Tools of Algebra:
Expressions, Equations & Inequalities

Fast Track GRASP Math Packet

Part 1

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Welcome!

Congratulations on deciding to continue your studies! We are happy to share this study packet on expressions, equations and inequalities. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we often see on high school equivalency exams. If you study these topics carefully, while also practicing other basic math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn the math.

At the end of the packet, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanation and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We also hope you will share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **http://www.acces.nysed.gov/hse/hse-prep-programs-maps**

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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Introduction

History of Algebra

4	9	2
3	5	7
8	1	6

Algebra was developed by different civilizations many, many years ago. More than 3000 years ago, Egyptians in North Africa used algebra when building the pyramids, conducting business, and measuring how stars and planets moved across the sky. Around the same time, the Babylonian civilization in present-day Iraq used algebra to measure land and do business. The ancient Greeks used geometry and drawing to further develop algebra. In India, mathematicians were solving algebraic equations more than 1500 years ago. 1000 years ago in China, mathematicians explored different kinds of algebra. They also invented some of the earliest magic squares, like the ones shown on the right. Try adding numbers in rows or columns. What do you notice?

Living about 1200 years ago in what is now Iraq, a famous Islamic mathematician and astronomer named Al-Khwarizmi (pronounced *al-kwar-is-me*) wrote a book of instructions for solving equations. The full title of the book could be translated as “The big book of finding solutions by balancing and completing equations.” The book was written as a practical guide for using algebra to answer questions in business, land ownership, inheritance, and construction. The title of Al-Khwarizmi’s book included the Arabic word, *al-jabr*, which is where the word *algebra* comes from. *Al-jabr* referred to how a value from one side of an equation can be moved to the other side, finding a solution while keeping the equation in balance.

Al-Khwarizmi also advocated the use of Hindu-Arabic numerals 1, 2, 3, 4, 5, 6, 7, 8, and 9, which are the same numerals we use today. He learned these symbols from Indian mathematics. At the time, Islamic mathematicians didn’t use letters or symbols in algebra. All algebra was done with regular words. A mathematician at the time might have written, “A number multiplied by two and added to five,” instead of writing $2x + 5$.

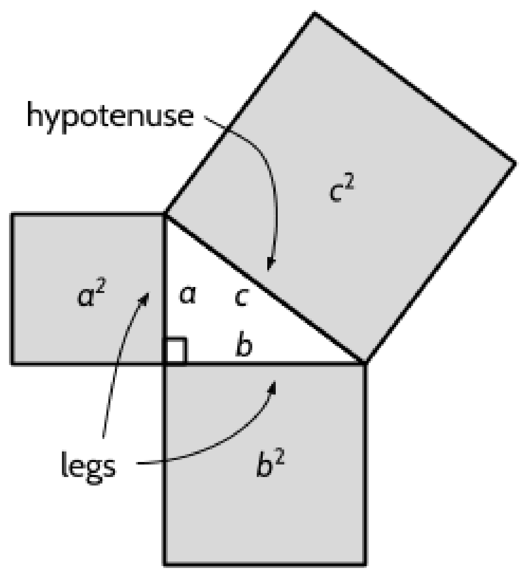
The use of letters as symbols in algebra started about 400 years ago. In England, Thomas Harriot, introduced the idea of using letters to represent unknown quantities. He wrote aa to mean a times a (an unknown number multiplied by an unknown number). These days, we would write this as a^2, which is a modern way of writing exponents developed by Rene Descartes, a French mathematician who also lived in the 17th century. Descartes used x, y, and z to refer to unknown quantities. This is where the common instruction to “Solve for x” came from.

Why did Descartes choose the letter x to represent an unknown number? One theory for why Descartes chose $x$is related to how books were printed at the time. Printing presses used individual wooden letters, with a limited number of each letter to use. Printers (the people who ran the printing press) often had a surplus of the letters x, y, and z since those letters were used less often to write French words. Some people think Descartes chose x because there were extras lying around.

What is Algebra?

Algebra has become a tool that is used in all types of mathematics. In school, it is often taught as a separate topic from geometry, statistics, probability, or even functions, which are themselves a type of algebra. However, algebra can be used to understand all of these kinds of math, as well as other topics, such as fractions, decimals, and percents. Algebra is a way of doing math that brings together all the other parts of mathematics.

So, what is algebra? Algebra is many things, so it can be hard to summarize in a simple definition. Algebra is the mathematics of recognizing patterns, making predictions, and making rules about how math works in every situation, not just in one specific case. It allows us to take what we know in one situation and apply it to many situations.



For example, in the **Lines, Angles, & Shapes: Measuring Our World** study packet, we learned that, in right triangles, the square of the length of one leg added to the square of the length of the other leg equals the square of the length of the hypotenuse. This was discovered many, many years ago by people from different cultures and time periods who

recognized patterns in the measurements of the sides of a right triangle[1].
Here are some possible measurements of the two legs and hypotenuse in a right triangle[2]:

$3^2 + 4^2 = 5^2$	$5^2 + 12^2 = 13^2$
$6^2 + 8^2 = 10^2$	$7^2 + 24^2 = 25^2$
$9^2 + 12^2 = 15^2$	$(1.5)^2 + (2)^2 = (2.5)^2$

This relationship of the measurements of the legs and the measurement of the hypotenuse will be true of any triangle you can draw, as long as it has a right angle. Knowing this pattern, we can represent the pattern with algebra. We can say that in a right triangle, if a is the length of one side, b is the length of the other side, and c is the length of the hypotenuse, then a^2 added to b^2 is <u>always</u> equal to c^2. We can write this as: $a^2 + b^2 = c^2$. Different measurements can be substituted for the variables a, b, and c and the equation will still be true.

The equation $a^2 + b^2 = c^2$ is an example of how we can use the tools of algebra. The letters *a*, *b*, and *c* can represent numbers. An *equation* is a number sentence with a quantity on one side that is equal in value to a quantity on the other side of the equals sign. In this case, $a^2 + b^2$ on one side is equal to c^2 on the other side of the equation. If we know two of these measurements, we can always find the third measurement. For example, we can use algebra to answer this question:

If $a = 9\ in$ and $c = 41\ in,$ what is the length of *b*?[3]

To find the length of side b, we can manipulate the numbers on both sides of the equation, while keeping the equation balanced, just like Al-Khwarizmi did 1200 years ago. These are some of the algebraic skills you will practice in this packet.

The algebraic equation $a^2 + b^2 = c^2$ helps us understand all triangles with a 90-degree angle. The rule is true for all right triangles. This shows how we can use algebra to explain patterns we see and prove rules about how mathematics works in general. Algebra helps us generalize and make rules for all mathematical situations. In this example, we use algebra to describe a pattern in geometry, but algebra can be used in all kinds of mathematics. To develop algebraic rules, we observe patterns, continue the pattern, and create rules. You will practice this skill more when you study functions in the other algebra packets.

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[1] You can read more about the Pythagorean Theorem in **Lines, Angles, & Shapes: Measuring Our World, Part 2**.
[2] You might test these calculations with a calculator to make sure they are all true.
[3] Look in the practice test questions section of Part 2 to find the answer.

Algebra as a Tool

Algebra can be used to solve real problems since the rules of algebra work in real life and numbers can be used to represent the values of real things. Many fields such as physics, engineering, and computer programming use algebra constantly. It is also useful to know in health care, construction, and business, especially accounting. Scientists use algebra to predict the size of a population that grows at a certain rate over many years. People working in business use algebra to calculate how an investment might grow. For example, algebra can be used to answer the question, *How much money would you have after 20 years if you invested $10,000 at 3% interest?*[4] Accountants use variables in algebraic formulas as part of spreadsheets that track the financial health of companies. In construction, algebra is used to make sure a concrete foundation is square and level and provides proper support for the building above.

Depending on the job you have, you may not have to use algebra, but understanding algebra can often be helpful in making the job easier or finding answers in more efficient ways. Understanding algebra gives you access to the tools that professionals use to be successful in their work. You might also find yourself using algebra on the job when you wouldn't expect it. For example, algebraic formulas are used in the healthcare field, where calculating the correct amount of medicine is very important.

Algebra is also useful in solving problems in many kinds of school math. For example, you can use algebra to find the volume of different geometric figures using these formulas from the HSE exam reference sheet.

Cylinder:	$V = \pi r^2 h$
Pyramid:	$V = \frac{1}{3}Bh$
Cone:	$V = \frac{1}{3}\pi r^2 h$
Sphere:	$V = \frac{4}{3}\pi r^2$

In this packet, we will practice using the tools of algebra. We will learn to use *variables*, *expressions* and *equations*, and then use those tools to solve a variety of problems. For example, we will practice using formulas like these for volume.

[4] You can learn more about calculating growth in investment in **The Power of Exponents**.

Language and Skills Practice

Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

equality · equation · evaluate · expression · generalize · operation · term · variable

In this first activity, you will think about each word and decide how familiar you are with it. For example, think about the word "cube." Which of these statements is true for you?

- I know the word "cube" and use it in conversation or writing.
- I know the word "cube," but I don't use it.
- I have heard the word "cube," but I'm not sure what it means.
- I have never heard the word "cube" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "area" might look when you're done:

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
cube	✓				like a box, all the sides are the same length

Complete the table on the next page.

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
equality					
equation					
evaluate					
expression					
generalize					
operation					
term					
variable					

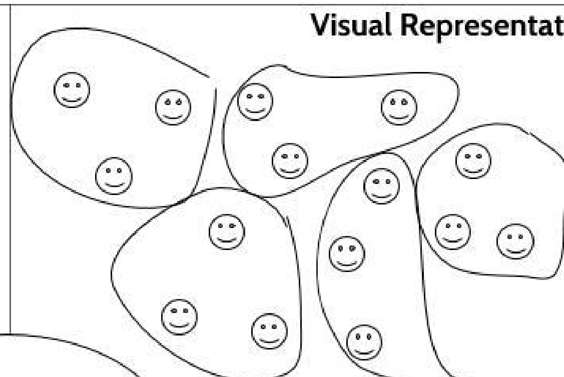
Using Graphic Organizers to Learn Vocabulary

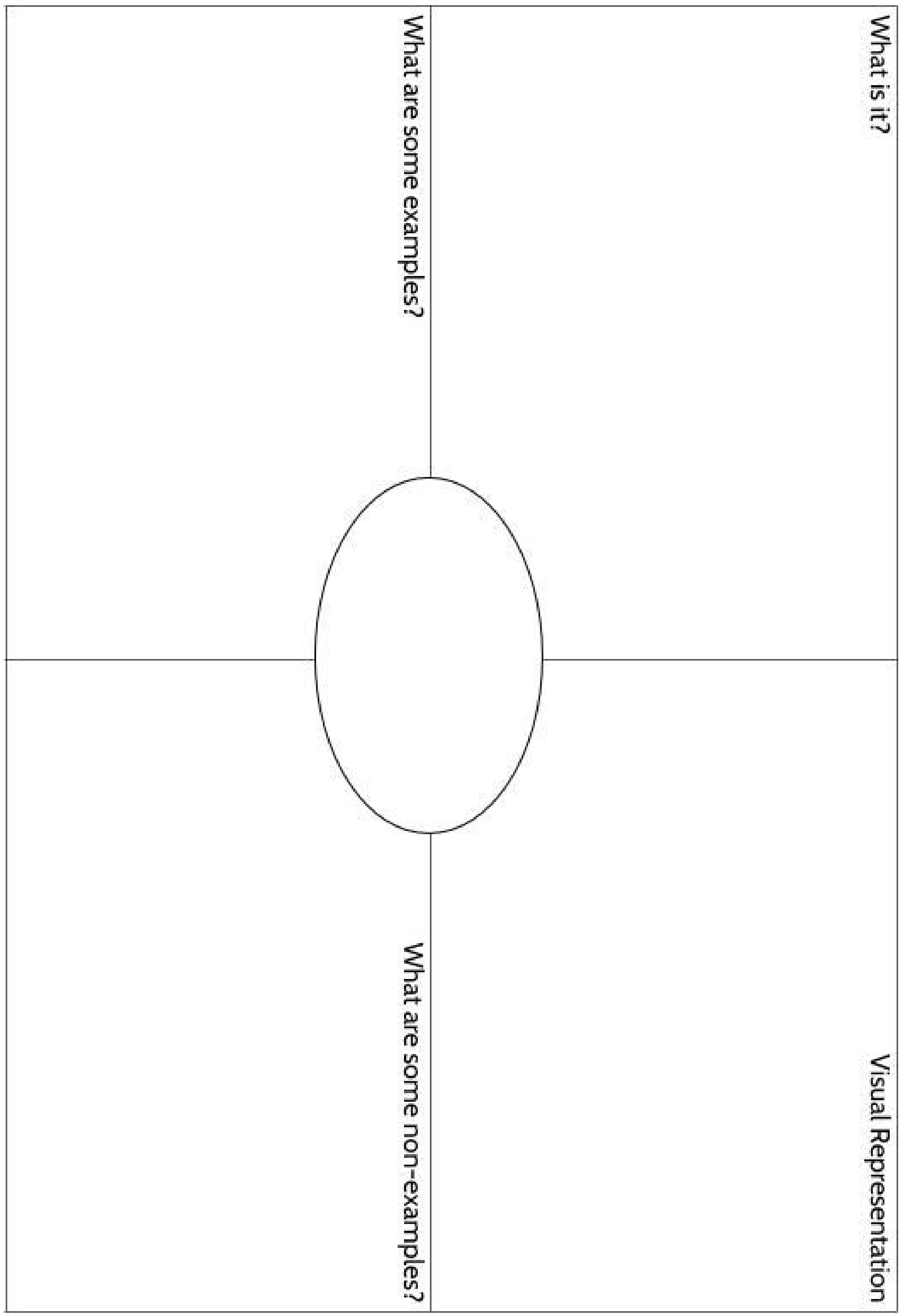
In order to learn math vocabulary, we need to practice using words in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.

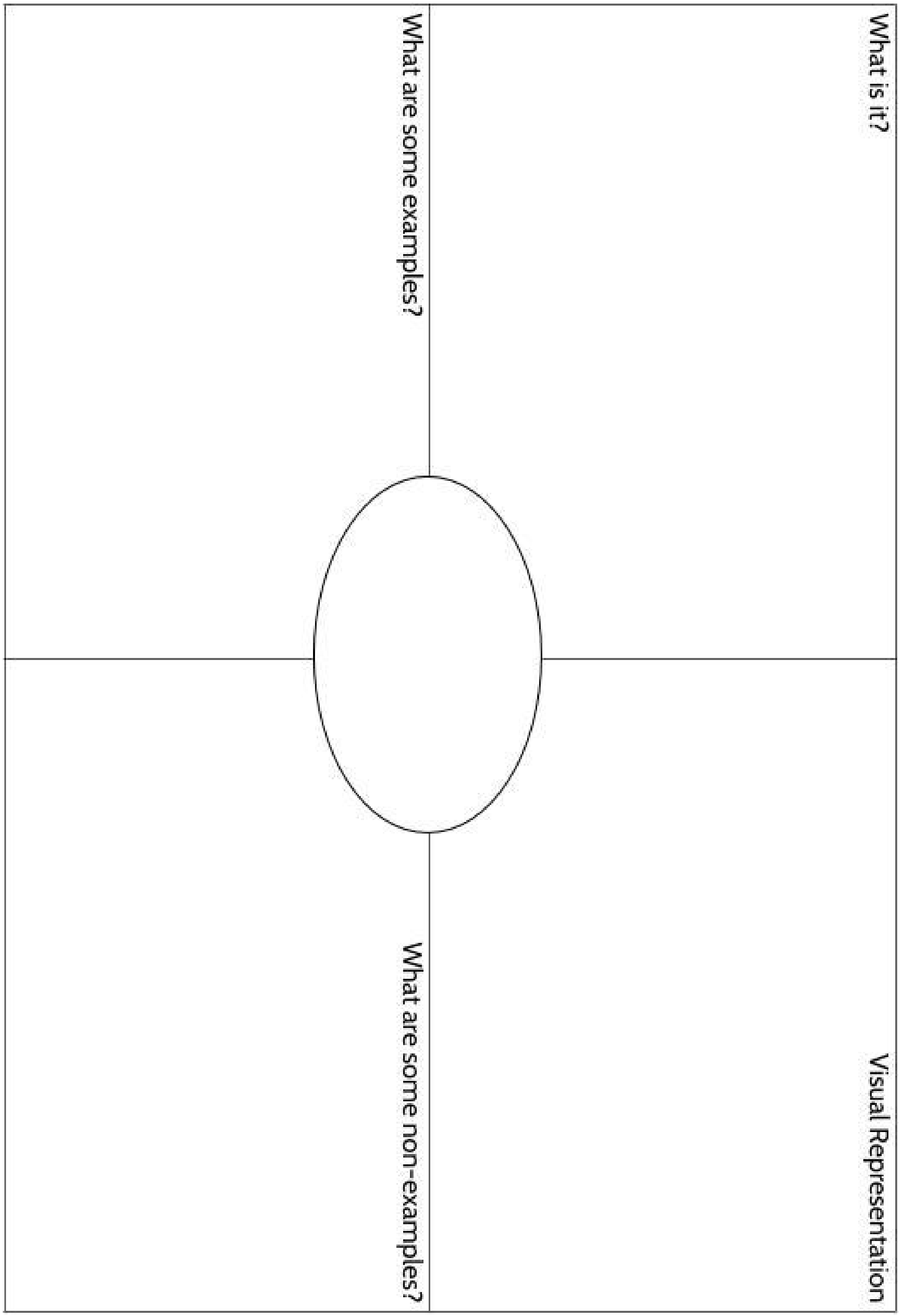
To start, choose a word from the packet and complete the graphic organizer:

- **What is the definition of the word?** You can look at the vocabulary review on page 111 for help. Write the definition in your own words to really make the word yours.
- **Make a visual representation.** You can make a drawing or diagram that will help you remember what the word means.
- **What are some examples of the word you're studying?** Below you can see that there are examples of *quotients*, which are the answers to division problems.
- **What are some non-examples of this word?** These are things that are **not** the word you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

quotient

What is it?	**Visual Representation**
A quotient is the result of dividing one number by another. It is the answer to a division question.	
What are some examples?	**What are some non-examples?**
15 divided by 3 equals **5** 66 ÷ 6 = **11** 63/18 = **3.5** 5, 11 and 3.5 are quotients in these calculations. population ÷ area = **density**	4 times 6 equals 24 18 + 5 = 23 17 – 2.5 = 14.5 3.5 × 18 = 63


What is it?
What are some examples?
Visual Representation
What are some non-examples?


What is it?
What are some examples?
Visual Representation
What are some non-examples?

The Multiplication Table

The multiplication table is something you might have first seen a long time ago. It is full of patterns and algebra.

1) Complete this multiplication table.

×	0	1	2	3	4	5	6	7	8	9
0										
1	0									
2	0	2								
3	0	3	6							
4	0	4	8	12						
5	0	5	10	15	20					
6	0	6	12	18	24	30				
7	0	7	14	21	28	35	42			
8	0	8	16	24	32	40	48	56		
9	0	9	18	27	36	45	54	63	72	

2) Did you find any shortcuts that made it easier to fill out the multiplication table? What were the shortcuts?

You may have noticed the following:

- Any number multiplied by 0 is 0. For example: $9 \times 0 = 0$

$\times$	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0

- Any number multiplied by 1 is itself. For example: $7 \times 1 = 7$

$\times$	0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8	9

- The numbers in the bottom left part of the multiplication table are repeated in the top right. For example: 3×6 is 18 and 6×3 is also 18.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	35	45	54	63	72	81

- The diagonal numbers (0, 1, 4, 9, 16, 25, etc.) are the result of multiplying a number by itself: 0×0, 1×1, 2×2, 3×3, etc. These are called square numbers.

0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9
0	2	4	6	8	10	12	14	16	18
0	3	6	9	12	15	18	21	24	27
0	4	8	12	16	20	24	28	32	36
0	5	10	15	20	25	30	35	40	45
0	6	12	18	24	30	36	42	48	54
0	7	14	21	28	35	42	49	56	63
0	8	16	24	32	40	48	56	64	72
0	9	18	27	35	45	54	63	72	81

3) **Find the smallest multiple of 9 that has only even[5] digits.**

Here are the first ten multiples of 9:

$9,\ 18,\ 27,\ 36,\ 45,\ 54,\ 63,\ 72,\ 81,\ 90$

- 36 is a multiple of 9, but it has only one even digit: 6. The first digit, 3, is odd.
- 45 is also a multiple of 9, but the second digit, 5, is odd.

Can you find a multiple of 9 where all of the digits are even?

There is something you may have noticed about multiples of 9: The digits always add up to a multiple of 9. Here are a couple examples: 54 → 5 + 4 = 9 and 198 → 1 + 9 + 8 = 18. Try adding the digits together of other multiples of 9 to see if it also works.

You can use this fact to see if any number is divisible by 9. For example, is the number 1935 divisible by 9? We can check it by adding all the digits of the number together: 1 + 9 + 3 + 5 = 18. Since 18 is a multiple of 9, then 1935 is divisible by 9. To double-check, you could divide 1935 by 9. The answer is 215 (with no remainder).

