# Lines, Angles & Shapes: Measuring Our World

*Fast Track GRASP Math Packet*

## Part 1

*The Brooklyn Bridge, completed in 1883*

Version 1.3
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## Welcome!

Congratulations on deciding to continue your studies! We are happy to share this study packet on several topics in geometry, including lines, angles, and three forms of measurement. We hope that that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we often see on high school equivalency exams. If you study these topics carefully, while also practicing other basic math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn the math.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanation and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We also hope you will share what you learn with your friends and family. If you find something interesting in here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **http://www.acces.nysed.gov/hse/hse-prep-programs-maps**

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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# Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

**area** **angle** **circle** **measure** **parallel** **perimeter** **similar** **volume**

In this first activity, you will think about each word and decide how familiar you are with it. To start, think about the word "*area*" Which of these statements is true for you?

- I know the word "*area*" and use it in conversation or writing.
- I know the word "*area*," but I don't use it.
- I have heard the word "*area*" but I'm not sure what it means.
- I have never heard the word "*area*" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "area" might look when you're done:

| Word | I know the word and use the word | I know the word but don't use it | I have heard the word, but I'm not sure what it means | I have never heard the word | My best guess at the meaning of the word (or use the word in a sentence) |
|---|---|---|---|---|---|
| area | ✓ | | | | A place or location, like a neighborhood or town |

Complete the table on the next page.

This activity is designed to help you start thinking about some of the important words you will find in this packet. As you go through the activities in this packet, you will learn more about these words, what they mean, and how to use them. You will learn more precise definitions that may come up during your high school equivalency exam.

There is also a glossary with the definitions of useful vocabulary at the end of Part Two.



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| Word | I know the word and use the word | I know the word but don't use it | I have heard the word, but I'm not sure what it means | I have never heard the word | My best guess at the meaning of the word (or use the word in a sentence) |
|---|---|---|---|---|---|
| area | | | | | |
| angle | | | | | |
| circle | | | | | |
| measure | | | | | |
| parallel | | | | | |
| perimeter | | | | | |
| similar | | | | | |
| volume | | | | | |



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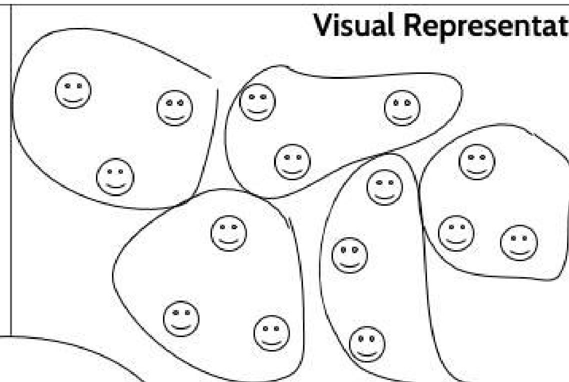
## Using Graphic Organizers to Learn Vocabulary

In order to learn math vocabulary, we need practice using it in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.

To complete the graphic organizer, you will choose a word and then answer four questions:

- What is the definition of the word? You can look at the vocabulary review at the end of **Lines, Angles, & Shapes: Measuring Our World, Part 2** for help. Try to write the definition in your own words to really make the word yours.
- Make a visual representation. You can make a drawing or diagram that will help you remember what the word means.
- What are some examples of the word you're studying? Below you can see that there are examples of *quotients*, which are the answers to division problems.
- What are some non-examples of this word? These are things that are **not** the word you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

| **What is it?** | **Visual Representation** |
|---|---|
| A quotient is the result of dividing one number by another. It is the answer to a division question. |  |
| **What are some examples?**<br>15 divided by 3 equals **5**<br>66 ÷ 6 = **11**<br>63/18 = **3.5**<br>5, 11 and 3.5 are quotients in these calculations.<br>population ÷ area = **density** | **What are some non-examples?**<br>4 times 6 equals 24<br>18 + 5 = 23<br>17 – 2.5 = 14.5<br>3.5 × 18 = 63 |

