotter keeps their hands on the weight and just lifts, he or she will be the one who gets stronger, not the weight lifter. A good spotter watches the person lifting weights and lets them do the work. When the lifter gets stuck, the spotter offers words of support and encouragement. If the lifter still can't proceed, the spotter helps, just enough to get them past, sometimes only using a few fingers, and does the least amount of lifting they can. The spotter keeps the lifter able to work and develop beyond instances of struggle. It may be helpful to think of our work with students in a similar way. You can't get stronger or develop perseverance watching someone else lift weights. Students have to learn to work through struggle, not stop and wait for someone else to do the work when they get stuck. As a general rule, we should try to never take the pencil out of a student's hands.

This can be one of the hardest things for us as teachers to do. It is very tempting to just show students how to solve a problem as soon as they get stuck. It is often what they want us to do and if we do show them, they will be thankful and happy, which makes us feel great. But when we do that, what are we teaching them about their ability and independence? How are we preparing them to keep going the next time they struggle?

We should be honest with our students and tell them that we are preparing them for HSE exams and college and life, all of which will give them problems they've never seen before. In math class, we need to build our tolerance to uncertainty and struggle. We need to separate ourselves from the notion that math problems are like sitcom problems, solved quickly and neatly to perfect resolution. Our students need to understand that struggle is not a bad thing. Too many adult students interpret struggle as a deficit on their part. As soon as they start to struggle, they put down their pencils and say things like, "I just don't get it. I'm not good at math." Reacting to struggle that way makes it more difficult to learn, since working through struggle is a necessary part of the learning process.

When students are struggling, we should aspire to only ask them questions—and to ask as few questions as it takes to get them moving on their own.

- When students ask a question about one of the conditions that make the problem "problematic", encourage them and reflect question back to them
- Answer most questions with *"Good question. What do you think?"*
- When students start to shut down, get them talking. Ask them to describe the situation in their own words. Ask them what they've tried so far.
- When students are stuck, suggest a strategy—for example, *"Can you draw a picture?"* or "*What could the answer be? Is there a way you can check that? What have you tried from our list of problem-solving strategies?"*

CHALLENGING, NON-ROUTINE PROBLEMS

> **"A problem is defined…as any task for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method." –Hiebert (1997)**

The problems found in the math framework fit the above definition and the criteria for a good problem below:

- The problem should allow for different approaches/solution methods.
- The problem should have a low entry and a high ceiling, meaning it should allow for students at different levels to approach the problem in a way that makes sense for them (you might have a lower level student who is able to work on the problem drawing a picture, whereas a more advanced student might create a chart or an equation).
- Students should be unable to proceed immediately towards a solution.
- It should promote discussion, both of different approaches and of targeted math concepts or problem-solving strategies.

Many teachers have gotten the message that the Common Core and HSE math is more rigorous and more difficult than what students faced on the GED. This increased rigor is often understood as more advanced mathematical topics. This is only part of the story. The other part of the story is that students are going to need to face problems that will require them to make choices, try different things, change course if necessary, adapt and be flexible in their thinking and know for themselves when they are done and if they are correct. Instead of being centered around worksheets where students are answering a lot of questions, this framework is built around students working on one problem at a time. The problems allow for teachers to bring in formal mathematics after students have brought their rich thinking, sense-making and communication to bear. The problems are designed to draw the mathematics out of our students and build from there.

CORE PRINCIPLE #4:

Facilitate and Plan for Collaborative Learning

> **"No matter how kindly, clearly, patiently, or slowly teachers explain, teachers cannot make students understand. Understanding takes place in students' minds as they connect new information with previously developed ideas, and teaching through problem-solving is a powerful way to promote this kind of thinking. Teachers can help guide their students, but understanding occurs as a by-product of solving problems and reflecting on the thinking that went into those problem solutions." –Diana Lambdin**

EMPHASIZE METHODS OVER ANSWERS

Learning math is a collaborative activity. This includes group work and student presentations. After students work on a problem, we shift our attention to exploring our methods. While you are interacting with students during their group work, be on the look out for which strategies you'd like to discuss and start thinking about the order in which you want those presentations to go. In general, you want to look at the more concrete strategies first and then move towards the more abstract ones. Start with the strategy that you think is most accessible to all of your students and then look at the ones that connect to the specific math content you want to explore.

Students tend to think that math ends once you have the answer. We can help them see beyond that by taking seriously the learning that happens after students already have the answer to the problem. It is not uncommon for student presentations and the discussion of different solution methods to take just as long as it took for students to work on the problem, if not longer. There is a lot of mathematics to be learned after the problem is done. If we honor that, students will learn to honor it as well. And once they realize how much they get out of the discussions, it can make a large impact in their ideas about how people learn mathematics.

When students present their strategies, we want other students to engage with them. We want to help students make their thinking understood and we want other students to understand the thinking of their classmates.