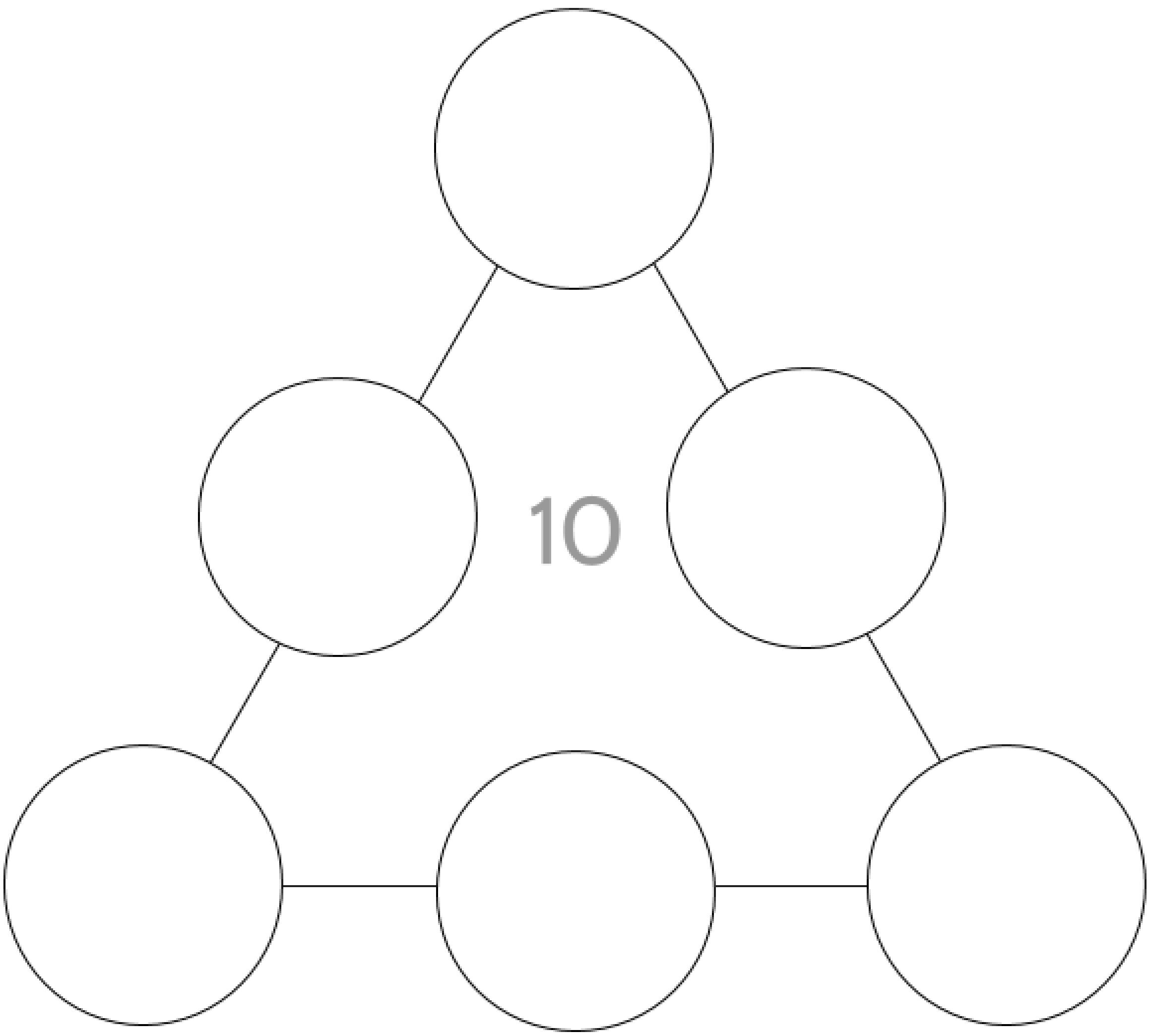
4) Is the number 4568 a multiple of 9? Add the digits 4 + 5 + 6 + 8 to find out.

[5] An even number is a number that can be divided by 2. For example, the following numbers are even: 2, 4, 6, 8, 10, 12, 14, 16, etc.

Magic Triangle

This puzzle is similar to the magic square first invented by the Chinese more than 2000 years ago. In the magic triangle, we place the numbers 1 through 6 in the circles, using each number only once.

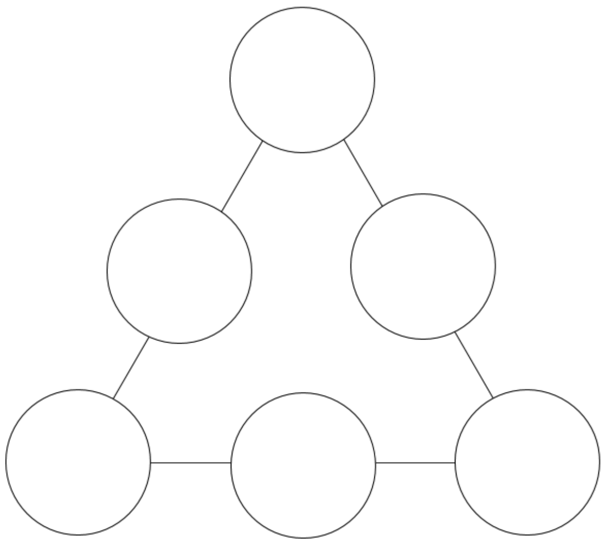
5) To solve the puzzle below, the numbers in the three circles on each side should add up to the same sum. For the triangle below, each side should add up to 10. If you can, use a pencil!



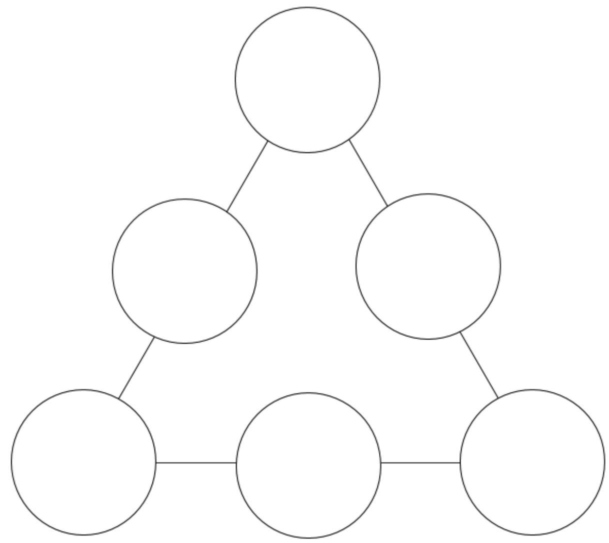
1 2 3 4 5 6

10 is one of the four possible sums for the sides of a magic triangle with the numbers 1, 2, 3, 4, 5, and 6. Can you find the three other possible numbers the sides can add up to?

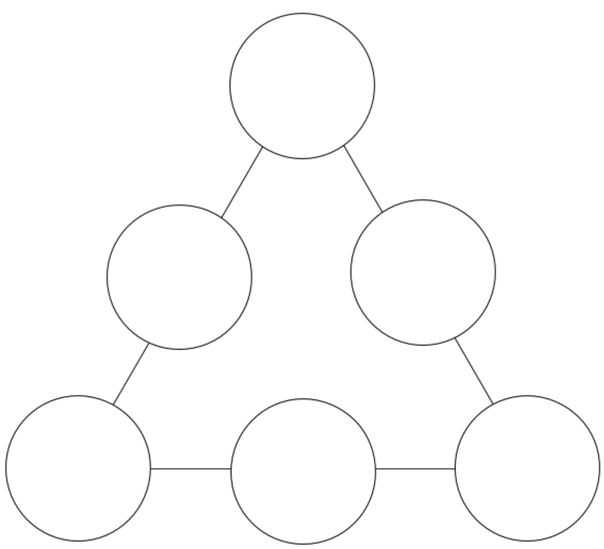
Here are some blank triangles to use for practice.



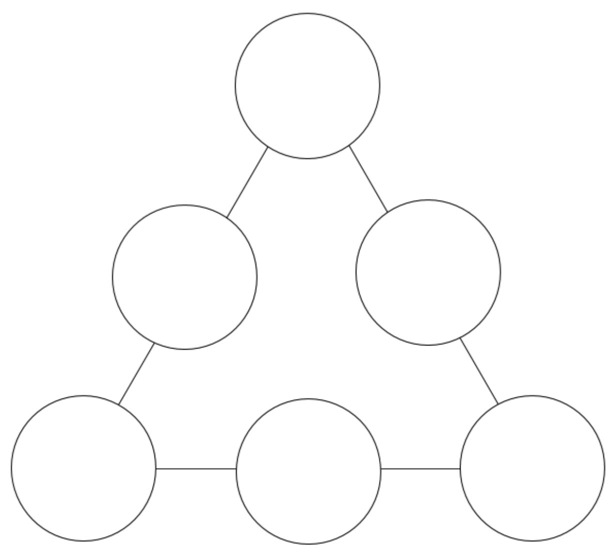
1 2 3 4 5 6



1 2 3 4 5 6



1 2 3 4 5 6



1 2 3 4 5 6

Answer Key - Language and Skills Practice

1)

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	35	45	54	63	72	81

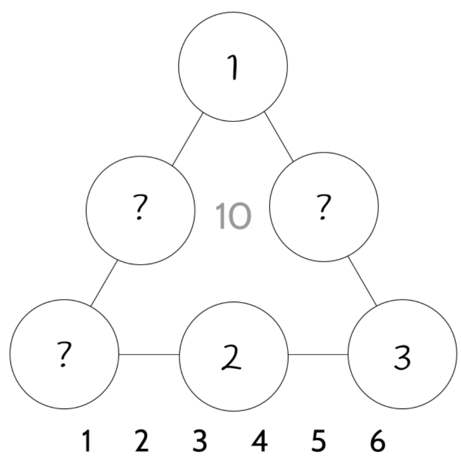
2) Look at page 15 for some things we noticed.

3) 288. Look at the table below to see all the multiples of 9 up to 360. Do you think there are other multiples of 9 where all the digits are even?

Table of the First Forty Multiples of 9			
$9 \times 1 = 09$	$9 \times 11 = 99$	$9 \times 21 = 189$	$9 \times 31 = 279$
$9 \times 2 = 18$	$9 \times 12 = 108$	$9 \times 22 = 198$	**$9 \times 32 = 288$**
$9 \times 3 = 27$	$9 \times 13 = 117$	$9 \times 23 = 207$	$9 \times 33 = 297$
$9 \times 4 = 36$	$9 \times 14 = 126$	$9 \times 24 = 216$	$9 \times 34 = 306$
$9 \times 5 = 45$	$9 \times 15 = 135$	$9 \times 25 = 225$	$9 \times 35 = 315$
$9 \times 6 = 54$	$9 \times 16 = 144$	$9 \times 26 = 234$	$9 \times 36 = 324$
$9 \times 7 = 63$	$9 \times 17 = 153$	$9 \times 27 = 243$	$9 \times 37 = 333$
$9 \times 8 = 72$	$9 \times 18 = 162$	$9 \times 28 = 252$	$9 \times 38 = 342$
$9 \times 9 = 81$	$9 \times 19 = 171$	$9 \times 29 = 261$	$9 \times 39 = 351$
$9 \times 10 = 90$	$9 \times 20 = 180$	$9 \times 30 = 270$	$9 \times 40 = 360$

4) 4568 is not a multiple of 9. You can check this by adding 4 + 5 + 6 + 8, which equals 23. The number 23 is not a multiple of 9, so 4568 is not a multiple of 9 either. You can also check with a calculator to see that 4568 ÷ 9 = 507.555... Since the answer to this division problem is not a whole number, 4568 is not a multiple of 9.

5) Here's a head start on a solution. Can you find the other missing numbers?



There are four different magic triangles possible, each with a different sum that each side must add up to. You can make magic triangles with sums of 9, 10, 11, or 12.

Equations and Expressions

Equality

In this packet, we will be working with different kinds of *number sentences*. In some of these number sentences, we see that something is equal to something else. For example, $8 + 2$is equal to 10. We can show this with the number sentence $8 + 2 = 10$. A number sentence which shows one thing equal to something else is called an *equation*.

We will also work with other kinds of number sentences where the left side doesn't have to be equal in value to the right side. For example, $8 + 2$is bigger than 9. We can write this in a number sentence like this: $8 + 2 > 9$. A number sentence where the two sides don't always equal each other is called an *inequality*.

We will start this section with the equals sign $=$, which is an important part (maybe the most important part) of understanding algebra.

1) What number is missing from this number sentence? How do you know?

$$7 + 3 = \square + 4$$

2) Is this number sentence true?

$$15 = 7 + 8$$

3) Fill in the blanks on both sides to make a true number sentence.
Use any numbers you like.

____________________ = ____________________

Some number sentences are true and some are not. Test your knowledge of true number sentences on the next page.

Number Sentences - True or False?

Look at each of the number sentences in the first column and decide whether it is true or false. Then explain your choice.

	Number Sentence	T or F	Why?
4)	$5 \times 0 = 5$	F	Any number multiplied by zero is zero.
5)	$12 \times 1 = 12$		
6)	$7 + 0 = 7$		
7)	$8 + 4 = 7 + 5$		
8)	$2 + 5 = 5 + 2$		
9)	$7 = 7$		
10)	$7 = 2 + 5$		
11)	$7 = 9 - 2$		
12)	$4 \times 6 = 6 \times 4$		

Here are a few examples of equations (number sentences using an $=$ sign):

$$9 + 6 = 15 \qquad\qquad 8 - 3 = 5 \qquad\qquad 5 \times 6 = 30$$

Interestingly, when algebra was first developed by Arabic mathematicians more than 1000 years ago, they didn't use symbols like $+$, $-$, or $=$. The *plus sign* is a little more than 650 years old, the *minus sign* is about 550 years old, and the *equals sign* is about 450 years old. The mathematicians who originally created algebra didn't write with symbols. Instead, they wrote in full sentences in everyday language, which may have looked something like this:

> The number six added to the number nine is equal in value to the number fifteen.
>
> The number three subtracted from the number eight is equivalent to the value of the number five.
>
> Taking a quantity of six items and repeating them five times will produce a quantity of thirty.

Reading the math in these full written sentences is hard to follow, right? It is much easier to look at a number sentence such as $5 \times 6 = 30$ and understand the symbols at a glance. The symbols we use for math are tools that were invented by people to make mathematics more efficient. Just like a hammer is a tool that was invented for driving nails and building houses, mathematical symbols are tools to make communication with mathematics easier. It is amazing how much mathematics ancient people were able to create without the tools we have now.

Here are some of the symbols we will use as tools in this packet:

$$+ \quad - \quad \times \quad * \quad \cdot \quad \div \quad / \quad \sqrt{} \quad > \quad < \quad \geq \quad \leq \quad = \quad \neq \quad (\;)$$

It's okay if you aren't familiar with all of these symbols yet. We will introduce each symbol as we use it in the packet.

Equations

Up to now, we have been using “number sentence” or "equation” to refer to something like this: $2 + 5 = 7$. The word *equation* comes from the word *equal*. It is also related to the word *equate*, which means “to make equal” or “to treat as equivalent.” So, the word *equation* literally means the process of making things equal. In mathematics, an equation shows that something on one side of the equals sign is equal in value to something on the other side.

Not all equations are true. Some of the following equations are true and some are false:

True:	**False**:
$7 = 6 + 1$	$15 + 1 = 15 \times 1$
$7 = 7$	$12 + 0 = 0$
$2 + 5 = 7$	$2 + 5 = 7 + 1$
$8 + 4 = 7 + 5$	$8 + 4 = 12 + 5$

The left side of an equation must equal the right side for the equation to be true. $7 = 6 + 1$ is a true equation because 7 is equal in value to 7.

Balance

You can think of an equation as being like a scale for measuring weight. When you put things with the same weight on each side, the scale will be balanced. Imagine you put an apple on the left side of the scale. In order for the scale to be balanced, you would have to put something that weighs the same as the apple on the right side.

Miloš Jurišić (CC BY-SA 3.0)

Equations are similar. The balance point of an equation is the equals sign. In order for an equation to be true, both sides of the equation must have the same value. True equations are balanced. An unbalanced equation is false.

Look back at the equations in the **False** column above. Can you see why each of these equations is false?

You should also look at your answers in **Number Sentences - True or False?** a few pages back. If the equation is balanced, then it is true.

Balanced and Unbalanced Equations

The true and false equations from the previous page are rewritten below. The true equations are shown above the balanced scale in the middle. They are balanced, since they have a value on the left side of the equation that is equal to the value on the right side.

$15 + 1 > 15 \times 1$
$12 + 0 > 0$

$7 = 6 + 1$
$7 = 7$
$2 + 5 = 7$
$8 + 4 = 7 + 5$

$2 + 5 < 7 + 1$
$8 + 4 < 12 + 5$

The false equations on the previous page are unbalanced in two ways. They either have a larger value on the left or a larger value on the right. For example, the equation $15 + 1 = 15 \times 1$ (written with an equals sign) is not true, since 16 is not equal to 15. However, the number sentence $15 + 1 > 15 \times 1$ is true, since $15 + 1$ is greater than 15×1. The symbol $>$ means *is greater than*. An unbalanced number sentence can be true if it is written with the correct inequality symbol.

$>$ is greater than

$<$ is less than

For example, the number sentence $2 + 5 < 7 + 1$ is true, since $2 + 5$ is less than $7 + 1$.

Insert $=$, $>$, or $<$ to make true number sentences.

13) $8 \times 4 \quad 8 + 4$

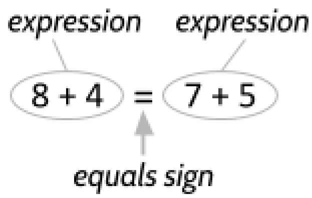
14) $9 + 5 \quad 5 + 9$

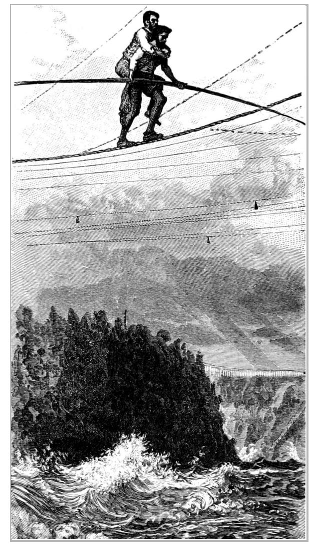
15) $5 + 3 \quad 8 + 2$

In this packet, we will focus on true equations or balanced number sentences. We will come back to inequalities in the 2nd packet.

Anatomy of an equation

Everything on the left side of an equation is an *expression* (one or more numbers and symbols with a value) and everything on the right side is another expression.





An expression can be just one number or could be a combination of numbers and symbols. All equations are made of expressions set equal to each other. An expression has to be connected to another expression to make a number sentence.

We now have an updated definition for an *equation*: **An equation is a number sentence that shows two expressions are equal by using the equals sign**.

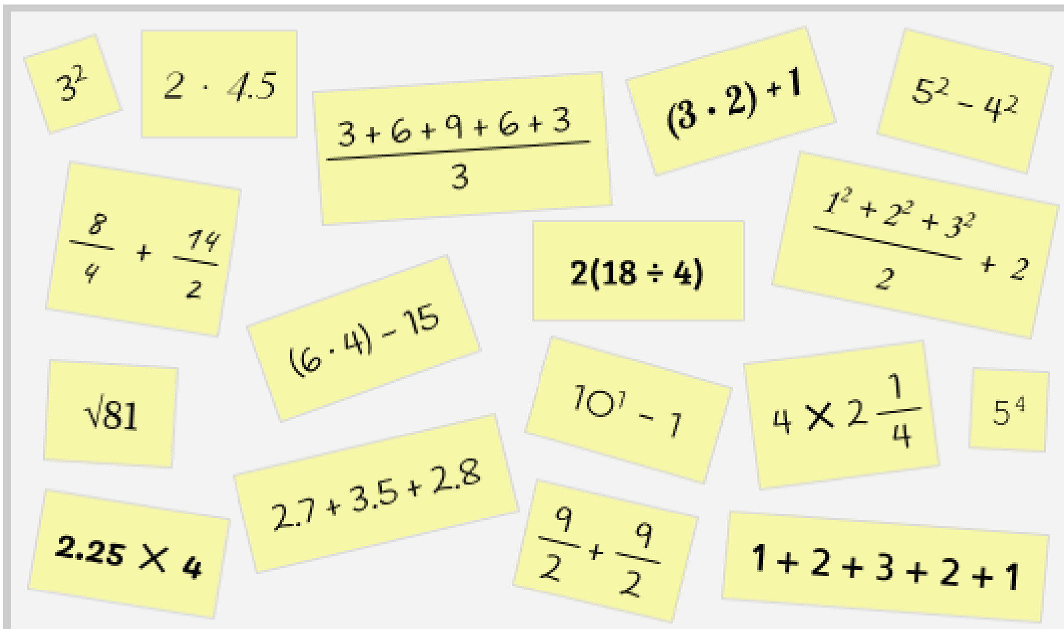
To decide if an equation is true or false, *evaluate* (calculate the value of) the expression on the left side of the equals sign and compare it to the value of the expression on the right side.

If the two expressions are equal in value, then the equation is true.

	Equation	Value of the left side	Value of the right side	T or F
16)	$8 + 4 = 7 + 5$	12	12	T
17)	$7 + 6 + 3 + 4 = 8 + 1 + 2 + 9$			
18)	$25 + 32 = 26 + 30$			
19)	$25 \times 2 = 2 \times 25$			
20)	$3^2 = 3 \times 2$			
21)	$1.5 \times 3 = .75 \times 6$			

Notes from a Math Class

The image below is from a classroom activity called *Number of the Day*. The teacher gave the class a number and asked students to write expressions that would have the same value as that number. These are some of their responses:



22) What do you think the number of the day was? Why?

23) Do you notice any mistakes? What are they?

24) Write three new expressions that are equal to the number of the day above.

Each of the expressions from the previous page should be equal in value to the number of the day. Since they are all equal to the number of the day, they should also be equal to each other. We should be able to take any two of these expressions and make a true equation:

$$\boxed{3^2} = \boxed{1+2+3+2+1}$$

Is it a true equation? The left side of the equation has a value of 9 since 3^2 means 3×3. And the right side of the equation $(1 + 2 + 3 + 2 + 1)$ also has a value of 9. Since $9 = 9$, the equation is true.

Check to see if the following equations are true or false.