quotient



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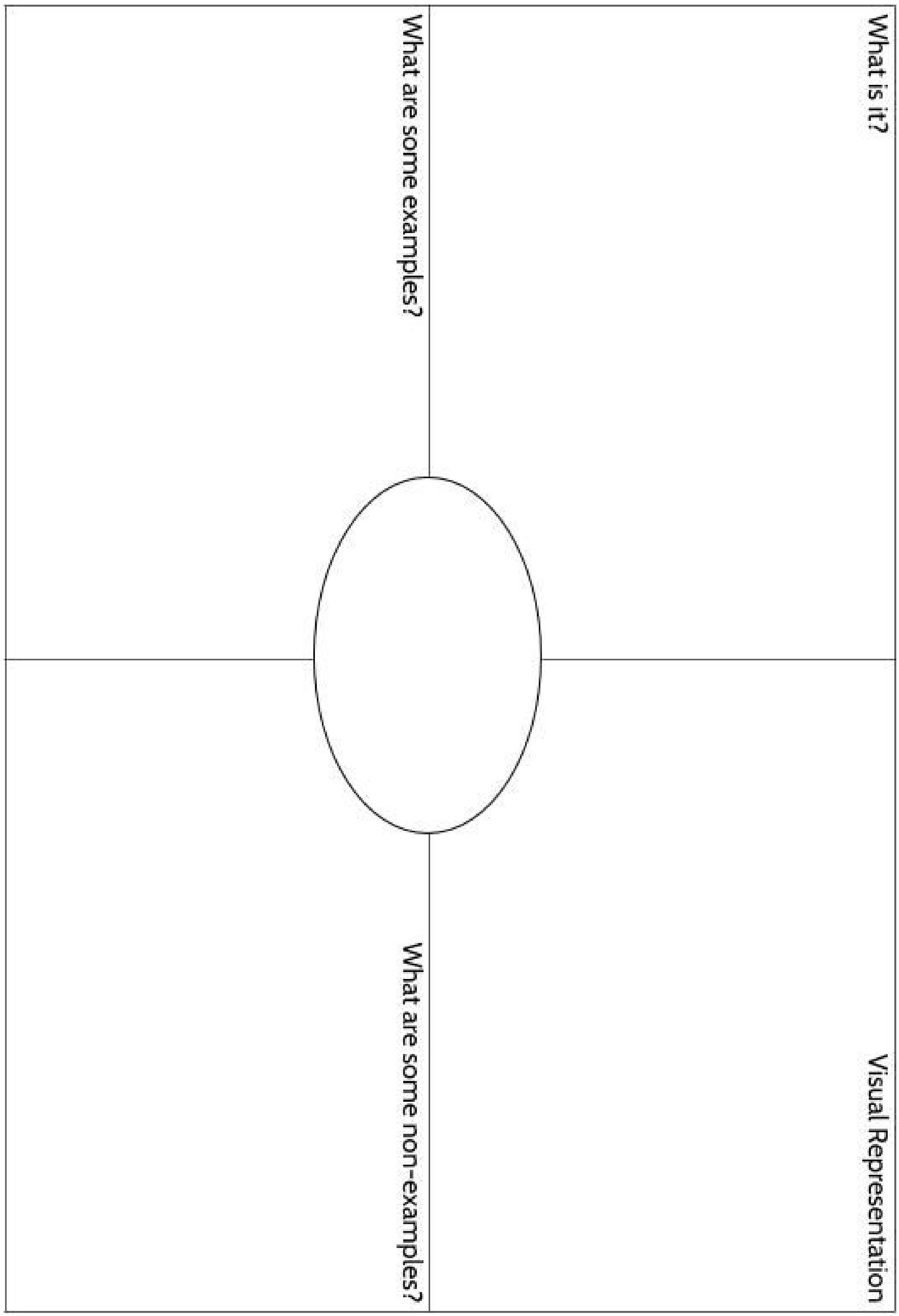




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What is it?

What are some examples?

Visual Representation

What are some non-examples?



