The Border Problem

1. Put picture of Border Problem on the board.
2. Say, “I’d like you to work on this problem mentally – no writing or talking. I’d like you to figure out how many squares are in the colored in portion, without counting one by one.”
3. Ask, “So what did everyone come up with?” As students share their answers, write down each one, but do not reveal which one is correct.
4. It is likely that some students will say either “40” or “38” as possible answers. “40” is a common error – it involves multiplying the 10 blocks on each side by 4 sides. The problem with this method is it involves counting each corner square twice. Of course, you should not say that. Ask one of the students who say “40” to come up and demonstrate their method first. Sometimes they will catch their mistake when they have to explain it, using the diagram. If not, when you ask the class to comment on the method, someone will raise counting the corners twice.
5. If every student gets “36” and no one says either “40”, ask “By a show of hands, how many of you had 40 first? Where does that 40 come from?” Do the same if 38 doesn’t come up as an answer.
6. Methods for coming up with “36” – As students explain their method, it will likely fit one of the following mathematical sentences. Keep track of each method, by writing the student’s name and the sentence that describes what the student did.
   Here are six methods:
   a. 4 x 10 – 4
   b. 10 + 9 + 9 + 8
   c. 10 + 10 + 8 + 8
   d. 4 x 8 + 4
   e. (10 x 10) - (8x8)
   f. 9 x 4
7. If students do not raise any of these, you can work the process in reverse. After students have shared all the methods that they used, you can put one of the above mathematical sentences on the board and ask students to explain it, using the picture.
8. Once you have all 7 methods, ask students to compare the different methods.
   You might ask them:
   Which of these methods is similar? How are they similar? How are they different?
9. Without the benefit of a picture, ask students to visualize a similar square, but one that is 6 x 6. Ask them to calculate how many squares are in the colored-in portion. If students raise their hands quickly, tell them to keep them down, so others do not get discouraged. Say you are more interested in how they calculate the answer - so if they have one already, ask them to try other methods.
10. Going back to the mathematical sentences in Step 6, ask how each student whose name is written next to the sentence would do the 6 x 6 problem.
11. The final step is how can we write each method so we can figure out the colored-in portion for any sized square.