	Equation	Value of the left side	Value of the right side	T or F
25)	$(6 \cdot 4) - 15 = \frac{3+6+9+6+3}{3}$			
26)	$2.25 \times 4 = 4 \times 2\frac{1}{4}$			
27)	$\frac{8}{4} + \frac{14}{2} = 2 \cdot 4.5$			
28)	$5^2 - 4^2 = (3 \cdot 2) + 1$			
29)	Use two of the expressions to make your own equation:			

Creating and Solving Equations I

30) Directions: Use only the digits 1 to 7, at most one time each, to make a true equation.

$$\square\square = \square\square + \square\square$$

Examples:

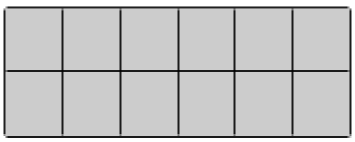
$63 = 21 + 45$ is false because 63 doesn't equal 66.

$66 = 21 + 45$ is true but the digit 6 is used more than once, so it doesn't count.

Area Models for Multiplication

Equivalent expressions can help us do more efficient mental calculations, especially when we multiply. They can also help us understand properties of numbers. To see how this might be true, let's look at a method for multiplication called an area model. As a reminder, *area* refers to the number of squares it takes to cover a surface.

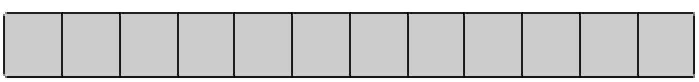
Rectangle A is 2 squares wide and 6 squares long. The area is 12 squares. Count the squares to make sure.

A.


Another way to find the number of squares is to multiply the length by the width: 6×2 is 12. You could also multiply the width by the length since 2×6 also equals 12. Each of the following equations is true.

$$6 \times 2 = 12 \qquad 2 \times 6 = 12 \qquad 6 \times 2 = 2 \times 6$$

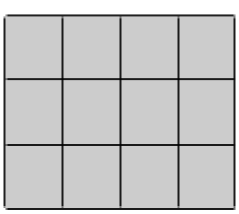
Rectangle B could represent 1×12 or 12×1.

B.


Each equation below gives the number of squares in the area of the long rectangle above.

$$1 \times 12 = 12 \qquad 12 \times 1 = 12$$

31) Look at Rectangle C. Write two equations that show the area of the rectangle.

C.


(_____)(_____) = _____

(_____)(_____) = _____

Symbols for Multiplication

Multiplication is often written with the symbol × as in 5×2, but it is not the only way to show multiplication. In algebra, other symbols are usually used for multiplication instead of ×. Can you guess why?

Other ways to show multiplication are:

- A middle dot ·
- An asterisk *
- Parentheses ( )

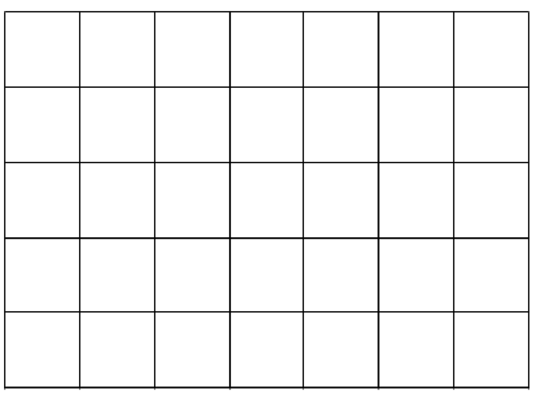
Mathematicians often avoid using × because it can be confused with the x used as a variable in algebra.

Each of the five equations below mean: *2 multiplied by 6 equals 12*

2 × 6 = 12   2 · 6 = 12   2 * 6 = 12   (2)(6) = 12   2(6) = 12

By the way, the × symbol was first used about 400 years ago, in the 17th century.

32) What is the area of the rectangular grid below?

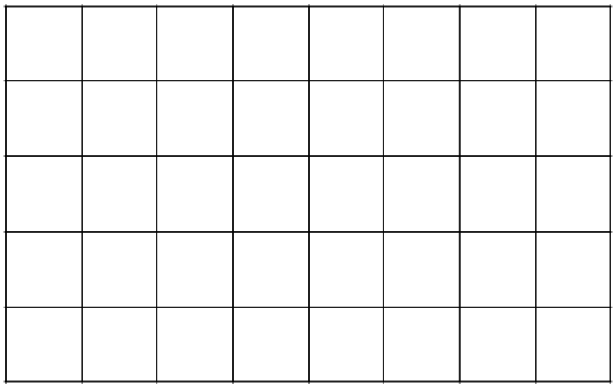


Write two possible equations for the area of the rectangle:

__5__ * __7__ = ______

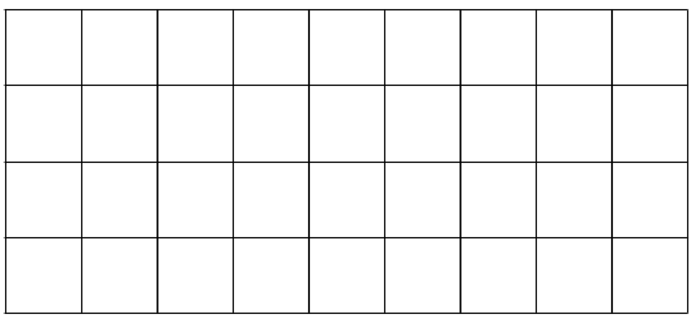
______ * ______ = ______

33) What is the area?



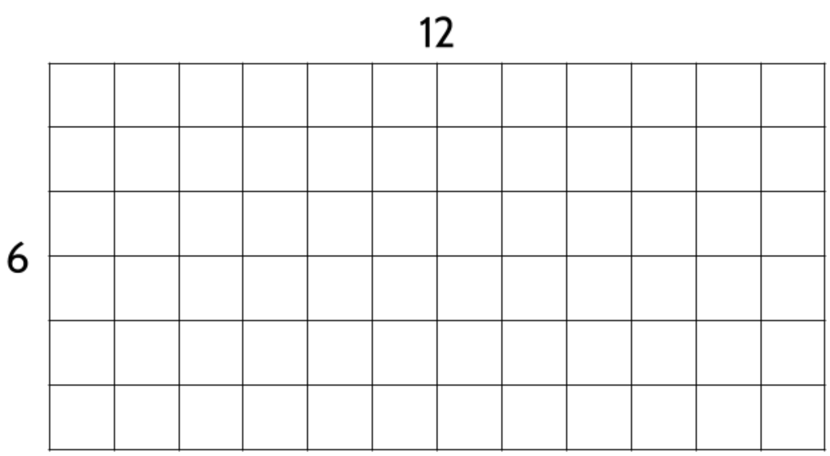
Write two possible equations for the area of the rectangle:

34) What is the area?



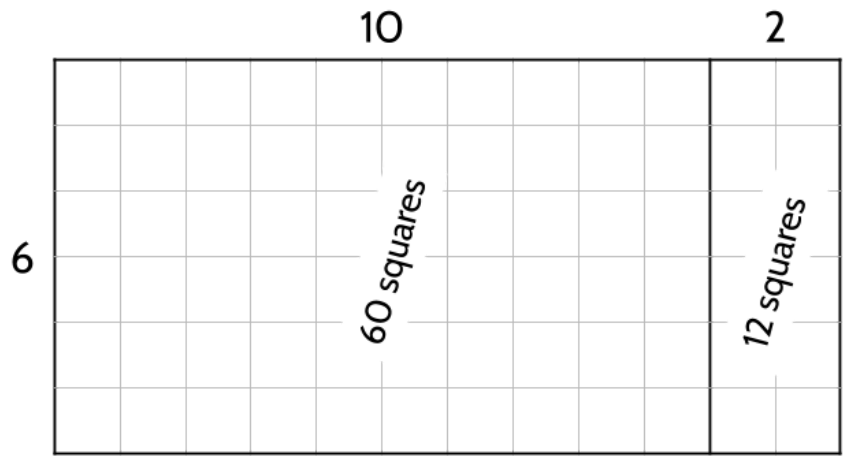
Write two possible equations for the area of the rectangle:

35) What is the area of the rectangular grid below?



36) One way to find out the area is to count all the little squares. How many squares did you count? __________

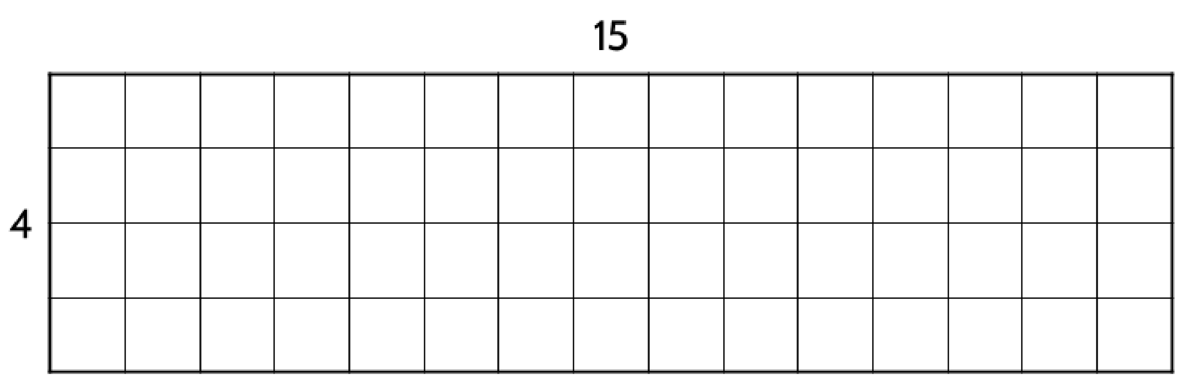
37) You could also split the big rectangle into two smaller rectangles, then find the area of those smaller rectangles. Multiply 6 × 10 to get the number of squares on the left and 6 × 2 to get the number of squares on the right. Then add the two areas together to get the total area (60 + 12 = 72).



$6 \cdot 10 =$__________ How many total squares are there?

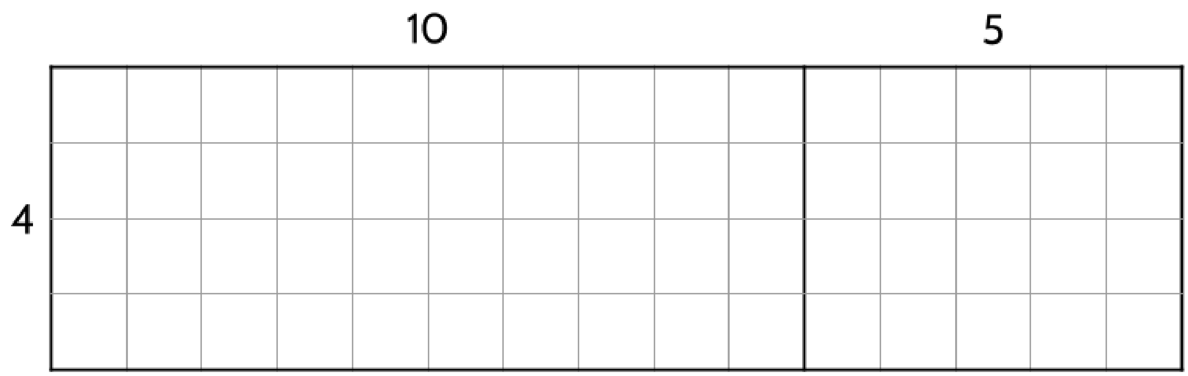
$6 \cdot 2 =$__________ $(6 \cdot 10) + (6 \cdot 2) =$ _________ squares

38) What is the area of this rectangle?



$(4)(15) =$__________ ← The parentheses here mean multiplication: 4×15

39) What is the area of the two rectangles below, added together? (It should be the same as the area of the big rectangle above.)



$4 \cdot 10 =$ ___________ $4 \cdot 5 =$ ___________ $(4 \cdot 10) + (4 \cdot 5) =$ ___________

40) Is the following equation true? How do you know?

$4 \cdot 15 = (4 \cdot 10) + (4 \cdot 5)$

The area of the rectangle on the previous page can be written in many different ways. Each of the following expressions are equivalent:

$$4 \cdot 15 \qquad (4 \cdot 10) + (4 \cdot 5) \qquad 4(10 + 5)$$

Look at the third expression: $4(10 + 5)$. To evaluate this expression and find the total area of the rectangle, you can either add 10 + 5 and then multiply by 4, or you can multiply 4 × 10 and add it to 4 × 5. Shown below, two numbers can be multiplied in the usual way on the left or we can *distribute* the multiplication like shown on the right.

Normal multiplication	Multiplication Based on the Distributive Property
4 × 15 = 60	4 × (10 + 5) = 4 × 10 + 4 × 5 = 40 + 20 = 60
6 × 12 = 72	6 × (5 + 7) = 6 × 5 + 6 × 7 = 30 + 42 = 72

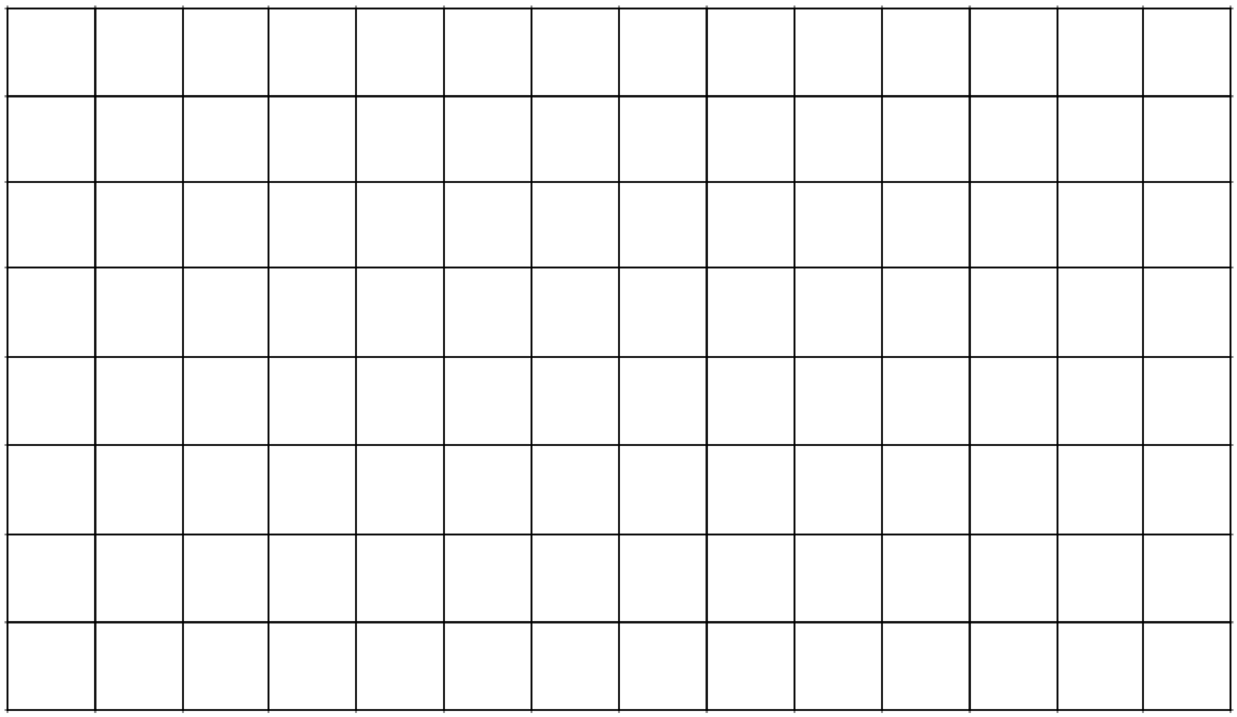
The reason why $(4 \cdot 10) + (4 \cdot 5)$ and $4(15)$ are equal is because of the *distributive property of multiplication*. *Distribute* means to share or spread around. In the examples above, the multiplication is shared with the quantities that are added together inside the parentheses.

When calculating the area of the rectangle above, we split the width of 15 into 10 + 5. It can be useful to look for multiples of 10 and split them away from the rest of the rectangle. It is easier to separately multiply numbers by 10 and 5 than it is to multiply numbers by 15.

41) Think about the expressions with multiplication below. Which of these are easier to calculate in your head?

7 × 13   7 × 10   7 × 3

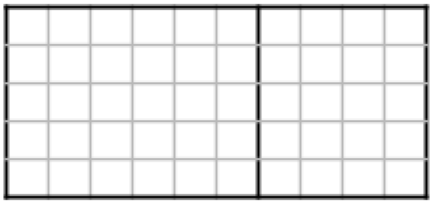
42) Without counting, find the area of the rectangle below. We recommend splitting it up into a couple smaller rectangles.



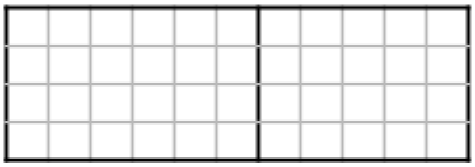
43) Write two equations that can be used to calculate the area of the rectangle above.

44) Do you see any examples of the distributive property or “sharing” of multiplication in your equations? Please explain.

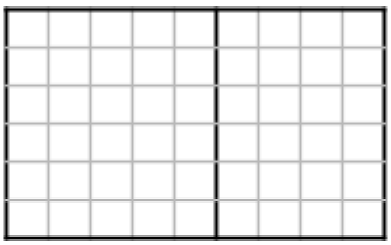
45) For each of the following rectangles, complete the equation by writing a different expression that shows the same area.



$5(10) =$ 5(6+4)



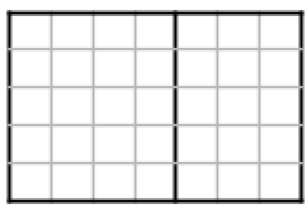
$4(11) =$ ______________



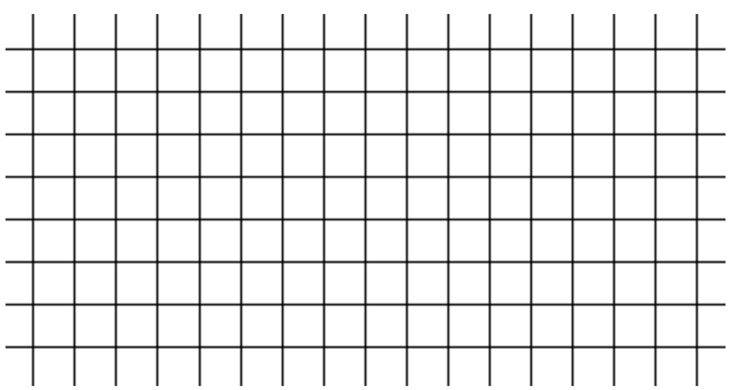
$6(9) =$ ______________

46) Which expression matches this rectangle?

A. $4(5 + 3)$
B. $4(3 + 5)$
C. $3(4 + 5)$
D. $5(4 + 3)$



47) Draw a rectangle that represents the expression $6(10 + 2)$.



48) Paul's teacher asked the class to find the value of the following expression:

$$5(3 + 8)$$

Here is an excerpt from Paul's notes. Can you help Paul understand his mistake?

$5(3 + 8)$

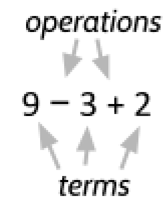
$5 \times 3 + 8$

$15 + 8$

23

Use the graph paper below if it is helpful.

Order of Operations



The individual quantities in a mathematical expression or equation are called *terms*. Here are some examples of terms: 2, 3, 9, x, and $3y$. A term is either a single number or variable, or numbers and variables multiplied together.

The actions we perform on terms are called *operations*. For example, the expression $9 - 3 + 2$ has three terms and two operations.

The four basic operations are addition, subtraction, multiplication, and division. Exponents and roots are also mathematical operations. In one equation, you might be asked to perform several different operations. Does the order in which we do these calculations matter?

For example, what is the value of the expression $13 - 2 \cdot 5$? Does it matter what order we perform the operations subtraction and multiplication?

We could subtract, then multiply:	Or multiply, then subtract:
$13 - 2 = 11$ then$11 \cdot 5$	$2 \cdot 5$ then $13 - 10$

49) Do you get the same answer both times? Why or why not?

50) There are also two different ways to calculate the value of the expression $9 + 6 \cdot 3$. You will get a different total value depending on whether you do addition or multiplication first. What are the two different values?

How do we know which order to use? This is an issue mathematicians had to deal with, because it is confusing if people get different answers with the same calculations. The *order of operations* tells us what to solve first, second, third, and so on... in an expression, so that we all get the same answer in the end.

When you are writing your own expressions and equations, there are tools you can use to control which calculation you want to happen first. This can help prevent confusion when calculating the value of your expression, especially if you want someone else to get the same answer you did. *Parentheses* ( ) can be used to control the order of operations. Parentheses group parts of the expression together so that they are calculated first.

51) Let's imagine a situation where 3 people each have 10 dollars and then each spends 5 dollars. How much money do they have left in total?

52) Which of these calculations gives us the right amount of money left over at the end?

$3 \cdot (10 - 5)$	$(3 \cdot 10) - 5$
$3 \cdot 5$	$30 - 5$
15	25

To answer the question of how money is left, we could think about what each person has at the end. They each started with 10 dollars and spent 5, so they each have 5 dollars left. There are 3 people, so 3 times 5 equals 15.

The parentheses around the subtraction show that this operation should happen first. When you are writing expressions and equations, you can use parentheses to control the order in which calculations happen.

53) What if you wanted $9 - 3 + 2 = 4$ to be a true equation? Rewrite the equation and add parentheses to make it true.

Along with parentheses, other grouping symbols include the *fraction bar*—— and the *radical* $\sqrt{}$ (used for roots). These symbols indicate that the grouped operations should be done before other operations. Here are a few examples of how these symbols can be used:

$\frac{5+5+2}{6} = 2$	The numbers on the top of the fraction bar should be added together before being divided by 6. (The fraction bar means the value on top is divided by the value on bottom.)
$\sqrt{29 - 4} = 5$	Inside the radical, 4 should be subtracted from 29 to get 25 before taking the square root.