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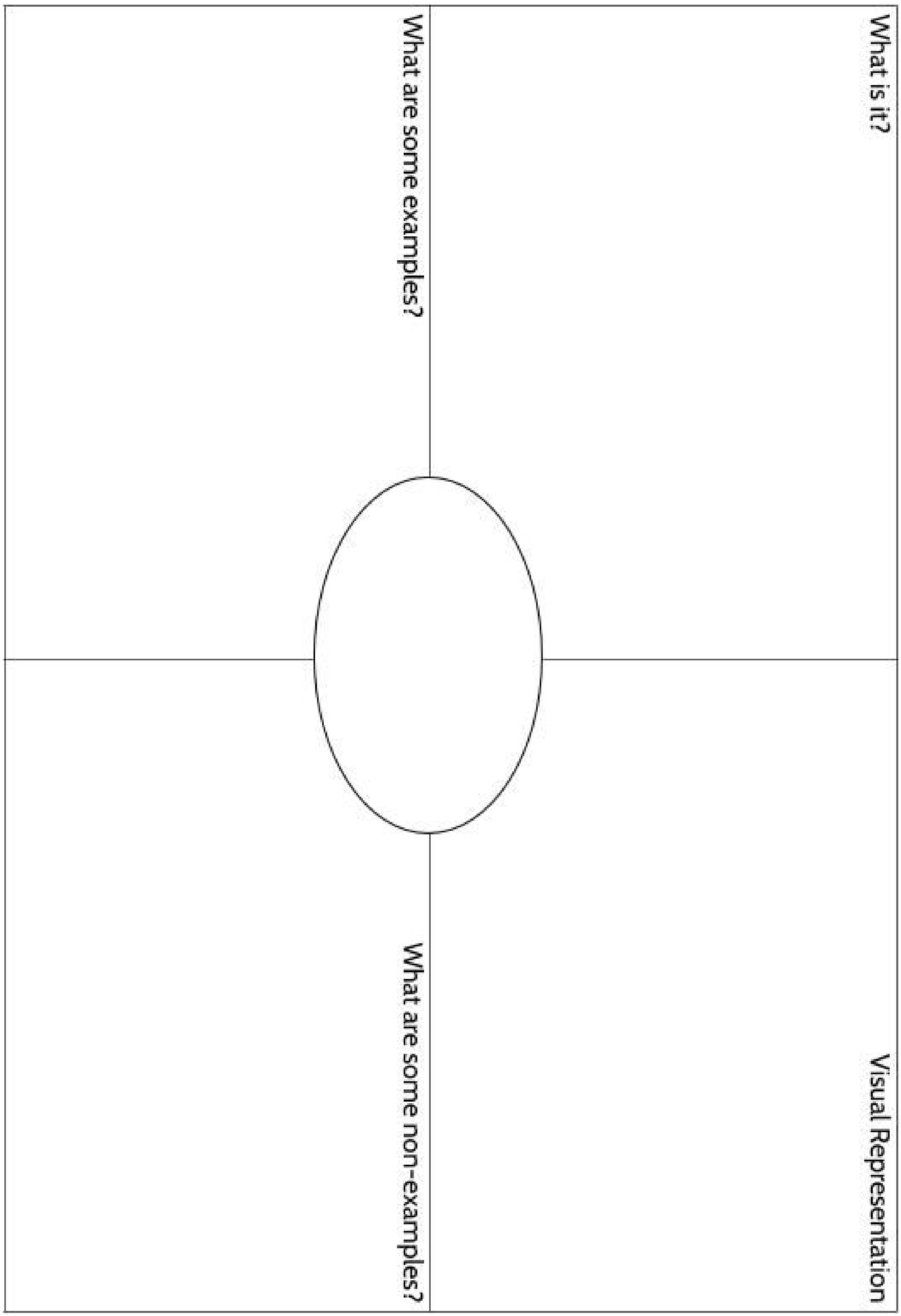




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| | |
|---|---|
| What is it? | Visual Representation |
| What are some examples? | What are some non-examples? |



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# Introduction

### Geometry in History

The word *geometry* is only about 900 years old, but human beings have been studying geometry for over 5000 years. The word *geometry* comes from two Greek words - *ge* which means “land” or “earth” and *metria*, which means “a measuring of.” So the word *geometry* means “a measuring of the land” or “the measuring of the Earth.”

Many of the words we use in geometry and in measurement come to us from the Ancient Greeks, who lived from about 3000 years ago until the end of the ancient world. The Ancient Greek civilization is thought of as a time of political, philosophical, artistic, and scientific achievement that influences our world even today.

According to the Greek historian Herodotus, who wrote about 2500 years ago, geometry began thousands of years before his time, in Egypt. It was from the Egyptians that the Greeks learned to measure the land and Earth around them.