Here are some questions that can help achieve these goals:

- Constantly ask, *Can you show us how you did that?*
- When a student presents their thinking and part of their reasoning is unclear, ask them to tell the class more about what they did there.
- When students present their thinking, give other students an opportunity to ask questions—if they don't have any, ask at least one question. You can also ask the rest of the class questions about the strategy. You want them to realize that being able to explain what was done in their own words is the threshold for understanding.
- After a student explains their thinking, ask someone else in class to explain a potentially confusing aspect of the student's thinking.
- After a presentation, ask students to turn to a partner and take 2 minutes to talk about something they appreciate about the strategy. Share out afterwards, having students speak directly to the student(s) whose strategy they are appreciating.
- Once a few strategies have been discussed, you can start asking questions like, *How are these strategies different? How are they similar?*

Help everyone realize who they should be listening to during student presentations. One strategy is to sit down in the seat of the student who is presenting. This will take the focus off of you, as the teacher. Let students do the talking, including clarification of their ideas.

FOCUS RESPONSIBILITY WITH STUDENTS.

- Don't exert authority by saying what is right or wrong.
- Respond to most student explanations with, *What do the rest of you think?*
- Ask, *How do we know this answer is correct?*
- Model thinking and powerful methods. When students have done all they can, the teacher can demonstrate other approaches. If this is done at the beginning, however, students will simply imitate the method and not appreciate why it was needed. Whenever possible, teachers should draw from presented student work.

CORE PRINCIPLE #5

Make Time For and Encourage Metacognition and Self-Regulated Learning

We need to give students time to reflect on their sense-making process. After doing math (working on problems, discussing those problems and analyzing different strategies), students need time to pair share and/or write to help them think about what happened. You can have students write in math journals, or you can give them exit tickets, with the few questions you want them to respond to. Either way, you want to collect the responses as often as you can, provide feedback and return them to students.

Here are some examples of things you want students to be thinking about:

- What was challenging about this problem? Where did you get stuck? What did you do when you got stuck?
- What strategies worked for you?
- What was the best mistake you made today? What did you learn from that mistake?
- How did you get started on the problem?
- How is this problem similar to other problems we have worked on?
- What was the best question asked in class today—it could have been asked by you, another student or the teacher. How did that question help you?
- What do you want to remember about your work on this problem?
- What are two things you want to remember about today's class? What questions do you still have?
- How do you think what we did today is connected to what we were doing last week?
- If you could go back in time to the moment before you started working on this problem, what advice would you give yourself?
- What did you learn from working on this problem? What did you learn from explaining your strategy? What is one thing you learned from someone else's strategy?

- What did <student's name> do to help make her thinking and strategy clear to us?
- What general problem-solving strategies can we add to our class list?
- In ten sentences, summarize what we did today for a student who is not here.

When appropriate, we also want students to know why we are making the choices we are making as teachers. When we discuss different solution methods and go from concrete to more abstract, we can tell students we do that because that is how people learn (and ask them how it felt). When students ask us if their answer is correct and we answer by asking them, "What do you think?", we should explain that we are not trying to frustrate them, but instead to help them be independent and develop their ability to judge the validity of their answers.

CORE PRINCIPLE #6:

Problem-Solving Strategies are Integrated into Content Learning

In addition to the math content, we want our students to build a toolbox of problem-solving strategies. We want students to think of these strategies as things they can do to help themselves make sense of challenging problems. These strategies help with perseverance when we frame them as "things you can do when you have no idea what to do."

SOME EFFECTIVE PROBLEM-SOLVING STRATEGIES:

- Draw a picture/Make a visual representation
- Organize your thinking/Make a chart
- Guess and check
- Work backwards
- Act it out
- Work on a similar and simpler problem
- Use a model
- Create an equation

To see the classroom video, **Respecting Problem-Solving Strategies: The Handshake Problem,** visit the CUNY HSE Curriculum Framework web site at http://literacy.cuny.edu/hseframework.

Basic Structure of a Math Lesson Plan

Here is the basic structure that each of the lessons and teacher supports in the math section follow.

1 Launch

Math lessons begin with a launch, which prepares students for the core problem of the day. The launch can be a discussion, writing assignment, open-ended question, photograph, video, or anything else that helps get students thinking about the context of the problem to come. It should be accessible enough that every student feels comfortable contributing.

2 Problem-Posing

Teacher engages students in a task. This might just be giving out the core problem. It might be giving students the situation—without the actual math problem/question—and asking them to create a visual representation of the situation. It might be giving students a problem and a list of problem-solving strategies and asking students to circle all of the strategies they think might be helpful in making sense of the problem.

3 Student Work

Student have the opportunity to work on their own at first, and then in groups. The first phase of the student work is their engaging with the problem. The second phase is having students work out how they are going to explain what they did to the rest of the class. I learned a great process for this from Billy Wharton, a New York City teacher. Billy gives students pieces of newsprint/butcher paper to work on and has them fold the paper in half. On one half, they do all of their work—this is the messy side. On the other half, they rewrite and add labels to their process so that it will be clear to others during their presentation. Many students will need to do this several times before they get better at it.

4 Student Presentations and Debrief

Students present their reasoning and problem-solving process to their classmates and analyze each other's work.

5 Teacher Summary

This is an opportunity for teachers to make explicit connections to the mathematical content objectives for the day and student work on the core problem. This is when teachers might introduce vocabulary and formal notation (*"You know that relationship you noticed? Well, in mathematics there is a name for it…"*). It is also a moment where a teacher might speak to particular mathematical habits of mind they saw that they want to celebrate.

6 Reflection

Students are given time to look back over the day's class. It is a moment for students to consider what they did and be explicit about what they learned. Whatever form the reflection takes, teachers can use this reflection as a final formative assessment for day.