When you use algebra, you will be calculating the value of different expressions:

$$(3 + 2)^2 - 5 + 4 \times 5 \qquad 3 \cdot \frac{4^2 \cdot 2}{(5+3)} \qquad (6 - 2) \cdot \sqrt{16 + 9}$$

Some expressions will have many operations and groupings, and as we saw earlier, you can get different answers depending on the order that you do your calculations.

To prevent confusion, mathematicians needed a standard *order of operations* for consistency. The most powerful operations should be completed first. Exponents increase or decrease faster than multiplying, which increases or decreases faster than addition. This is part of the natural order of operations. You could say that exponents and roots are naturally more powerful than multiplication and division, which are each more powerful than addition and subtraction.

Order of Operations

The first rule is that some operations are prioritized over others so they come first. The more powerful operations happen first.

Sometimes we want to use a different order of calculation, so we use grouping symbols to say "do this first" even if it is not the most powerful operation.

The operations are prioritized in the order below:

1. Operations within parentheses and other groupings such as the *radical* $\sqrt{}$ and the *fraction bar* ——.
2. Exponents and roots
3. Multiplication and division
4. Addition and subtraction

The second rule is that we read expressions and equations from left to right in the same way we read a book. If two operations have the same priority (multiplication and division, for example) we do operations on the left before operations on the right.

54) Evaluate the expression $(3 + 2)^2 - 5 + 4 \times 5$ using the order of operations. What is the value of this expression?

Using the order of operations, the value of $(3 + 2)^2 - 5 + 4 \times 5$ can be calculated like this.

Calculate inside parentheses: $3 + 2 = 5$	$(3 + 2)^2 - 5 + 4 \times 5$
Calculate the exponent: $5^2 = 25$	$5^2 - 5 + 4 \times 5$
Find the product (multiplication): $4 \times 5 = 20$	$25 - 5 + 4 \times 5$
Find the difference (subtraction): $25 - 5 = 20$	$25 - 5 + 20$
Find the sum (addition): $20 + 20 = 40$	$20 + 20$
	40

There is one thing to be careful of here. Look at the step after multiplication in this expression: $25 - 5 + 20$.

55) We could do subtraction first and then addition. What answer did we get?

56) Or we might try addition first and then subtraction. What answer would we get?

Here's something you should remember: Addition and subtraction have the same priority in the order of operations, so in this case we go from left to right in doing the calculations. This means that in the expression $25 - 5 + 20$ you should do the subtraction before the addition. The correct answer should be 40, not 0.

Multiplication and division also have the same priority, so if you have a choice between multiplication and division, do the operation on the left first.

57) What is the value of $8 \div 2 \times (3^2 - 5)$? Show your calculations below.

Fix These Equations!

The equation $5 \times 13 - 2 = 55$ is not true.

5×13 is 65 and $65 - 2$ is 63, not 55.

$$5 \times 13 - 2 = 55$$

$$65 - 2 \neq 55$$

However, we can make the equation true by putting $13 - 2$ in parentheses so that it is calculated first.

$$5 \times (13 - 2) = 55$$

$$5 \times 11 = 55$$

Each equation below is false. Add parentheses to each equation to make it true.

58) $12 = 3 \times 6 - 2$

59) $11 - 2 \times 4 + 1 = 1$

60) $11 - 2 \times 4 + 1 = 37$

61) $23 = 3 + 7 \times 2 + 3$

62) $12 - 2 \times 5 + 1 = 60$

63) $4 - 1^2 - 5 = 4$

64) $8 + 2 \times 4 - 1 = 14$

65) $12 - 8 \times 1 + 7 = 32$

66) $8 - 2 + 6 \div 3 = 4$

67) $7 + 3^2 = 100$

68) $24 + 16 \div 8 - 4 = 10$

69) $20 \div 7 - 2 + 5^2 \times 3 = 79$

Four-Function Calculators

Most new calculators use the order of operations correctly, including the TI-30XS calculator used on the high school equivalency exam. However, older four-function[6] calculators (shown on the right) don't "understand" the order of operations.

For example, if you enter the expression $13 - 2 \times 5$ into a four-function calculator, the answer displayed at the end would be 55 instead of 3. This is because these calculators make the calculations in the order you enter the expression. After you enter$13 - 2$, the display will show 11 and when you enter $\times\ 5$ the display will show 55.

To get the right value for $13 - 2 \times 5$, you would have to enter 2×5 first and then $13 - 10$.

How would you enter the following expressions to get the correct value on a four-function calculator? Describe in words how you would enter each expression into the calculator.

70) $4 + 6 \times 5$

Multiply 6 by 5, then add 30 to 4

71) $(7 + 3^2) \div 2$

72) $3 \times (2 + 3)^2$

73) $(2 + \frac{32}{8}) \cdot (7 - 2)$

[6] These older models are called four-function calculators because they can do the functions of addition, subtraction, multiplication and division.

Matching Equivalent Expressions

The word *equivalent* is an adjective that means something is equal in value to something else. For example, 4 quarters are equivalent to 1 dollar. Or you could say that 10 dimes are equivalent to 100 pennies. They are worth the same amount, so their values are equivalent.

74) Evaluate each expression, then match expressions with equivalent values.

IV.	A. $\frac{5 \cdot 6}{2}$	I. $6^2 - 5^2$
___	B. $(5 - 2) \times (3 + 1)$	II. $2^2 + 3^2$
___	C. $(\sqrt{36} \cdot 5) - (2^2 + 3 \cdot 4)$	III. $2^2 + 2^3$
___	D. $2^3 - 3 + 6$	IV. $1 + 2 + 3 + 4 + 5$
___	E. $\frac{100}{10} + \sqrt{9}$	V. $1^2 + 2^2 + 3^2$

Now, rewrite each pair of expressions above as an equation. After you write the equation, confirm that it is true by finding the value of the expressions on both sides of the equals sign.

75) $\frac{5 \cdot 6}{2} = 1 + 2 + 3 + 4 + 5$ 15 = 15; True

76)

77)

78)

79)

Four 4's

80) Can you find every number between 1 and 20 using only four 4's and any operation?

Example: $\sqrt{4} + \sqrt{4} + \frac{4}{4}$ is equal to 5.

$$\sqrt{4} + \sqrt{4} + \frac{4}{4}$$
$$2 + 2 + 1$$
$$5$$

You can use addition, subtraction, multiplication, division, exponents, square roots, parentheses, and decimal places. We recommend that you use a calculator to evaluate the value of your expressions. Some values have been done for you.

1.

2.

3.

4.

5. $\sqrt{4} + \sqrt{4} + \frac{4}{4}$

6.

7.

8.

9.

10.

11. $(4 \div .4) + (4 \div 4)$

12.

13. $(4 - .4) \div .4 + 4$

14.

15.

16.

17.

18.

19. $(4 + 4 - .4) \div .4$

20.

Find as many numbers as you can, but don't worry about trying to find them all.

Review - Expressions and Order of Operations

Each of the following equations is missing one side. Write an expression that balances each equation. Your expressions should include at least one operation (addition, subtraction, multiplication, division, exponents, or roots). The first one is done for you.

81) $\sqrt{25} + 1$ **=** 4×1.5 (Be creative. This is just one possibility.)

82) **=** $8 \cdot 5 + 9$

83) $2(4.2 + .8)$ **=**

84) **=** $\frac{(4-3)}{2}$

85) $8 \cdot 125$ **=**

Insert numbers so that the following equations are true. There is more than one way to do this correctly. You might choose different numbers that also create true equations.

86) $8 + 5 =$ __9__ + __4__

87) $14 + 18 =$ _____ + _____

88) $54 +$ _____ $= 63 +$ _____

89) $42 - 7 =$ _____ − _____

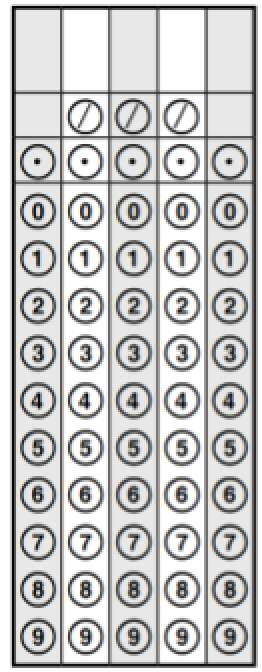
90) $20 -$ _____ $= 15 -$ _____

91) _____ + _____ = _____ + _____

92) _____ − _____ = _____ − _____

93) _____ + _____ = _____ − _____

94) What is the value of the expression $2(5 - 2) + 3(1+ 3^2)$? Enter your answer in the grid to the right.



95) A student took four tests and got an average of 85. Which two expressions could represent this situation? Choose two.

A. $\frac{95+75+90+80}{4}$

B. $4(95 + 75 + 90 + 80)$

C. $(95 + 75 + 90 + 80) \div 4$

D. $95 + 75 + 90 + 80 \div 4$

E. $4 \cdot 95 + 75 + 90 + 80$

96) If the expression $25 - 4^2 + \frac{6}{2}$ is evaluated, what would be done last?

A. subtracting

B. squaring

C. adding

D. dividing

97) Which expression makes the following equation true?

$5 \times$ ______ $= \frac{5}{16}$

A. $\frac{1}{5}$

B. $\frac{1}{16}$

C. $\frac{5}{16}$

D. $\frac{16}{1}$

98) Which of the following equations is <u>not</u> true?

A. $2.7 + 3.5 + 2.8 = (6 \cdot 4) - 15$

B. $5^2 - 4^2 = 3^2$

C. $10^1 - 1 = 5^4$

D. $4 \cdot 2\frac{1}{4} = 2.25 \cdot 4$

99) Which expression could you add to the left side to make this equation true?

______________ = 23

A. $(15 - 5) \cdot (2 + 3)$

B. $15 - 5 \cdot 2 + 3$

C. $15 - (5 \cdot 2 + 3)$

D. $(15 - 5) \cdot 2 + 3$

100) Consider the steps Ismael used to evaluate the expression $8(20 + 5) - 10$.

Expression:	8(20 + 5) – 10
Step 1:	160 + 5 – 10
Step 2:	165 – 10
Step 3:	155

In which step, if any, did Ismael make an error?

A. Step 1

B. Step 2

C. Step 3

D. Ismael did not make an error.

Answer Key - Expressions and Equations

1) The answer is 6. Sometimes, people think that the answer should be 10 because 7 + 3 is 10. However, the equals sign means the value of the left side (7 + 3) is equal to the value of the right side (+ 4).

2) Yes.

3) There are an infinite number of correct answers. 🙂

4) False. You might also say $5 \times 0 = 0$because 0 5's means there are no 5's.

5) True. Any number multiplied by 1 is itself. Examples: $8 \times 1 = 8$, $45 \times 1 = 45$, $2.5 \times 1 = 2.5$...

6) True. $7 + 0 = 7$ is true because when you add 0 to a number, the number remains the same. You might have a different way of saying the same thing.

7) True. Both sides add up to equal to 12. $12 = 12$ so the number sentence is true.

8) True. $2 + 5 = 5 + 2$ is true because you are adding the same two numbers on both sides; the numbers are just reversed. $2 + 5$ is equal to $5 + 2$.

9) True. The sentence can be read, "7 is equal to 7." They are equal because they are the same number.

10) True. 7 is equal to $2 + 5$. It may seem strange to have $2 + 5$ on the right side of the equals sign, but both $7 = 2 + 5$ and $2 + 5 = 7$ are both correct ways to write this number sentence.

11) True. The value of the left side (7) is equal to the value of the right side ($9 - 2$).

12) True. When you multiply numbers, the order doesn't change the answer. 4×6 is equal to 6×4. A few other examples: $7 \times 5 = 5 \times 7$, $8 \times 4 = 4 \times 8$, $9 \times 2 = 2 \times 9$

13) 8 $\times$ 4 **>** 8 + 4

14) 9 + 5 **=** 5 + 9

15) 5 + 3 **<** 8 + 2

	Equation	Value of the left side	Value of the right side	T or F
16)	$8 + 4 = 7 + 5$	12	12	T
17)	$7 + 6 + 3 + 4 = 8 + 1 + 2 + 9$	20	20	T
18)	$25 + 32 = 26 + 30$	57	56	F
19)	$25 \times 2 = 2 \times 25$	50	50	T
20)	$3^2 = 3 \times 2$	9	6	F
21)	$1.5 \times 3 = .75 \times 6$	4.5	4.5	T

22) The number of the day was 9. Almost all of the expressions have a value of 9.

23) A couple expressions don't equal 9. The students must have made a mistake on these expressions:

- $(3 \cdot 2) + 1$ equals 7
- 5^4 equals 625

24) There are many, many different ways to write an expression that has a value of 9. Be creative. You can use addition, subtraction, multiplication, division, exponents, roots, etc.

	Equation	Value of the left side	Value of the right side	T or F
25)	$(6 \cdot 4) - 15 = \frac{3+6+9+6+3}{3}$	9	9	T
26)	$2.25 \times 4 = 4 \times 2\frac{1}{4}$	9	9	T
27)	$\frac{8}{4} + \frac{14}{2} = 2 \cdot 4.5$	9	9	T
28)	$5^2 - 4^2 = (3 \cdot 2) + 1$	9	7	F
29)	You can use almost any of the expressions from the board and set them equal to each other. Don't use $(3 \cdot 2) + 1$or 5^4 since these expressions aren't equal to 9.			

30) Here are a few solutions. Let us know if you find others.

$12 + 35 = 47$ $21 + 53 = 74$ $12 + 63 = 75$ $21 + 36 = 57$

31) $3 \times 4 = 12$
$4 \times 3 = 12$

32) $5 \times 7 = 35$
$7 \times 5 = 35$

33) $5 \times 8 = 40$
$8 \times 5 = 40$

34) $4 \times 9 = 36$
$9 \times 4 = 36$

35) 72

36) 72

37) $6 \cdot 10 = 60$
$6 \cdot 2 = 12$
$(6 \cdot 10) + (6 \cdot 2) = 72$

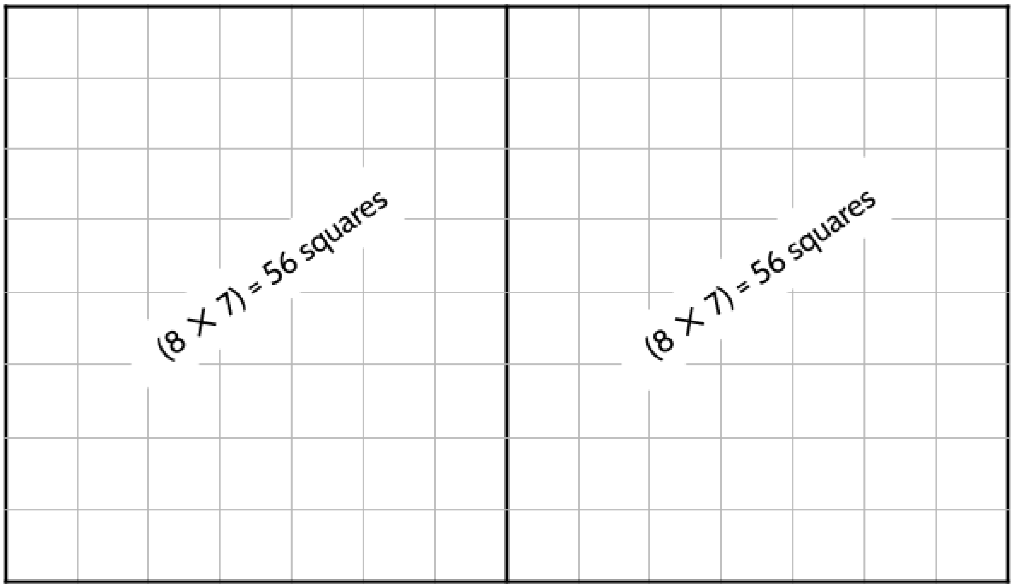
38) $(4)(15) = 60$

39) $4 \times 10 = 40$
$4 \times 5 = 20$
$(4 \times 10) + (4 \times 5) = 60$

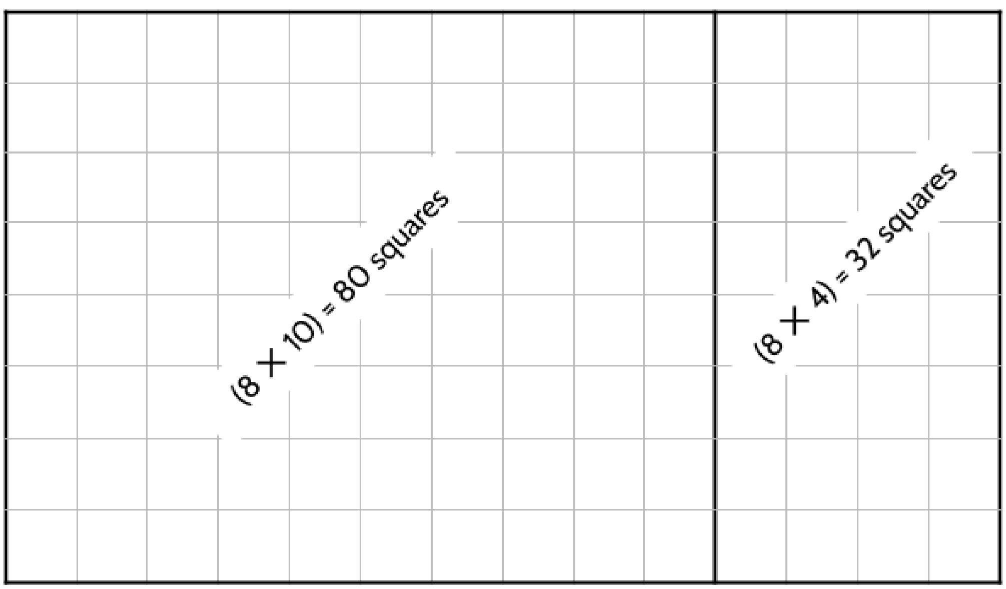
40) Yes. 4×15 has a value of 60 and $(4 \times 10) + (4 \times 5)$ also has a value of 60.

41) I can't speak for you, but I find it easier to figure out 7×10 and 7×3. It's harder for me to calculate 7×13 in my head.

42) The rectangle is 8 units high and 14 units wide. There are many ways to split it up and get the same answer. Yours might be different than the ones below. Here are a few examples:



$(8 \cdot 7) + (8 \cdot 7) = 112$ squares



$(8 \cdot 10) + (8 \cdot 4) = 112$ squares

43) There are many different equations that could be used. One equation would be for the large rectangle without splitting it up:

$8 \cdot 14 = 112$

You could also write many different equations to show the large rectangle split up in different ways. Here are a few examples:

$(8 \cdot 5) + (8 \cdot 9) = 112$
$(8 \cdot 6) + (8 \cdot 8) = 112$
$(8 \cdot 8) + (8 \cdot 6) = 112$

You could even split it into three or more rectangles:

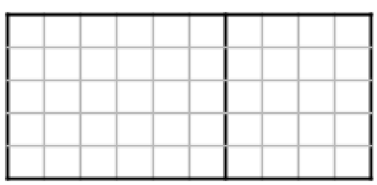
$(8 \cdot 5) + (8 \cdot 5) + (8 \cdot 4) = 112$

Can you imagine how the following equation would be drawn on a rectangular grid?

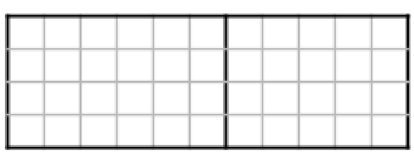
$(4 \cdot 7) + (4 \cdot 7) + (4 \cdot 7) + (4 \cdot 7) = 112$

44) For example, in the equation $(8 \cdot 5) + (8 \cdot 9) = 112$, the multiplication of 8 is distributed to the 5 and the 9. Another way to write it would be $8(5 + 9) = 112$.

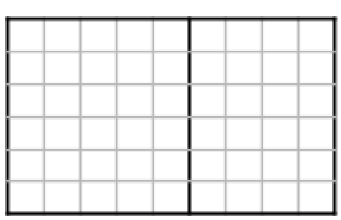
45) There are many expressions that could show the same area. A few are shown for each rectangle. All of the expressions are equal to the area of their rectangle.



$$\begin{aligned} 5(10) &= 5(6 + 4) \\ &= (5 \cdot 6) + (5 \cdot 4) \\ &= 30 + 20 \\ &= 50 \end{aligned}$$



$$\begin{aligned} 4(11) &= (4 \cdot 6) + (4 \cdot 5) \\ &= (4 \cdot 6) + (4 \cdot 5) \\ &= 24 + 20 \\ &= 44 \end{aligned}$$

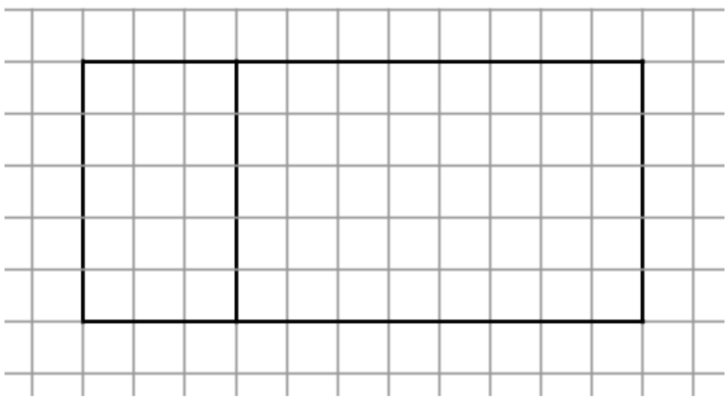


$$\begin{aligned} 6(9) &= (6 \cdot 5) + (6 \cdot 4) \\ &= (6 \cdot 5) + (6 \cdot 4) \\ &= 30 + 24 \\ &= 54 \end{aligned}$$

46) D

47) This is one way to draw the rectangle that matches the expression $6(10 + 2)$.