When you think of Egyptian geometry, you might think of the great pyramids, built 4500 years ago and still standing today. However, the Egyptian study of land and development of systems of measurement are even older than that. Some of the earliest recorded measurements of length and area come from Ancient Egypt and are over 5000 years old.

The Egyptians were very practical in their use of geometry. There are word problems from the ancient world dealing with concrete matters—things like, “How many bricks would be needed to make a ramp of a certain size?” (Yes, humans have been working on word problems in math for thousands of years!)

The Egyptians’ study of geometry allowed them to collect taxes, divide land, make square corners, and use blueprints. It allowed them to build wonders and organize their civilization.

Geometry is not important because it is old, but it is old because it is important. It is old because it was a fundamental way for humans to understand their environment and to build a world around them.



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**Precision and Estimation**

We may not be building pyramids, but each of us is constantly measuring in our lives.

As you work on the activities in this packet, you will make observations and learn how to make precise calculations and measurements. You will also practice answering the kinds of geometry questions that you will see on the high school equivalency exam.

Sometimes in life, precise measurements are required. For example, you can't build a strong foundation of a building without being able to make a proper square. But we also do a lot of estimating in measurement, often without numbers at all.

Imagine you are driving a car. Your brain is constantly measuring the distance between your car and the car in front of you.

Imagine you are having a party in your apartment and you expect there to be dancing. You can think of the number of people coming and how much space they'll need to dance. And you can move the furniture to clear a dance floor without a ruler or tape measure in your hand.

After dinner, when you look through your tupperware to choose the right size container for your leftovers, how do you know which one to choose?

The situations above describe three categories of measurement (length, area, and volume) that we will explore throughout this packet.

**Length, Area, and Volume**

The next few pages will introduce you to the way we will be using these types of measurement as we study the geometry that is found all around us, as well as on the high school equivalency exam.



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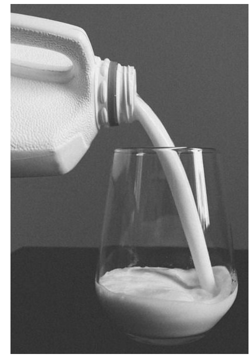
| Length | Area | Volume |
|---|---|---|
|   |  |  |
| **The distance between two points** | **The size of a surface** | **The space inside an object** |

**LENGTH**

Whenever we talk about measuring the distance between two points, we are talking about a measure of length. The length of my arm means the measurement from the top of my arm down to my fingertip.

If you can imagine measuring something with a tape measure, measuring from one end to the other, it is a measure of length. Even if you would need a really, really, long tape measure, it is a measure of length. For example, can you imagine measuring the distance from where you are sitting to the moon with a tape measure? That distance is a measure of length.

Write three examples of a measurement that is the distance between two points.

- 
- 
- 



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**AREA**

When we measure the size of a surface, we are using area. It might be helpful to think of a paint can when it comes to area. If you can imagine painting something with a brush, it is a measure of the size of that surface. We can imagine painting the surface of a wall. Even if it is something you could not actually paint, if you can imagine painting it's surface, then it is a measure of area. For example, we can't really paint an entire soccer field, but we can imagine covering it's surface with paint.

Write three examples of a measurement that is the size of a surface.

- 
- 
- 

**VOLUME**

When we measure how much space there is inside an object, we are using volume. If you can imagine pouring liquid into something, then it can be measured using volume. Can you pour water *into* a table top? No. But you can imagine pouring liquid into a box, or a bowl, or even your mouth. All of those things can be measured using volume.

Write three examples of a measurement that is talking about the space inside an object.

- 
- 
- 



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## LENGTH, AREA AND VOLUME

Many objects can be measured using all three—length, area, and volume.


Let's look at a can of ground coffee.

*What part of this can could you measure with a tape measure?*

*What part of this can could you paint?*

*What measurement are we talking about if we measure how much space there is inside this can?*


One example of length is the the height of the can. That might be important if you wanted to know how how many cans you could stack on a shelf. Imagine measuring the can from the top to the bottom with a tape measure. That is a measurement of length.

One example of area would be the surface of the can. That would be helpful if you were designing the label and needed to know how much room you would have. Imagine painting the outside of this coffee can. That is a measurement of area.