48) Paul should have gotten a value of 55. He could have drawn a rectangle like the following to show $5 \cdot 3 + 5 \cdot 8 = 55$.



Paul should have distributed 5 to 3 and 8 (not just the 3).

$5(3 + 8)$
$5 \cdot 3 + 5 \cdot 8$
$15 + 40$
55

49) No, because if you subtract first, you will get 55 as an answer. If you multiply first, you will get 3 as an answer.

50) For the expression $9 + 6 \cdot 3$, if you add first, you will get 45 as your answer. If you multiply first, you get 27 as your answer.

51) \$15

52) $3 \cdot (10 - 5)$
$3 \cdot 5$
15

53) $9 - (3 + 2) = 4$

54) 40

55) 40

56) 0

57) 16

58) $12 = 3 \times (6 - 2)$

59) $11 - 2 \times (4 + 1) = 1$

60) $(11 - 2) \times 4 + 1 = 37$

61) $23 = (3 + 7) \times 2 + 3$

62) $(12 - 2) \times (5 + 1) = 60$

63) $(4 - 1)^2 - 5 = 4$

64) $8 + 2 \times (4 - 1) = 14$

65) $(12 - 8) \times (1 + 7) = 32$

66) $8 - (2 + 6 \div 3) = 4$ **or** $(8 - 2 + 6) \div 3 = 4$

67) $(7 + 3)^2 = 100$

68) $(24 + 16) \div (8 - 4) = 10$

69) $20 \div (7 - 2) + (5^2 \times 3) = 79$

70) Multiply 6 by 5, then add 30 to 4

71) Multiply 3 by 3, then add 7, then divide by 2

72) Add 2 to 3, then multiply by 5, then multiply by 3

73) Divide 32 by 8, then add to 2. Remember the value 6. Subtract 2 from 7 to get 5. Multiply 6 by 5.

74) A. IV, B. III, C. V, D. I, E. II

75) $\frac{5 \cdot 6}{2} = 1 + 2 + 3 + 4 + 5$ 15 = 15; True

76) $(5 - 2) \times (3 + 1) = 2^2 + 3^2$ 12 = 12; True

77) $(\sqrt{36} \cdot 5) - (2^2 + 3 \cdot 4) = 1^2 + 2^2 + 3^2$ 14 = 14; True

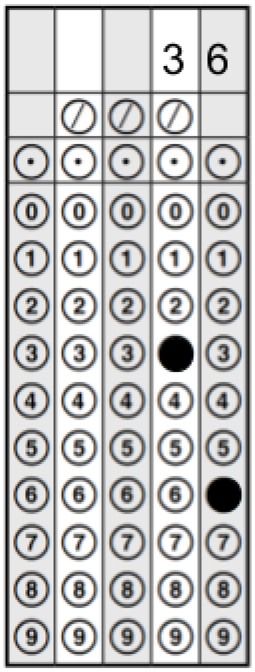
78) $2^3 - 3 + 6 = 6^2 - 5^2$ 11 = 11; True

79) $\frac{100}{10} + \sqrt{9} = 2^2 + 3^2$ 13 = 13; True

80) The Four 4's is a classic problem. We'll share a solution for each number below, but there are many solutions for each number. You can do an Internet search for "four 4s" to find many solutions, web pages and videos.

1. $4 \times 4 \div 4 \div 4 = 1$
2. $\frac{4}{4} + \frac{4}{4} = 2$
3. $\sqrt{4 \cdot 4} - \frac{4}{4} = 3$
4. $4 + 4(4 - 4) = 4$
5. $(4 + 4 \cdot 4) \div 4 = 5$
6. $4 + \frac{4+4}{4} = 6$
7. $4 + 4 - \frac{4}{4} = 7$
8. $4 + 4 - 4 + 4 = 8$
9. $4 + 4 + \frac{4}{4} = 9$
10. $(44 - 4) \div 4 = 10$
11. $(4 \div .4) + (4 \div 4) = 11$
12. $(44 + 4) \div 4 = 12$
13. $(4 -.4) \div .4 + 4 = 13$
14. $4 \cdot 4 - 4 \div \sqrt{4} = 14$
15. $4 \cdot 4 - \frac{4}{4} = 15$
16. $4 + 4 + 4 + 4 = 16$
17. $4 \cdot 4 + \frac{4}{4} = 17$
18. $4 \cdot 4 + 4 - \sqrt{4} = 18$
19. $(4 + 4 -.4) \div .4 = 19$
20. $4(4 + \frac{4}{4}) = 20$

82-94) There are many possible correct answers for these questions. As long as the value of the left side of the equation is equal to the value of the right side of the equation, the number sentence will be true.

95) 

96) A, C

97) C

98) B

99) C

100) D

101) A

Thinking in an Algebraic Way

Algebra is the science that teaches how to determine unknown quantities by means of those that are known. - Leonhard Euler (1707-1783)

A Magic Trick

1) We're going to learn a math magic trick. Try it on yourself first.

    Choose a number from 1 to 20. Double it, add 10, divide by 2, and then subtract the number you started with. I'm going to guess the number you end with.

    Try out the calculations. Show your work below:

    I bet you got 5. How did I know? Choose a different starting number. Try out the calculations. Show your work below:

    Finally, I'll choose a number and try the trick. My number is **15**.

    $15 \times 2 = 30$

    $30 + 10 = 40$

    $40 \div 2 = 20$

    $20 - 15 = 5$

    Do you think 5 will be the answer no matter what number you start with?

Later in the packet, we will use algebra to analyze this math magic trick. In the meantime, try it with your friends and family. Let them choose a starting number and "predict" their final answer is 5. See if you can figure out why it works.

Guess My Number

In the previous activity, you found that you will always get 5 after doing certain calculations no matter what number you start with. In this activity, I have a different mystery number for each question. I am going to do some calculations with the mystery number and tell you the result. Your challenge is to figure out the mystery number I started with in each description.

For each mystery number, explain how you know what it is.

2) When I double my mystery number, I get 26. What is my number?

   Your number is 13, since 13 ✕ 2 = 26. Double 13 is 26.

3) If I add 5 to my number, I get 18. What is my number?

4) When I subtract 10 from my number, I get 35. What is my number?

5) My number divided by 4 is 7. What is my number?

6) If I subtract 3 from my number, I get 6.5. What is my number?

7) If I add my number to my number and then add 5, I get 17. What is my number?

8) If I divide my number by 2, I get 12.5. What is my number?

9) If I subtract my number from 20, then multiply by 2, I get 10. What is my number?

10) Write your own mystery number puzzle.

Open Number Sentences

We can write a description of a mystery number in everyday written language like this:

When I multiply my mystery number by 3, I get 24.

Or we can describe the mystery number in an open number sentence or equation like this:

$\square \cdot 3 = 24$

$\square \cdot 3 = 24$ can be called an *open number sentence* because it is open to different values to replace $\square$. The box is an example of a *variable*, since we can *vary* (change) what value we enter in the box. We can decide if the number sentence is true or false after we enter a value for $\square$. Some values will make the number sentence true and others will make the sentence false.

For example, we could try replacing $\square$ with the numbers 6 through 10 below:

Number Sentence	Value of the left side	Value of the right side	T or F
$6 \cdot 3 = 24$	18	24	F
$7 \cdot 3 = 24$	21	24	F
$8 \cdot 3 = 24$	24	24	T
$9 \cdot 3 = 24$	27	24	F
$10 \cdot 3 = 24$	30	24	F

It is possible to substitute different values for $\square$, but the goal is to make a true number sentence. $8 \cdot 3 = 24$is the only true number sentence above.

Solving an equation means finding values for the variables which make the equation true.

11) What value could you give $\square$ to make this number sentence true?

$5 \cdot \square + 3 = 38$

Matching Statements and Open Number Sentences

12) In this activity, you will match equations with the mystery number descriptions in everyday language.

3	A.	Double my number is 26.	1.	$\square \div 2 - 10 = 2$
____	B.	Twice my number plus 5 is 19.	2.	$\square \div 2 = 19$
____	C.	When I subtract 10 from half my number, I get 2.	3.	$2 \cdot \square = 26$
____	D.	My number multiplied by 2 is 19.	4.	$\square + \square - 5 = 19$
____	E.	2 subtracted from my number is 20.	5.	$\square - 2 = 20$
____	F.	If I add my number to my number and then subtract 5, I get 19.	6.	$2 \cdot \square + 5 = 19$
____	G.	Half of my number is 19.	7.	$20 - \square = 2$
____	H.	If I subtract my number from 20, I get 2.	8.	$2 \cdot \square = 19$

13) Find the mystery number that makes each open sentence true. Write the mystery number for each $\square$ above.

Solving Equations

Solving equations is similar to finding the mystery number from an everyday language description. The goal is to find the value that makes the equation true.

	Everyday Language	Equation
Description of Mystery Number	When I subtract 10 from my number, I get 35. What is my number?	$\square - 10 = 35$
Solution	The mystery number is 45.	$\square = 45$

For each equation below, find the value of $\square$ that makes the equation true. Write the solution below as another equation.

Example: $\square + 5 = 25$

$\square = 20$

14) $8 + 4 = \square + 5$

15) $2 + \square = 7$

16) $2 + 5 = 5 + \square$

17) $\square = \square$

18) $\square + 0 = 20$

19) $7 = \square + 1$

20) $1 + 6 = 2 + \square$

21) $46 + 77 = 44 + \square$

22) $98 + 56 = 99 + \square$

23) $123 + \square = 126 + 87$

24) $416 + 107 = 420 + \square$

25) $\square + 967 = 234 + 964$

26) $369 + 83 = \square + 88$

How Many Values Make the Equation True?

In the equations on the previous page, there is only one value for each $\square$ that makes the equation true. For example, in the equation $\square + 0 = 20$, the box must have a value of 20. However, sometimes there is more than one value that makes the equation true and in some cases there is no value that makes the equation true.

For the following equations, you have to decide how many different values would make the equation true. If there is only one value, write the value. If there is more than one value, give a few different examples that make the sentence true.

27) Complete the table.

Equation	Zero, One, or More than One Value	Explanation
$\square + 0 = \square$	More than one value.	The variable could be many numbers. For example, $6 + 0 = 6$ and $25 + 0 = 25$ are both true.
$\square + 6 = 18$		
$\square \cdot 0 = 0$		
$\square \cdot 5 = 5 \cdot \square$		
$\square + 34 = \square - 7$		
$\square \cdot 1 = \square$		
$\square \cdot \square = 16$		

Note: Within each equation, $\square$ must have the same value, but can have different values in different equations.

28) Complete the table.

Equation	Zero, One, or More than One Value	Explanation
$\frac{\square}{\square} = 1$		
$\square + 35 = 35 + \square$		
$4 \cdot \square = 9 \cdot \square$		
$\square + 1 = \square$		
$\frac{\square}{1} = \square$		
$\sqrt{\square} = 5$		

29) How many values make the following statement true? Explain how you know.

A number added to itself and then added to ten is equal to the same number added to five and then doubled.

30) How many values make the following statement true? Explain how you know.

A number multiplied by three is equal to the same number added to twelve.

Evaluate Expressions

Finding the value of an expression is sometimes called *evaluating* an expression. For example, if you are asked to evaluate $5^2 + 3$, the answer would be 28.

You might also be asked to evaluate the value of an expression when substituting a number for the variable $\square$. For example, the value of the expression $\square + 5$ when $\square = 8$ is 13.

31) If $\square = 5$, what is the value of the expression $(3 \cdot \square)^2$?

A. 30

B. 45

C. 75

D. 225

32) If $\square = 6$, what is the value of the expression $3 \times (\square - 2)$?

A. -6

B. 12

C. 16

D. 18

33) If $\square = 3$, what is the value of the expression $21 - 2(4 + \square)$?

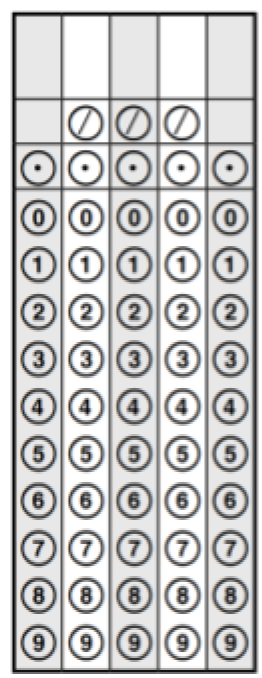
A. 7

B. 10

C. 16

D. 26

34) Evaluate the expression $(5 + \square)^2$ when $\square = 4$. Enter your answer in the gridded response area to the right.



35) Evaluate the expression $\frac{3 + 3^3}{\square}$ when $\square = 3$.

A. 4

B. 8

C. 10

D. 72

Answer Key - Thinking in an Algebraic Way

1) Your answer for this activity should include calculations for a few different starting numbers. Here are a few examples:

$\mathbf{5} \times 2 = 10$	$\mathbf{9} \times 2 = 18$	$\mathbf{12} \times 2 = 24$
$10 + 10 = 20$	$18 + 10 = 28$	$24 + 10 = 34$
$20 \div 2 = 10$	$28 \div 2 = 14$	$34 \div 2 = 17$
$10 - \mathbf{5} = 5$	$14 - \mathbf{9} = 5$	$17 - \mathbf{12} = 5$

We will look more at this math trick later in the packet.

2) Possible answer: Your number is 13, since 2 ✕ 13 = 26. Double 13 is 26.

3) 13

4) 45

5) 28

6) 9.5

7) 6

8) 25

9) 15

10) Optional: Send your mystery number puzzle to info@collectedny.org or @CollectEdNY on Twitter. Your puzzle may be featured on a website for teachers and students across New York State!

11) 7

12) The ⬜ in each number sentence below has been replaced with the mystery number.
   A. 2 · **13** = 26
   B. 2 · **7** + 5 = 19
   C. **24** ÷ 2 - 10 = 2
   D. 2 · **9.5** = 19
   E. **22** - 2 = 20
   F. **12** + **12** - 5 = 19
   G. **38** ÷ 2 = 19
   H. 20 - **18** = 2

13)

1. ❐ = **24**
2. ❐ = **38**
3. ❐ = **13**
4. ❐ = **12**
5. ❐ = **22**
6. ❐ = **7**
7. ❐ = **18**
8. ❐ = **9.5**

14) ❐ = 7

15) ❐ = 5

16) ❐ = 2

17) The mystery number can be any number.

18) ❐ = 20

19) ❐ = 6

20) ❐ = 5

21) ❐ = 79

22) ❐ = 55

23) ❐ = 90

24) ❐ = 103

25) ❐ = 231

26) ❐ = 364

27)

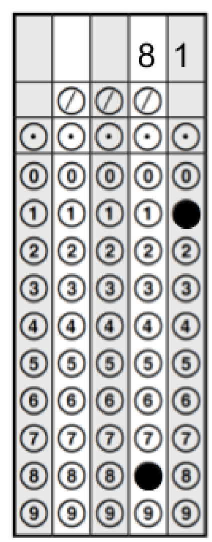
Equation	Zero, One, or More than One Value	Explanation
☐ + 0 = ☐	More than one value.	The variable could be many numbers. For example, 6 + 0 = 6 and 25 + 0 = 25 are both true.
☐ + 6 = 18	One value	12 is the only number the variable could be and have this equation be true.
☐ · 0 = 0	More than one value	When any number is multiplied by zero, the result will be zero, so the variable here can be any number and the equation would be true. For example, 6 groups of 0 is zero. 25 groups of 0 is zero.
☐ · 5 = 5 · ☐	More than one value	Any number times five is the same as five times that number
☐ + 34 = ☐ - 7	Zero values	There is no number that if you added 34 to it would be the same as that number minus 7.
☐ · 1 = ☐	More than one value	Any number times 1 is the same as that number. For example, 6 groups of 1 is 6. 25 groups of 1 is 25.
☐ · ☐ = 16	More than one value	If the variable were 4, then the equation would be true. For example, 4 x 4 = 16. But 4 is not the only possible value for the variable. -4 x -4 is also 16. For this equation there are two possible values for the variable: 4 and -4.

28)

Equation	Zero, One, or More than One Value	Explanation
$\frac{\square}{\square} = 1$	More than one value	Any number divided by itself is 1, so this equation would be true no matter what number the variable represents. For example, $\frac{6}{6} = 1$and $\frac{25}{25} = 1$
$\square + 35 = 35 + \square$	More than one value	Adding 35 to any number is the same as adding that same number to 35.
$4 \cdot \square = 9 \cdot \square$	One value	If the variable is 0, this equation is true. If the variable is 0, then both sides of the equal sign would be 0.
$\square + 1 = \square$	Zero values	There are no values for the variable that would make this equation true.
$\frac{\square}{1} = \square$	More than one value	Any number divided by 1 is that same number. For example, $\frac{6}{1} = 6$ and $\frac{25}{1} = 25$
$\sqrt{\square} = 5$	One value	The square root of 25 is the same as 5.

29) There is more than one value that makes this statement true. One strategy to know how many values would make this statement true is to try several numbers. For example, if the number was 6, 6+6 is 12, plus 10 is 22. 6+5 is 11, and doubled that makes 22. 22 is the same as 22. If we try 8, 8+8 is 16 plus 10 is 26. 8+5 is 13, and if we double 13, we also get 26. In fact, it turns out this statement is true for any number greater than zero.

30) This statement is true for one value, 6. Guess and check is a good strategy for answering this question. Let's try it with 4. Three times 4 is 12. Four plus 12 is 16, so 4 doesn't work. Let's try 10. Three times 10 is 30. Ten plus 12 is 22. My numbers are getting further apart, so let's try something between 4 and 10. Let's try 6. Three times 6 is 18 and 6 plus 12 is also 18. So this statement is true for 6. You'll learn other strategies for solving this type of problem as you work through this packet.



31) Choice D. 225

32) Choice B. 12

33) Choice A. 7

34) Gridded response shown on the right.

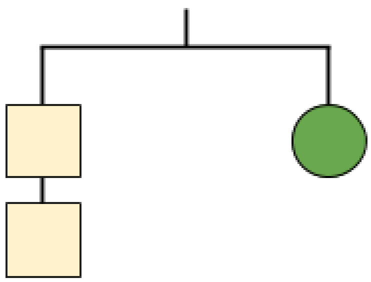
35) Choice C. 10

Balancing Equations

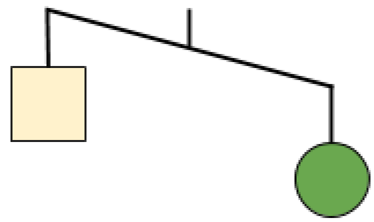
Hanger Diagrams

An equation that is true is a balanced equation, since both sides are equal. A hanger diagram is a way for us to visualize the balance of equations. Imagine hanging weights on both sides of the hanger. If the hanger is balanced, the left side is equal in weight to the right side. An unbalanced hanger is like a false equation since the two sides don't weigh the same.

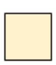
Balanced hanger

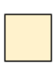


Unbalanced hanger

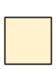


If we have equal weights on the ends of a hanger, then the hanger will be in balance. If there is more weight on one side than the other, the hanger will tilt to the heavier side. The balanced hanger shows 2 squares equal in weight to 1 circle. We can use the balanced hanger to understand the relationship between the weight of each object.

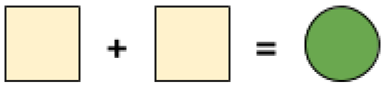
1) If  weighs 2 pounds, what is the weight of ?

2) If  weighs 7 pounds, what is the weight of ?

3) If  weighs 10 pounds, what is the weight of ?

4) If  weighs 4.5 pounds, what is the weight of ?

We could also write the balanced hanger diagram above as an equation with shapes:



We could also write an equation with letters as variables instead of the shapes and :

$v + v = w$

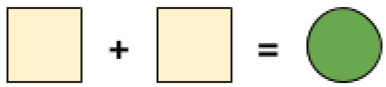
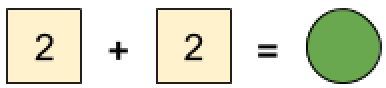
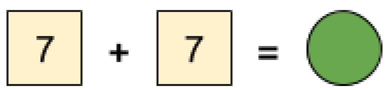
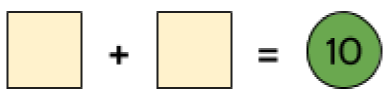
The variables v and w could represent any values. The goal is to make a true equation, so we could try substituting different numbers to see which ones make the equation true.

	If v or is...	If w or is...	T/F	Explanation
5)	1.5	3.5	F	The equation would be false, since 1.5 + 1.5 equals 3, not 3.5
6)	$\frac{3}{8}$	$\frac{6}{8}$		
7)	4.2	2.1		
8)	$\frac{3}{4}$	$1\frac{1}{2}$		
9)	$.08$	$.16$		
10)	$2\frac{4}{6}$	$1\frac{2}{3}$		

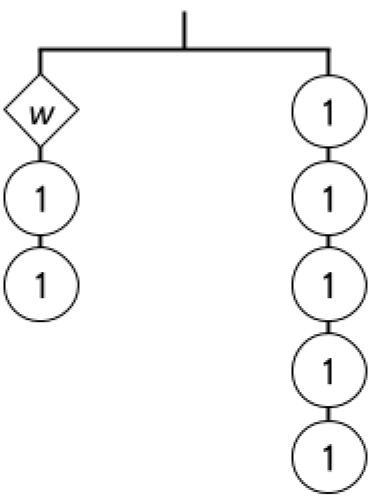
Write your own values for v and w that makes the equation true.

11)			T	

If we know the value of a variable, we can substitute values for them in the equation. The letter or blank shape means that we can substitute a value to make the equation true.

equation with shapes as variables: 	equation with letters as variables: $v + v = w$
equations with shapes and numbers   	equations with letters and numbers: $2 + 2 = x$ $7 + 7 = y$ $z + z = 10$

In the following exercises, you will look at different hanger diagrams and match them to the the right equation using letters as variables. On each balanced hanger, each shape has the same weight, but the shapes may have different weights in different hangers.



12) What is the value of w?

13) Which equation matches the hanger diagram?

A. $2w = 5$

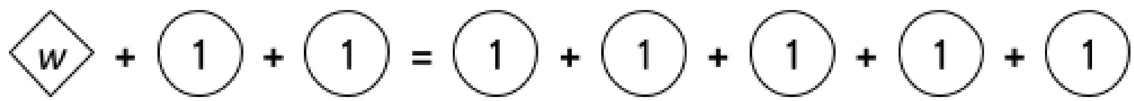
B. $w + 2 = 5$

C. $w - 2 = 5$

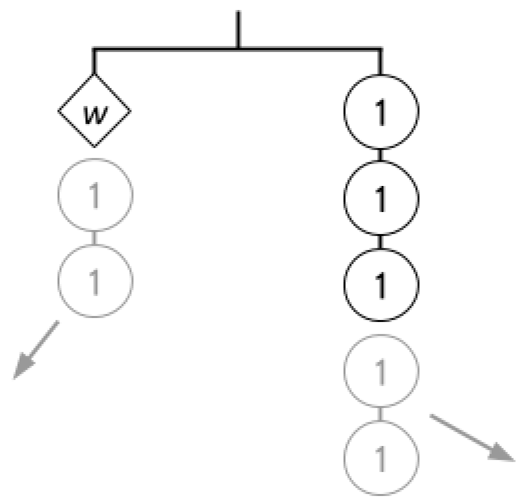
D. $w = 7$

Explain your choice:

We could write an equation with shapes to represent the hanger diagram above:



And we can also describe the hanger with an equation using letters as variables: $w + 2 = 5$.

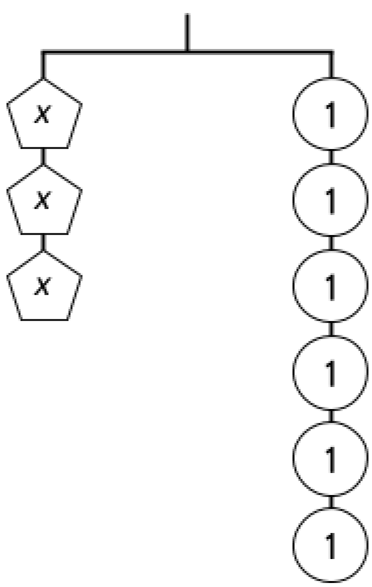


To solve the equation and find the value of w, remember to keep the hanger balanced while you make changes.

In the diagram below, we can remove two 1's from the left side and two 1's from the right side while keeping the hanger balanced. If we remove the same amount of weight from each side, the hanger will still be in balance.

This leaves w balanced with three 1's, so the solution is $w = 3$. If the variable is alone on one side of a balanced hanger, it is equal to the weight of the other side.

Let's look at another hanger diagram with a different kind of solution.



14) What value for x keeps the hanger balanced?

15) Which equation matches the hanger diagram?

A. $x + 3 = 6$

B. $x = 6$

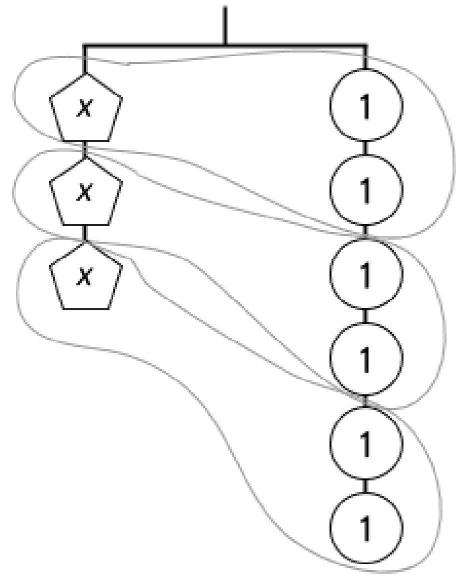
C. $3x = 6$

D. $6x = 3$

Explain your choice:

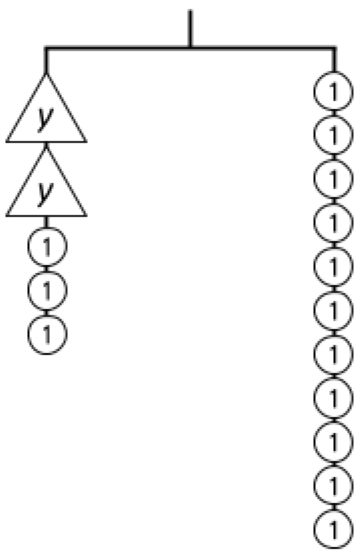
We can write an equation with shapes for the hanger above:

Or an equation with letters as variables: $3x = 6$.



To solve the equation and find the value of x, we can think about how much each x weighs. Since there are three x's and six 1's, we can split the x's into three groups and the 1's into three groups as well. Each x has the same weight as two 1's, so the solution is $x = 2$.

Look at the hanger diagram below.



16) What is the value of y?

17) Which equation matches the hanger diagram?

A. $2y + 3 = 11$

B. $2y = 11$

C. $3y + 2 = 11$

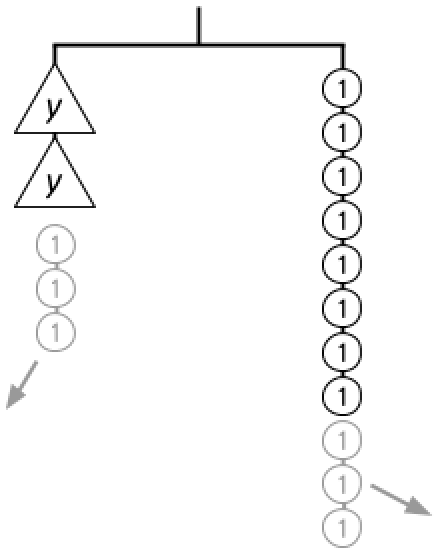
D. $y + 3 = 11$

Explain your choice:

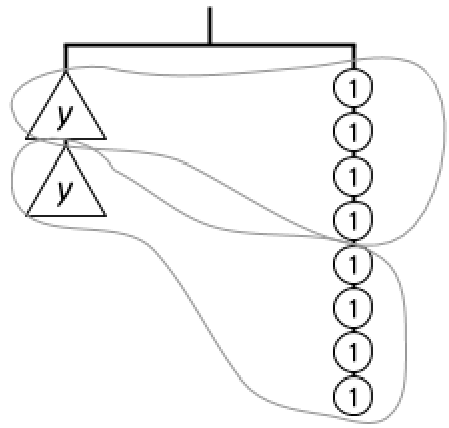
The written equation for the hanger diagram above is $2y + 3 = 11$. The two y's together with three 1's are equal in weight to eleven 1's.

Finding a value for y takes two steps.

1. First, we can remove three 1's from both sides of the hanger.



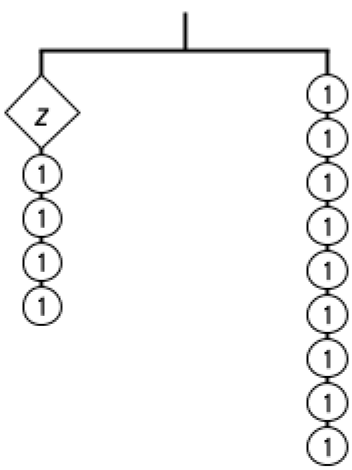
2. Then we can split the left side and the right side into two groups.



This shows that each y has the same weight as four 1's. The solution is $y = 4$.

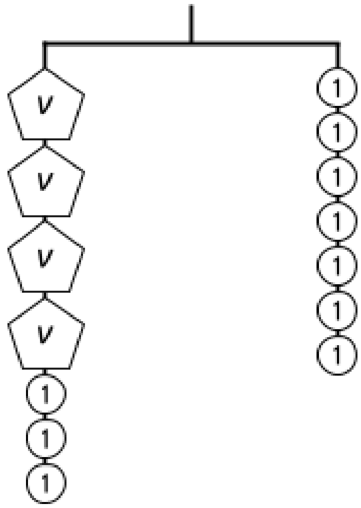
For the following hanger diagrams, write a matching equation and then find the value for the variable that keeps the equation balanced.

18)



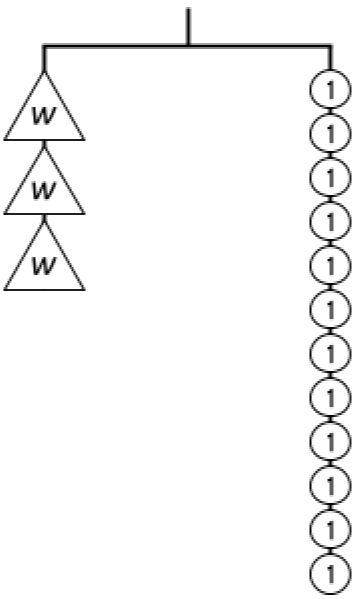
Write a matching equation.

What value of z keeps the hanger balanced?

19) 

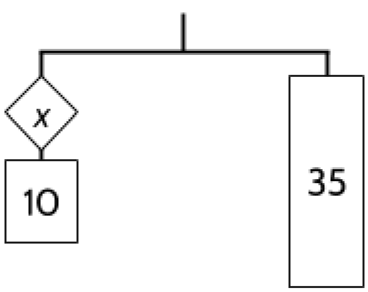
Write a matching equation.

What value of v keeps the hanger balanced?

20) 

Write a matching equation.

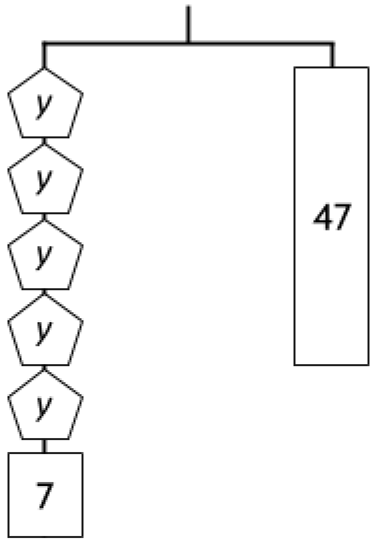
What value of w keeps the equation balanced?

21) 

Write a matching equation.

What value of x solves the equation?

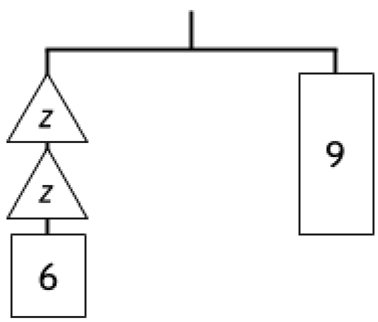
22)



Write a matching equation.

What is the solution to the equation?

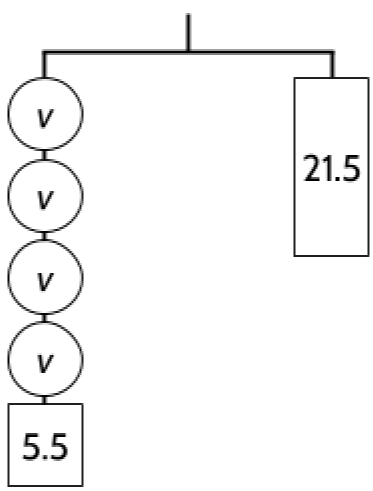
23)



Write a matching equation.

What value of z balances the equation?

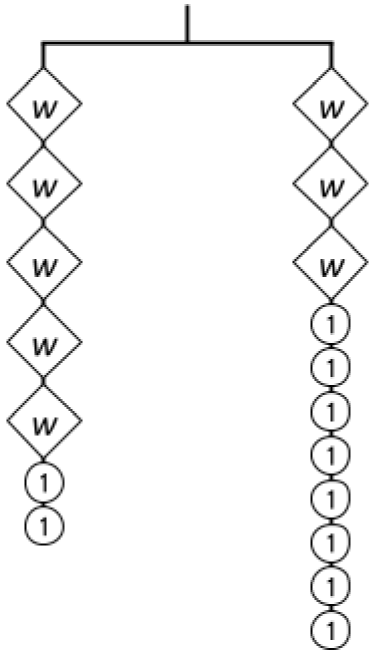
24)



Write a matching equation.

What is the solution to the equation?

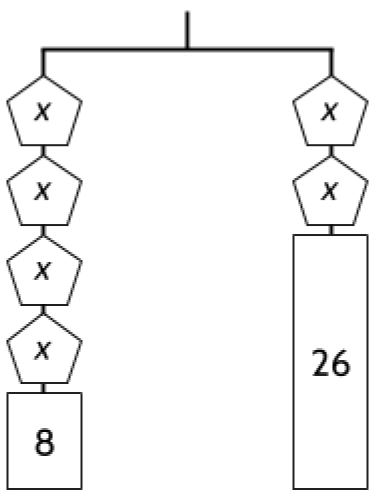
25)



Write a matching equation.

What value of w balances the equation?

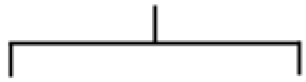
26)



Write a matching equation.

What value of x balances the equation?

27)

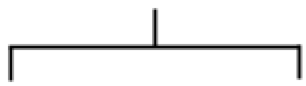


Draw a hanger diagram for this equation:

$x + 5 = 9$

What value of x balances the equation?

28)

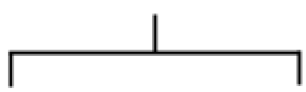


Draw a hanger diagram for this equation:

$3y + 4 = 25$

What value of y makes the equation true?

29)



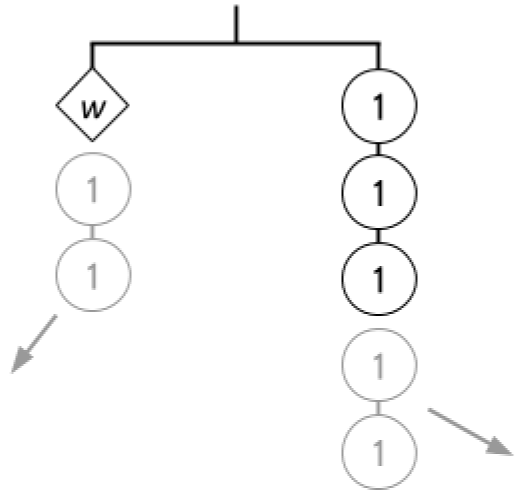
Draw a hanger diagram for this equation:

$6z + 4 = 3z + 10$

What is the value of z?

For more practice with hanger diagrams, go to http://solveme.edc.org.

Strategies for Solving Equations



Looking back at the hanger diagrams you completed, we can see important strategies for solving equations while keeping them balanced. For example, we solve the equation $w + 2 = 5$by subtracting 2 from each side of the equation. The solution to the equation is $w = 3$. You can see this visually by removing two units from each side of the hanger diagram.

30) Solve $y + 7 = 21$ for y.

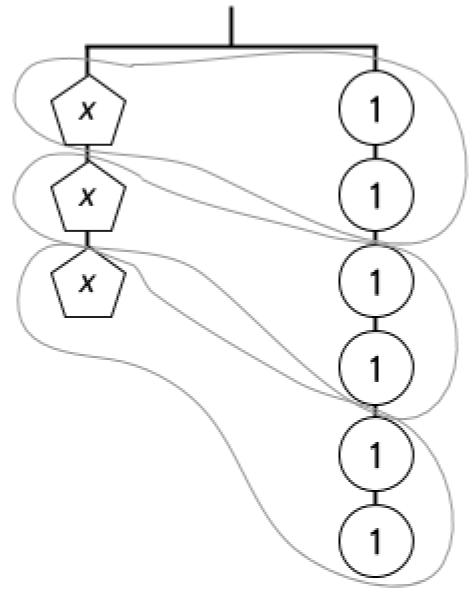
31) Solve $z + 2.5 = 10$ for z.

32) What value for v makes $v + \frac{1}{4} = \frac{3}{4}$ true?

33) Solve $w + 17 + 5 = 32$ for w.

For the equations above, subtracting the same amount from both sides will help you solve the equation. This is a useful strategy for solving equations that involve addition. If you subtract the same amount from both sides, the equation will remain balanced.

We have also used strategies for solving equations that include multiplication. For example, to solve the equation $3x = 6$, we can ask ourselves if 3 x's equal 6 units, how many does one x equal? We know we have three groups of x. By dividing both sides by 3, we can see how many of the units on the right side go with each x on the left side. The solution to the equation is $x = 2$.



34) Solve $4y = 36$ for y.

35) Solve $10z = 1000$ for z.

For both of the equations above, dividing both sides of the equation by the same number will help you solve the equation.

36) Solve $5v = 65$ for v.

Dividing both sides by the same value is a useful strategy for solving equations that involve multiplication.

Inverse Operations

In the examples above, we used subtraction to solve equations using addition. Here are a few more examples:

Equation:	$w + 15 = 45$	$x + .5 = 3$
Subtract from both sides:	$w + 15 \mathbf{- 15} = 45 \mathbf{- 15}$	$x + .5 \mathbf{- .5} = 3 \mathbf{- .5}$
Solution:	$w = 30$	$x = 2.5$

We can do this because subtraction and addition are *inverse operations*. In other words, addition and subtraction are opposite of each other. They undo each other. For example, look at $w + 15 = 45$. We have a mystery number, w, and add 15 to get 45. How can we figure out what the mystery number is? Subtracting 15 is the opposite of adding 15. Subtracting from both sides will keep the equation balanced and leave w equal to $45 - 15$.

$$w = 45 - 15$$

For each of the following equations, use the inverse operation to solve for the variable.

37) $y - 5 = 20$

38) $z + \frac{1}{4} = 5$

39) $v - 7 = 15$

40) $w + .75 = 4$

Using addition and subtraction as inverse operations will also work when moving variables from one side of the equation to the other. For example, if you subtract the same variable from both sides, the equation will still be balanced:

Example with numbers	Example with variables
$8 + 4 = 12$ $8 + 4 - 4 = 12 - 4$ $8 = 12 - 4$	$a + b = c$ $a + b - b = c - b$ $a = c - b$

41) Solve $a + b = 20$ for a.

42) Solve $c - d = 100$ for c.

We have also used division to solve equations using multiplication. Here are a few more examples:

Equation:	$w \cdot 9 = 45$	$x \cdot 20 = 400$
Divide both sides:	$w \cdot 9 \div \mathbf{9} = 45 \div \mathbf{9}$	$x \cdot 20 \div \mathbf{20} = 400 \div \mathbf{20}$
Solution:	$w = 5$	$x = 400$

We can do this because multiplication and division are inverse operations. Each one undoes the other. For example, if you start with 5, multiply by 10 and then divide by 10, you will have 5 at the end.

5×10 is 50.
$50 \div 10$is 5.

For each of the following equations, solve for the variable.

43) $y \cdot 5 = 20$

45) $v \div 3 = 15$

44) $z \div 6 = 5$

46) $w \cdot .25 = 4$

Using division as the inverse operation of multiplication will work even if you divide by a variable. For example, if you divide both sides of an equation by the same variable, the equation will still be balanced:

Example with numbers	Example with variables
$6 \cdot 5 = 30$ $6 \cdot 5 \div 5 = 30 \div 5$ $6 = 30 \div 5$	$a \cdot b = c$ $a \cdot b \div b = c \div b$ $a = c \div b$

47) Solve $a \cdot b = 24$ for a.

48) Solve $r \cdot t = d$ for r.

Creating and Solving Equations II

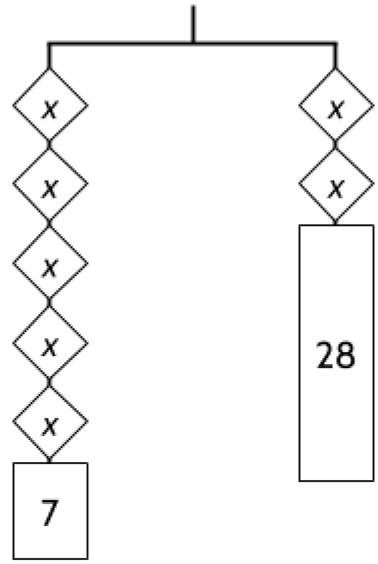
49) Directions: Use only the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value.

$$x + \square\square = \square\square$$

Example: $x + 23 = 45$
Solved: $x = 22$

Can you create an equation with a larger value for x?

Writing the Steps to Solve a Linear Equation



50) Consider the equation $5x + 7 = 2x + 28$. Your task is to explain the steps you would take, in order, to solve the equation for *x*.

Imagine that you are writing this for someone who is learning about this process for the first time. You should be as specific as possible, and write in complete sentences.

Writing an Equation for a Situation

Algebraic equations can be written to represent situations in our lives. Consider the following situation:

> Cleo works at a nursing home and gets paid \$12 per hour. Every week, \$40 is deducted from her wages to help pay for her health insurance. Cleo's weekly expenses are \$500 per week.

51) How many hours will Cleo need to work each week to pay her bills?

52) Fill in the blanks in the table to organize our work on the problem above.

Number of hours worked	hourly wage (\$12/hr.)	Money earned from wages	Deducting money for insurance (\$40)	Amount of paycheck
20	× 12	240	- 40	200
25	× 12		-40	
30	× 12		-40	
35	× 12		-40	
40	× 12		-40	
45	× 12		-40	
50	× 12		-40	

From the table, we can see that the number of hours and the amount of the paycheck are changing, but the hourly wage and the insurance deduction are staying the same. The amounts that are changing are variables (since they *vary* or change).

We could write an equation and use words as variables for the values that change:

$$number\ of\ hours\ worked \times 12 - 40 = amount\ of\ paycheck$$

We start with the number of hours on the left side of the table, make some calculations and end up with the amount of money on Cleo's paycheck on the right side of the table.

To make the equation shorter, we could write the following:

> Let ***h*** represent the number of hours worked.
> Let ***p*** represent the amount of the paycheck.

We include this statement so that other people will understand our formula. We will use *h* and *p* as variables in our formula. If we don't explain what they mean, it may be confusing to other people who want to understand our calculations.

And then we can write the formula like this:

$$h \times 12 - 40 = p$$

h is the number of hours Cleo works and *p* is the amount of money she takes home, depending on how many hours she works.

In the problem above, you were asked how many hours Cleo would need to work in order to bring home $500 per week in her paycheck. You can use our equation to find out the number of hours. The first step would be to replace *p* with $500, since that is how much money she needs for her bills:

$$h \times 12 - 40 = 500$$

This formula means that multiplying some number of hours by 12 and then subtracting 40 should be equal to 500.

53) What value of h would make this equation true (so that Cleo has enough money for her bills)?

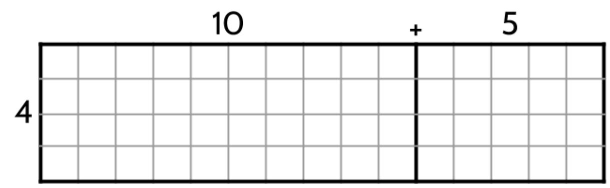
54) Imagine Cleo got a $1.50 raise in her hourly wage. How many hours would she need to work to pay her bills?

55) If Cleo was paid $15.00/hr, how much money would she be able to save each week if she worked 40 hours?

The Distributive Property and the Order of Operations

In an earlier activity, we used area models to look at multiplication. Since then, you have done a lot of work evaluating expressions and equations and using the order of operations. We are going to use the skills you have developed to explore a property of multiplication called the *distributive property.*

An Example Using Multiplication and Addition



The expression $4(10 + 5)$ could be used to calculate the area of a rectangle like the one on the right.

The length of the rectangle is $10 + 5$. Following the order of operations, we would add $10 + 5$ inside the parentheses before multiplying by 4. The parentheses group the addition so that it happens before the multiplication.

Our calculation would look like this:

Expression:	$4(10 + 5)$	
Step 1:	$4 \cdot 15$	do addition inside parentheses
Step 2:	60	multiply to find the value

However, we can also find the value without doing the addition first:

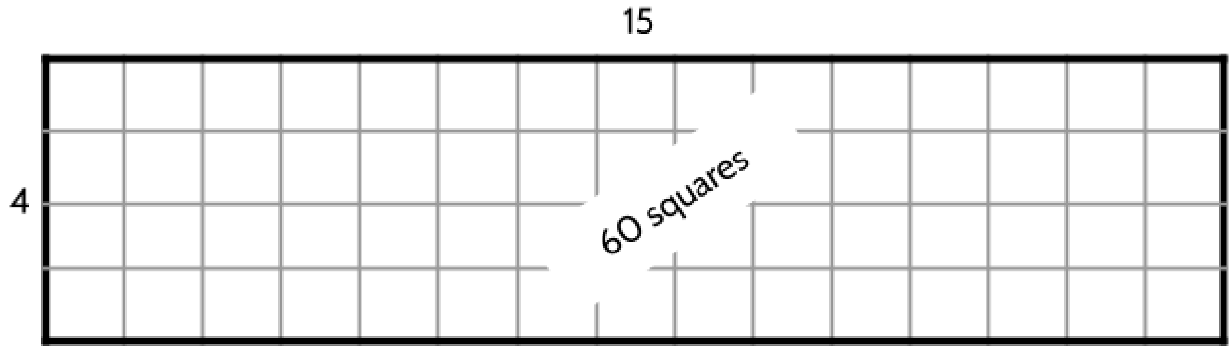
Expression:	$4(10 + 5)$	
Step 1:	$40 + 20$	multiply by both of the terms inside the parentheses separately ($4 \cdot 10$and $4 \cdot 5$)
Step 2:	60	add the products

The second way of calculating the value of $4(10 + 5)$ uses the distributive property of multiplication, which we looked at when multiplying with rectangles. The multiplication of 4 is *distributed* to 10 and 5 separately inside the rectangles. This allows us to multiply first and then add the two quantities together.

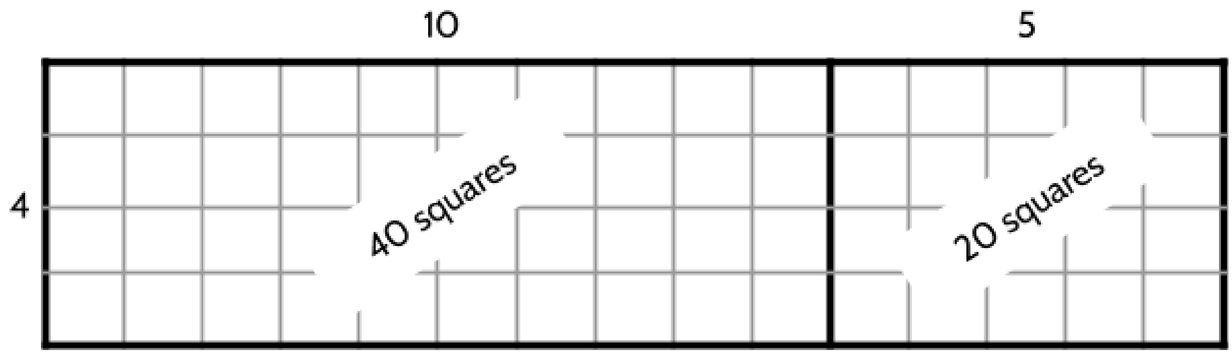
$$\begin{aligned} 4 \times (10 + 5) &= 4 \times 10 + 4 \times 5 \\ &= 40 + 20 \\ &= 60 \end{aligned}$$

When we calculated the area of rectangles, we saw that $4(10 + 5)$ is equal to $4 \cdot 10 + 4 \cdot 5$. There is a visualization of each expression below:

4(10 + 5)



4 · 10 + 4 · 5



From the rectangles above, we can see that $4(10 + 5) = 4 \times 10 + 4 \times 5$. We can look at the two rectangles and see that they are the same size. We can also add 40 to 20 to get 60.

In the equation $4(10 + 5)$, we can distribute the multiplication before adding and still get the same answer as when we add first and then multiply. Let's look at some examples.

56) Show that $5(20 + 3)$ is equal to $5 \cdot 20 + 5 \cdot 3$.

57) Complete the following table.

Expression	If We Group First	If We Distribute First
$4(10 + 2)$	$4 \cdot 12$ 48	$4 \cdot 10 + 4 \cdot 2$ 40 + 8 48
$6(8 + 2)$		
	$7 \cdot 9$	$7 \cdot 5 + 7 \cdot 4$
$8(5 + .5)$	$8 \cdot 5.5$	
Write your own expression.		
	$a(b + c)$	$a \cdot b + a \cdot c$

The distributive property of multiplication can be represented with the equation $a \cdot (b + c) = a \cdot b + a \cdot c$ which means that $a \cdot (b + c)$ is always equal to $a \cdot b + a \cdot c$ no matter what numbers are used for a, b, and c.

58) If $a = 4$, $b = 5$, and $c = 6$, what is the value of $a(b + c)$?

With the same values of a, b, and c, what is the value of $a \cdot b + a \cdot c$?

59) If $a = 5$, $b = 10$, and $c = 7$, is $a \cdot (b + c) = a \cdot b + a \cdot c$ a true equation? Explain below.

An Example Using Multiplication and Subtraction

Let's look at the expression $5 \cdot (10 - 4)$. This is how we can find the value of the expression using the order of operations:

Expression:	$5 \cdot (10 - 4)$	
Step 1:	$5 \cdot 6$	do subtraction inside parentheses
Step 2:	30	multiply to find the value

This is a way to find the value by using the distributive property instead:

Expression:	$5 \cdot (10 - 4)$	
Step 1:	$50 - 20$	multiply 5 by both of the terms inside the parentheses separately
Step 2:	30	subtract the products

In the second method, the multiplication of 5 is *distributed* to 10 and 4 separately inside the rectangles. Then 20 is subtracted from 50.

$$
\begin{aligned}
5 \times (10 - 4) &= 5 \times 10 - 5 \times 4 \\
&= 50 - 20 \\
&= 30
\end{aligned}
$$

Is $5 \times (10 - 4)$ a special case or is this always true? We should look at some other examples to see if we can distribute the multiplication before subtracting and still get the same answer as if we subtracted first and then multiplied.

60) Is $5(20 - 3)$ equal to $5 \times 20 - 5 \times 3$? How do you know?

61) Complete the following table.

Expression	If We Group First	If We Distribute First
$5(10 - 2)$	$5 \cdot 8$ 40	$5 \cdot 10 - 5 \cdot 2$ 50 – 10 40
$9(6 - 2)$		$9 \cdot 6 - 9 \cdot 2$
$4(2 -.5)$	$4 \cdot 1.5$	$4 \cdot 2 - 4 \cdot .5$
$2(10 - 1.5)$		
Write your own expression.		
	a(*b* - *c*)	*a* · *b* - *a* · *c*

The equation $a(b - c) = a \cdot b - a \cdot c$ shows that the distributive property is also true when multiplying quantities that are being subtracted inside parentheses.

62) If $a = 4$, $b = 9$, and $c = 3$, what is the value of $a(b - c)$?

With the same values of a, b, and c, what is the value of $a \cdot b - a \cdot c$?

63) If $a = 5$, $b = 10$, and $c = 7$, is $a(b - c) = a \cdot b - a \cdot c$ a true equation? Explain below.

An Example Using Division and Addition

Since multiplication and division are inverse operations, the distributive property can also be used with division. When dividing one expression by another expression, a fraction bar —— is often used instead of the division symbol ÷. For example, $\frac{50}{10}$ means $50 \div 10$ or 50 divided by 10. Here are some more examples of division using the fraction bar:

$\frac{8}{2} = 8 \div 2$ $\frac{60}{15} = 60 \div 15$ $\frac{1}{4} = 1 \div 4$

All of these are true equations. Expression on the left means the same thing as the expression on the right. It looks a lot like a fraction, right? That's because you can think of a fraction as a division problem, with the top divided by the bottom. The fraction bar means division.

By the way, you will also sometimes see the fraction bar as a diagonal line /. For example, you could write $50 \div 10$ as $50/10$. Or $\frac{1}{4}$ could be written as $1/4$. Both mean "1 divided by 4."

The fraction bar is also a way of grouping operations. According to the order of operations, any operations above or below the fraction bar should be done before the fraction bar division. Here is an example of operations grouped by a fraction bar:

$$\frac{6+18}{3}$$

Following the order of operations, we need to add 6 and 18 before dividing by 3.The whole calculation would look like this:

Expression:	$\frac{6+18}{3}$	
Step 1:	$\frac{24}{3}$	do addition above fraction bar
Step 2:	8	divide to find the value

However, there is a way to find the value by using the distributive property:

Expression:	$\frac{6+18}{3}$	
Step 1:	$\frac{6}{3} + \frac{18}{3}$	divide both numbers above separately by 3

Step 2: $2 + 6$ add the *quotients* (answer to a division problem)

64) Complete the following table.

Expression	If We Group First	If We Distribute First
$\frac{40+8}{4}$	$\frac{48}{4}$ or $48 \div 4$ 12 12	$\frac{40}{4} + \frac{8}{4}$ or $40 \div 4 + 8 \div 4$ 10 + 2 10 + 2 12 12
$\frac{30+4}{2}$	$\frac{34}{2}$ or $34 \div 2$	$\frac{30}{2} + \frac{4}{2}$ or $30 \div 2 + 4 \div 2$
$\frac{20+16}{4}$		$\frac{20}{4} + \frac{16}{4}$ or $20 \div 4 + 16 \div 4$
	$\frac{65}{5}$ or $65 \div 5$	
Write your own expression.		
	$\frac{b+c}{a}$	$\frac{b}{a} + \frac{c}{a}$

65) If $a = 6$, $b = 18$, and $c = 12$, what is the value of $\frac{b+c}{a}$?

With the same values of a, b, and c, what is the value of $\frac{b}{a} + \frac{c}{a}$?

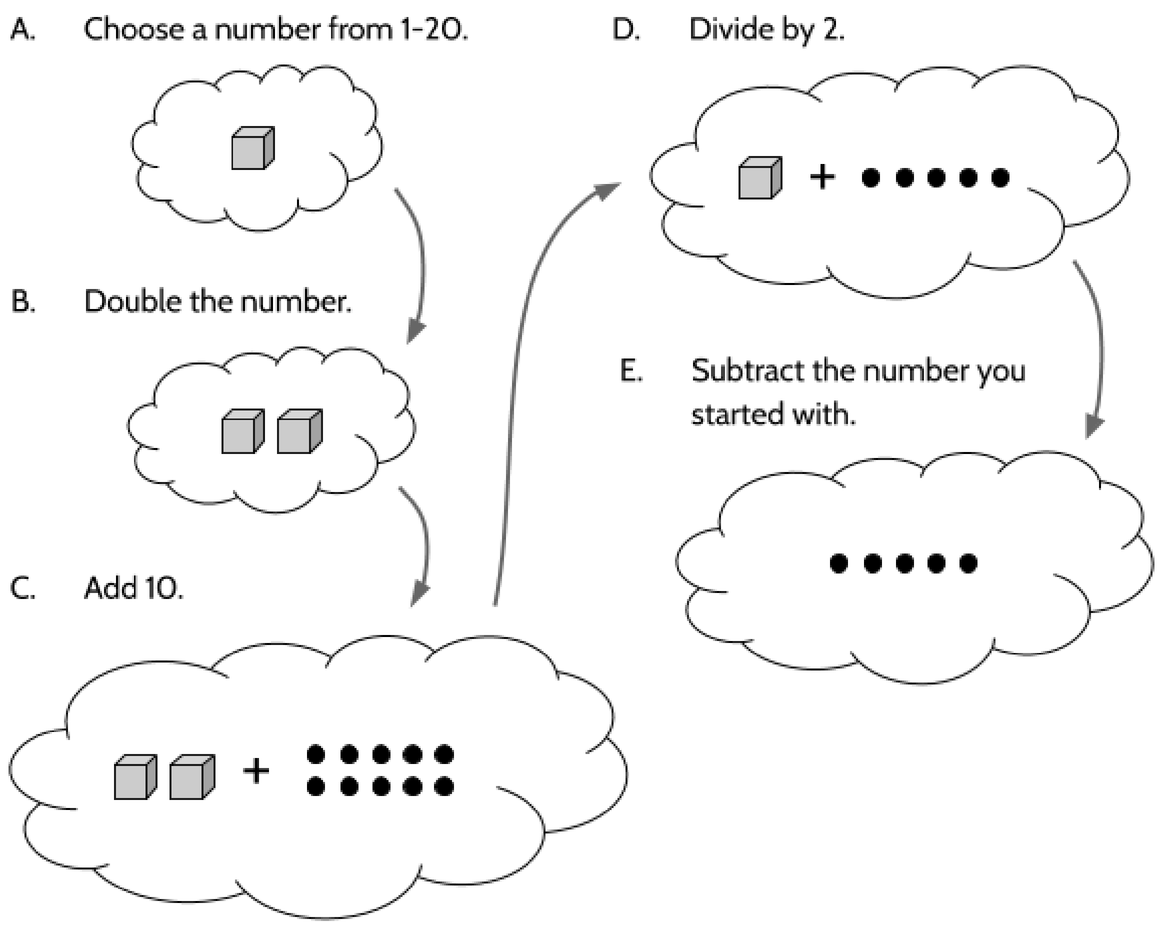
66) If $a = 3$, $b = 21$, and $c = 36$, is $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$ a true equation? Explain below.

The Math Behind the Magic Trick

Earlier in the packet, we practiced the following math trick.

> Choose a number from 1 to 20. Double it, add 10, divide by 2, and then subtract the number you started with.

We noticed that whatever number you choose, the final answer is always 5. Let's use pictures to understand what is happening in the trick:



67) Why is the result always 5?

Now, let's try applying some of the tools of algebra to see how the trick works. Since we don't know what number someone will choose, we will use a variable for the unknown number. It doesn't really matter what letter we use. I'm going to use n since it could stand for "number."

We can build up an equation by following the steps of the trick and turning it into math language as we go:

Any number between 1 and 20 →	n
Double the number →	$2n$
Add 10 →	$2n + 10$
Divide by 2 →	$\frac{2n+10}{2}$
Subtract starting number →	$\frac{2n+10}{2} - n$

The expression $\frac{2n+10}{2} - n$ means:

Double any number, add 10, divide by 2, and then subtract the number you started with.

We know one more thing. In our experience, whatever number we use, the final result is 5. This means the following equation seems to be true:

$$\frac{2n+10}{2} - n = 5$$

68) Using the equation above, explain why the result of the math trick is always 5.

69) Will the following expression work for a math magic trick? Why or why not?

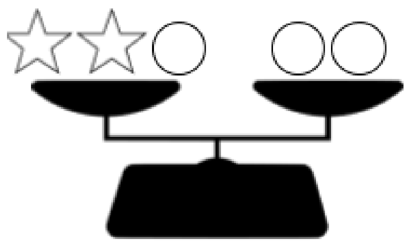
$$\frac{2n + 6 + n}{3} + 5 - n = 7$$

70) Challenge: Create your own math magic trick which always results in the same answer.

Balance Scales

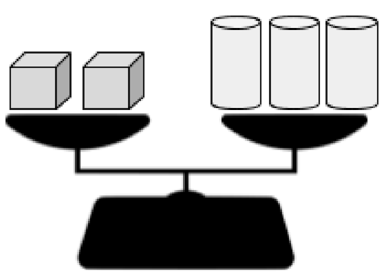
Balance scale puzzles are similar to hanger diagrams. Balance scales are a way of measuring the weight of items such as fruit or vegetables. An object with a known weight is placed on one scale and something with an unknown weight is placed on the other scale. If the two scales are balanced, then you know the weight is the same.

71) What can you say about the weight of the stars or the circles? Which weighs more? How do you know?

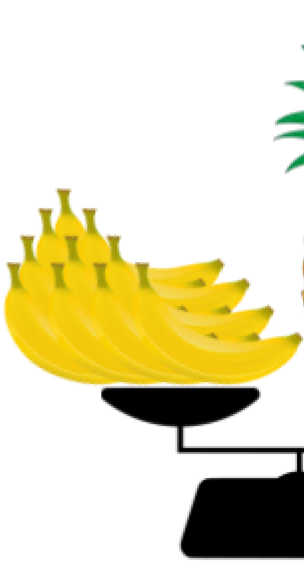


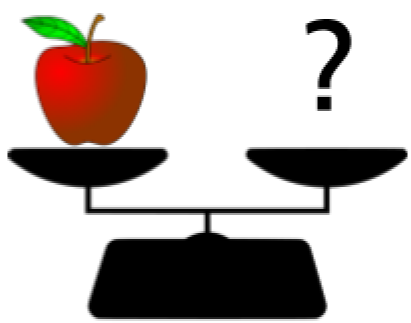
72) If v represents the weight of one star and w represents the weight of one circle, what equation represents the balance scale above?

73) What can you say about the weight of the cubes or the cylinders? Which weighs more? How do you know?



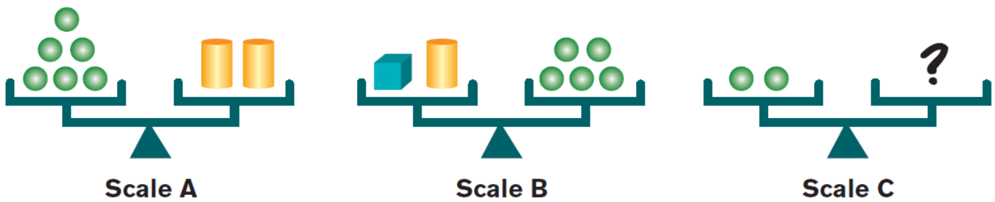
74) If x represents the weight of one cube and y represents the weight of one cylinder, what equation represents the balance scale above?





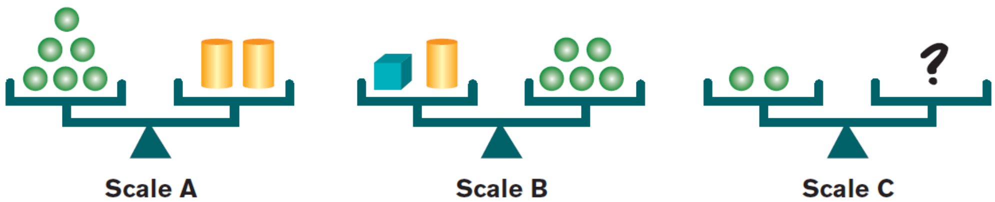
75) What could you put on the right side of the last scale to keep it balanced? Explain your thinking.

76) What objects should be placed on the right side of scale C to keep it balanced?

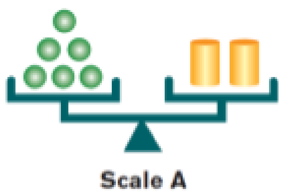


Explain your thinking below.

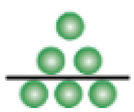
Problem:

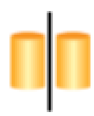


Explanation:

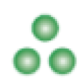


6 spheres weigh the same as 2 cylinders.



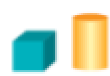


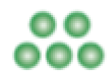
If we split the number of spheres and the number of cylinders in half, we can see that 1 cylinder weighs the same as 3 spheres.



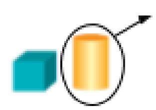
If we took 3 spheres off the left side and 1 cylinder off the right side, Scale A would still be balanced.

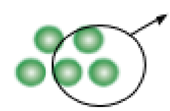
1 cube together with 1 cylinder weigh the same as 5 spheres.



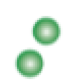


Since 1 cylinder is the same weight as 3 spheres (Scale A), Scale B will remain balanced if we take the cylinder off the left side and 3 spheres off the right side.

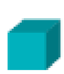




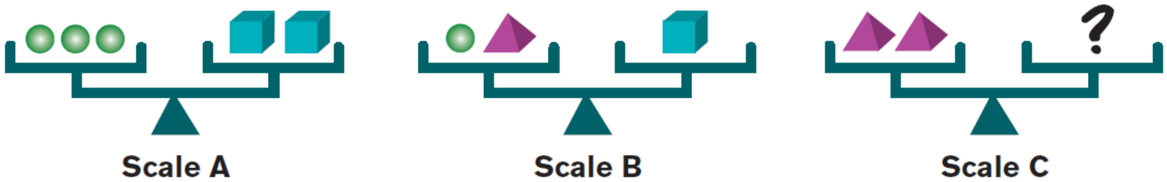
This leaves 1 cube on the left and 2 spheres on the right.

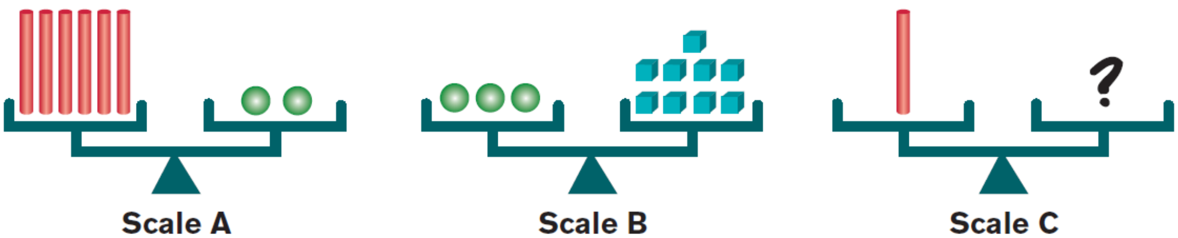


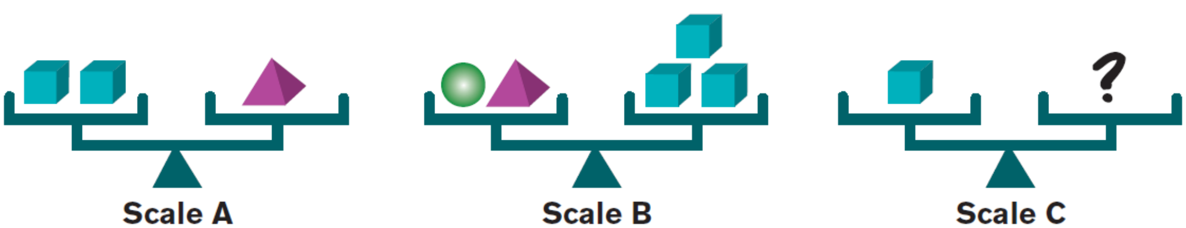
Solution:

Scale C needs 1 cube  on the right side in order to stay balanced with 2 spheres  .

In each of the following puzzles, find the right objects to place on the right side of Scale C to keep it balanced.

77) 

78) 

79) 

Restaurant Pricing

80) The Libra restaurant has a strange menu. The price of dessert is listed, but there is no price for the other menu items. Can you figure out the prices so you can order dinner?

Menu

One dinner costs as much as three salads.

Two salads cost as much as five iced teas.

Three iced teas cost as much as two desserts.

One dessert costs $2.25.

Please show all your work.

Answer Key - Balancing Equations

1) 4 pounds
2) 14 pounds
3) 5 pounds
4) 9 pounds

	If v or  is...	If w or  is...	T/F	Explanation
5)	1.5	3.5	F	The equation would be false, since 1.5 + 1.5 equals 3, not 3.5
6)	$\frac{3}{8}$	$\frac{6}{8}$	T	The equation would be true, since $\frac{3}{8}$+$\frac{3}{8}$ $= \frac{6}{8}$
7)	4.2	2.1	F	The equation would be false, since 4.2 + 4.2 equals 8.4, not 2.1
8)	$\frac{3}{4}$	$1\frac{1}{2}$	T	The equation would be true, since $\frac{3}{4}$+$\frac{3}{4}$ $= 1\frac{1}{2}$
9)	$.08$	$.16$	T	The equation would be true, since $.08 + .08 = .16$.
10)	$2\frac{4}{6}$	$1\frac{2}{3}$	F	The equation would be false, since $2\frac{4}{6} + 2\frac{4}{6} = 5\frac{2}{6}$not $1\frac{2}{3}$
11)	There are many possible answers that will make the equation true.			

12) 3
13) B
14) 2
15) C
16) 4
17) A
18) $z + 4 = 9$
    $z = 5$
19) $4v + 3 = 7$
    $v = 1$

20) $3w = 12$
$w = 4$

21) $x + 10 = 35$
$x = 25$

22) $5y + 7 = 47$
$y = 8$

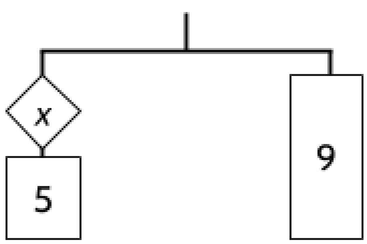
23) $2z + 6 = 9$
$z = 1\frac{1}{2}$

24) $4v + 5.5 = 21.5$
$v = 4$

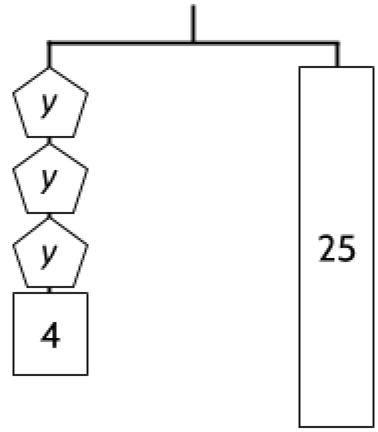
25) $5w + 2 = 3w + 8$
$w = 3$

26) $4x + 8 = 2x + 26$
$x = 9$

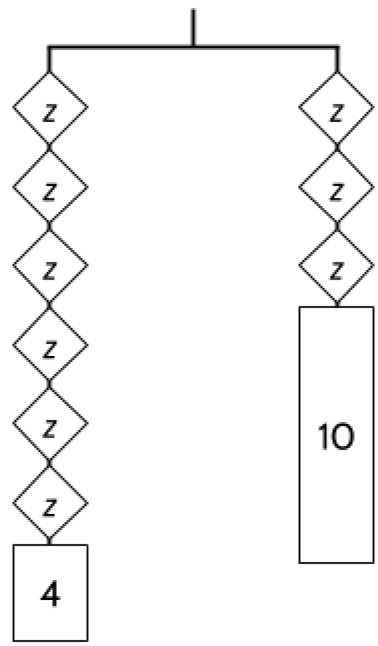
27) There are different ways to draw the hanger diagrams. Here is one way for each of these three problems.



$x + 5 = 9$
$x = 4$



28)
$3y + 4 = 25$
$y = 7$



29)
$6z + 4 = 3z + 10$
$z = 2$

30) $y = 14$

31) $z = 7.5$

32) $\frac{2}{4}$ or $\frac{1}{2}$

33) $w = 10$

34) $y = 9$

35) $z = 100$

36) $v = 13$

37) $y - 5 = 20$
$y - 5 + 5 = 20 + 5$
$y = 25$

38) $z + \frac{1}{4} = 5$
$z + \frac{1}{4} - \frac{1}{4} = 5 - \frac{1}{4}$
$z = 4\frac{3}{4}$

39) $v - 7 = 15$
$v - 7 + 7 = 15 + 7$
$v = 22$

40) $w + .75 = 4$
$w + .75 - .75 = 4 - .75$
$w = 3.25$

41) $a + b = 20$
$a + b - b = 20 - b$
$a = 20 - b$

42) $c - d = 100$
$c - d + d = 100 + d$
$c = 100 + d$

43) $y \cdot 5 = 20$
$y \cdot 5 \div 5 = 20 \div 5$
$y = 4$

44) $z \div 6 = 5$
$z \div 6 \cdot 6 = 5 \cdot 6$
$z = 30$

45) $v \div 3 = 15$
$v \div 3 \cdot 3 = 15 \cdot 3$
$v = 45$

46) $w \cdot .25 = 4$
$w \cdot .25 \div .25 = 4 \div .25$
$w = 16$

47) $a \cdot b = 24$
$a \cdot b \div b = 24 \div b$
$a = 24 \div b$

48) $r \cdot t = d$
$r \cdot t \div t = d \div t$
$r = d \div t$

---

49) Here is the equation with the largest value of x:
$x + 12 = 98$

50) There are different ways to explain how to solve this equation. If you are having trouble, look back at the explanation of hanger diagrams above.

51) 45 hours.

52)

Number of hours worked	hourly wage ($12/hr.)	Money earned from wages	Deducting money for insurance ($40)	Amount of paycheck
20	× 12	240	- 40	200
25	× 12	300	-40	260
30	× 12	360	-40	320
35	× 12	420	-40	380
40	× 12	480	-40	440
45	× 12	540	-40	500
50	× 12	600	-40	560

53) We can use our formula to calculate Cleo's paycheck:
$h \times 12 - 40 = p$
Cleo needs $500 for her bills, so we will replace p with 500 and then solve for h (the number of hours).
$h \times 12 - 40 = 500$
$h \times 12 - 40 + 40 = 500 + 40$
$h \times 12 = 540$
$h \times 12 \div 12 = 540 \div 12$
$h = 45$(Cleo would have to work 45 hours.)

54) Since Cleo's hourly raise would now be $13.50, we can use the revised formula below to calculate her paycheck:
$h \times 13.50 - 40 = p$
Cleo needs $500 for her bills, so we will replace p with 500 and then solve for a new value of h.
$h \times 13.50 - 40 = 500$
$h \times 13.50 - 40 + 40 = 500 + 40$
$h \times 13.50 = 540$
$h \times 13.50 \div 13.50 = 540 \div 13.50$
$h = 45$(Cleo would have to work 40 hours.)

55) We can use the following formula to calculate a paycheck at $15/hr:
$h \times 15 - 40 = p$
We know Cleo is working 40 hours, so we will replace h with 40. We don't know how much money she will make yet, so we will leave p as a variable to stand for the unknown amount on her paycheck.
$40 \times 15 - 40 = p$

$600 - 40 = p$
$560 = p$
Cleo would make \$560 on her paycheck. If she spends \$500 on expenses, she can save \$60 each week.

56) You might draw a rectangle to explain how to use the distributive property.

57)

Expression	If We Group First	If We Distribute First
$4(10 + 2)$	$4 \cdot 12$ 48	$4 \cdot 10 + 4 \cdot 2$ $40 + 8$ 48
$6(8 + 2)$	$6 \cdot 10$ 60	$6 \cdot 8 + 6 \cdot 2$ $48 + 12$ 60
$7(5 + 4)$	$7 \cdot 9$ 63	$7 \cdot 5 + 7 \cdot 4$ $35 + 28$ 63
$8(5 + .5)$	$8 \cdot 5.5$ 44	$8 \cdot 5 + 8 \times .5$ $40 + 4$ 44

58) 44 is the answer to both questions.

59) Yes. $5(10 + 7)$ is 85 and $50 + 35$ is also 85.

60) Yes. Show your calculations.

61)

Expression	If We Group First	If We Distribute First
$5(10 - 2)$	$5 \cdot 8$ 40	$5 \cdot 10 - 5 \cdot 2$ $50 - 10$ 40
$9(6 - 2)$	$9 \cdot 4$ 36	$9 \cdot 6 - 9 \cdot 2$ $54 - 18$ 36
$4(2 - .5)$	$4 \cdot 1.5$ 6	$4 \cdot 2 - 4 \cdot .5$ $8 - 2$ 6
$2(10 - 1.5)$	$2 \cdot 8.5$ 17	$2 \cdot 10 - 2 \cdot 1.5$ $20 - 3$ 17

62) 24 is the answer to both questions.

63) Yes. $5(10 - 7)$ is 15 and $50 - 35$ is also 15.

64)

Expression	If We Group First	If We Distribute First
$\frac{40+8}{4}$	$\frac{48}{4}$ or $48 \div 4$ 12 12	$\frac{40}{4} + \frac{8}{4}$ or $40 \div 4 + 8 \div 4$ $10 + 2$ $10 + 2$ 12 12
$\frac{30+4}{2}$	$\frac{34}{2}$ or $34 \div 2$ 17 17	$\frac{30}{2} + \frac{4}{2}$ or $30 \div 2 + 4 \div 2$ $15 + 2$ $15 + 2$ 17 17
$\frac{20+16}{4}$	$\frac{36}{4}$ or $36 \div 4$ 9 9	$\frac{20}{4} + \frac{16}{4}$ or $20 \div 4 + 16 \div 4$ $5 + 4$ $5 + 4$ 9 9
There is more than one way to break up 65. This is one way: $\frac{50+15}{5}$	$\frac{65}{5}$ or $65 \div 5$ 13 13	$\frac{50}{5} + \frac{15}{5}$ or $50 \div 5 + 15 \div 5$ $10 + 3$ $10 + 3$ 13 13

65) 5 is the answer to both questions.

66) Yes. $\frac{36+21}{3}$ is 19 and $\frac{21}{3} + \frac{36}{3}$ is also 19.

67) Think of the box as the number that we choose at the beginning. Do you see how the box disappears at the end, leaving 5 dots behind? That's the answer of 5 that we always get.

68) Here is an algebraic way to see it:

$\frac{2n+10}{2} - n = 5$ The original equation

$\frac{2n}{2} + \frac{10}{2} - n = 5$ Distribute the division so that $2n$ is divided by 2 and 10 is divided separately by 2.

$n + 5 - n = 5$ 2 divided by 2 is 1, so n is by itself. 10 divided by 2 is 5.

$5 = 5$ n minus n is 0, so the n's disappear. All that is left is 5.

69) Yes. Try it with some different numbers.

70) Optional: Send your mystery number puzzle to info@collectedny.org or @CollectEdNY on Twitter.

71) The circle weighs twice as much as the stars. If you take one circle away from each side, the scale will still be balanced and there will be 2 stars on the left and 1 circle on the right.

72) $2v + w = 2w$ or $2v = w$

73) The cubes weigh more since 2 cubes have the same weight as 3 cylinders.

74) $2x = 3y$

75) 3 bananas. In the first scale, 10 bananas have the same weight as 2 pineapples. Therefore, 5 bananas would have the same weight as 1 pineapple. If you replace the pineapple on the second scale with 5 bananas, the scale would still be balanced. If you then take away 2 bananas from both sides, you would find that 1 apple weighs the same as 3 bananas.

76) 1 cube  .

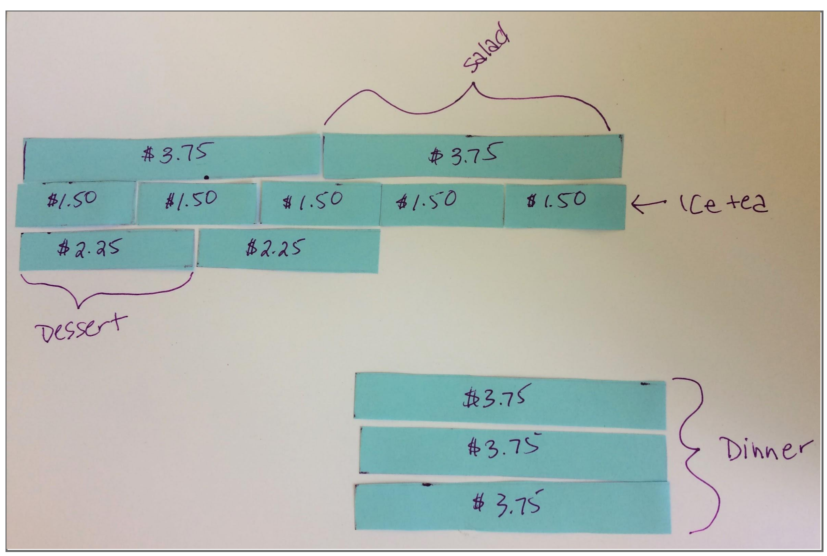
77) 1 sphere  .

78) 1 cube  .

79) 1 sphere  .

80) Dinner: $11.25, salad: $3.75, iced tea: $1.50, dessert: $2.25. Here are a couple solutions from two math teachers:

I doubled the one-dessert cost to find the cost of two. Since 2 desserts is the same cost as 3 iced teas, I divided that cost by three to find the cost of one iced tea. Then I multiplied the cost of one tea by 5 to find the cost of 5 teas. Since that's the same as 2 salads, I divided by 2 to get the cost of one salad. I multiplied the cost of one salad by three to get the cost of a dinner.



Vocabulary Review

You can use this section to look up words used in this math packet.

balance (noun)**:** a state of equilibrium; equal distribution of weight.

balance scale (noun)**:** a device for measuring the weight of objects

coefficient (noun): A number multiplied by a variable. For example, $4x$ is the same as 4 times x. $4$is the coefficient of x.

constant (noun): A value in an expression or equation that doesn't change. For example, in the expression $3w + 2$, the numbers 3 and 2 are both constants.

distributive property of multiplication (noun): a mathematical property that shows multiplying a number by a group of numbers added together is the same as doing each multiplication separately. For example, $3 \cdot (2 + 4)$ is equal to $3 \cdot 2 + 3 \cdot 4$.

equivalent (adjective)**:** having the same value. For example, 4 quarters and 20 nickels are equivalent. Eight hours is equivalent to 28,800 seconds.

equal sign (noun)**:** a symbol used to show symmetric balance between two values or quantities, one on each side of the equal sign. Can be read as "is equivalent to" or "is the same as."

equation (noun): A number sentence that shows two expressions are equal by using the equal sign. $2^3 = 8$ is an *equation*. $5x + 3$ is an expression, <u>not</u> an *equation*.

equilibrium (noun): When things are balanced. An equation can be in a state of equilibrium. A country could also be in a state of equilibrium, if opposing forces are balanced.

equivalent (adjective): Having the same value. 4^3 and 64 are *equivalent*.

estimate (verb): To make a rough guess at a number, usually without making written calculations

evaluate (verb): To calculate the value of something. If asked to *evaluate* 4^3, your answer should be 64.

expression (noun): Numbers and symbols that show the value of something. 100, $5x + 3$, and 2^3 are all *expressions*. $5x + 3 = 23$is an equation made up of two *expressions*.

exponent (noun): In a quantity represented as a power, the *exponent* shows how many times the base is multiplied. The exponent is shown as a smaller number up and to the right of the base. For example, in the power 2^3, the *exponent* is 3.

factor (noun): Whole numbers that are multiplied together to get another number. A number that can be divided into another number evenly, with no remainder.

factor (verb): To split a number into its factors (see above definition of factors).

generalization (noun): To look at specific examples and realize that something is true *in general*. For example, what happens if you divide a number by itself? Specific examples: $8 \div 8 = 1$ or $25 \div 25 = 1$ or $.75 \div .75 = 1$. *Generalization*: Any number divided by itself is 1.

inequality (noun): A number sentence that compares two values which do not have to be equivalent.

inverse (adjective): Opposite or reverse. Addition and subtraction are *inverse operations*.

multiple (noun): A number that can be divided by another number evenly, with no remainder. 25 is a *multiple* of 5.

numeral (noun): A symbol or name for a number. 12 and twelve are both *numerals*.

operation (noun): A mathematical process. The four basic operations are addition, subtraction, multiplication, and division. Exponents and roots are also operations.

parentheses (noun): These are parentheses: ( ). They are used for grouping operations that should be prioritized when calculating.

property (noun): A character or quality that something has. In science, we use the physical property to refer to color, texture, density, and other qualities of physical objects. In mathematics, properties refer to characteristics of numbers or operations. Example:

> *Commutative property*: Addition and multiplication are *commutative*, meaning the order of the operation doesn't matter. $2 + 7 = 7 + 2$ and $2 \cdot 7 = 7 \cdot 2$

product (noun): The result of multiplication. 4 times 5 gives a *product* of 20.

quotient (noun): The result of division. 20 divided by 5 gives a *quotient* of 4.

radical (noun): A symbol that means "root." *Radicals* are used for square roots, cube roots, and other roots.

square root symbol: $\sqrt{}$

rate (noun)**:** a ratio with two different quantities that are being compared.

Examples: *Speed* compares distance traveled to the amount of time that has gone by. *Population density* compares the number of people with the amount of space.

reciprocal (noun): If you multiply a number and its reciprocal, the answer will be 1. Another way of saying this is the *reciprocal* of a number is the result of dividing 1 by the number.

Examples: $3 \cdot \frac{1}{3} = 1$ $\frac{2}{3} \cdot \frac{3}{2} = 1$ $x \cdot \frac{1}{x} = 1$

right triangle (noun)**:** a right triangle is a triangle with one 90° angle. In a right triangle, the side opposite the right angle is called the *hypotenuse*. The two sides that form the right angle are called *legs*.

root (noun): The solution to an equation, usually similar to $a^2 = 25$ or $a^3 = 8$

square root: A square root of a number is a value that, when multiplied by itself, gives the number. The *square root* of 25 is 5.

term (noun): A single number or variable, or numbers and variables multiplied together. Terms are separated by + or – signs. For example, there are three terms in the equation $2x + 5(2) = 16$.

like terms: Two or more terms with the same variable and exponents which can be combined in an expression. Example: $10w$, $.5w$, and $\frac{1}{2}w$ are *like terms* and can be combined to make $11w$.

unlike terms: Terms that do not have the same variable and/or exponents and cannot be combined in an expression. Example: $8v$, $7v^2$, and $4x$ are *unlike terms*.

variable (noun): A letter or symbol that represents another value, either any number, a specific number, or a set of numbers. In the expression x^3, *x is a variable* that could mean any number. Variables can also represent other things. For example, in geometry, points and angles are represented by letters.

system of equations (noun): A set of two (or more) equations where the variables represent the same unknown values that make both equations true.

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