One example of volume is the measurement of how much space there is inside the can. How much coffee could we fit in there? That is a measurement of volume.



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Imagine the following situations and decide whether each is a measure of length, area, or volume.

Which would you use to measure the following: length, area, or volume?

1. The distance from the floor to the ceiling
2. The size of a coffee mug - how much liquid it will hold
3. Your height
4. The distance between my car and the car in front of you on the highway
5. The number of tiles needed to cover a bathroom floor
6. The space inside a Tupperware container
7. The wrapping paper necessary to wrap a gift
8. The distance between The Empire State Building and Niagara Falls
9. The size of a tupperware container needed to store leftovers
10. The amount of skin on your body
11. The total land in New York state
12. The amount of water a bathtub can hold
13. The size of a room

## Did You Know?

There are about 138 Egyptian pyramids. The oldest and tallest is called the Great Pyramid of Giza. It is about 200 feet taller than the Statue of Liberty and it would cover around 100 football fields! It is the only one of the Seven Wonders of the Ancient World still in existence today.



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## Length, Area, and Volume - Answer Key

1. length
2. volume
3. length
4. length
5. area
6. volume
7. area
8. length
9. volume
10. area
11. area
12. volume
13. The size of a room can actually be a measure of all three. If you are talking about the length of a room, you would be talking about the distance from one wall to the other. If you were talking about the area, you might be talking about how much floor space there is in the room. If you are talking about the volume, you might be talking about how much airspace there is in the room, or how much space there is to fill it with boxes.



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# The Building Blocks of Geometry

**Points** and **lines** are two of the building blocks of geometry. They help us describe the things in the world that we want to measure.

**Points** are the most fundamental elements in geometry. Points do not actually exist in the natural world—we can't touch them or see them, but we can use our experience of the natural world to understand them. A point has no length, area, or volume. It has no size; it has only location. Imagine using a pencil to draw the tiniest dot you can. Or a single tiny seed. Both of these are physical approximations of what we mean when we use the word *point* in geometry. In geometry, we represent a point with a dot and name it with a capital letter.

Example: • L The name of this point is *point L*.

Like points, ***lines*** in geometry do not exist in the natural world. We cannot see them or touch them, but like points we can use our experience in the world to understand them. Lines are made up of an infinite number of points continuing in opposite directions without end. We can think of a railroad track or of the strings of a guitar[1] as physical models of lines.

Lines can be named by using the letters of any two points on the line.

For example, the name of the line below is $\overleftrightarrow{SP}$ **or** $\overleftrightarrow{PS}$.


[1] *Photo of guitar by Baka9k - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=46595870*



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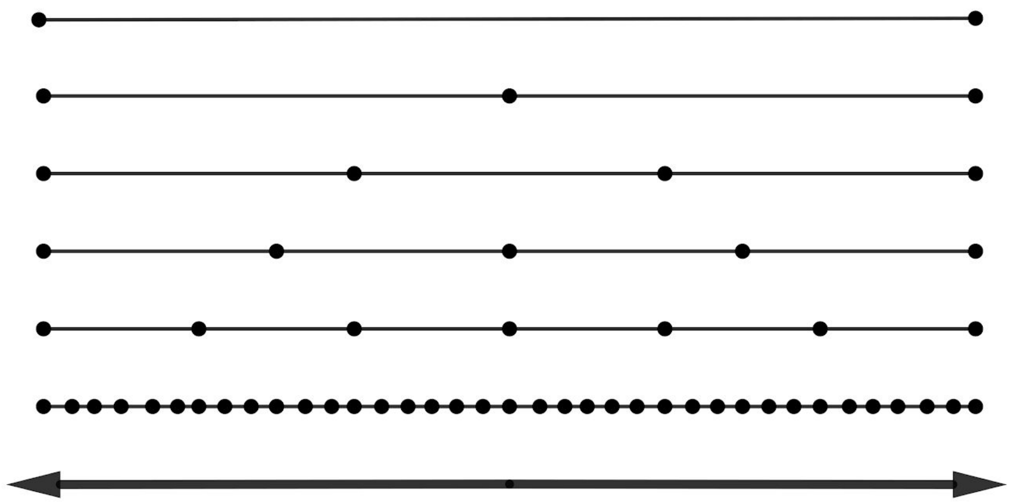
When we draw lines, we use arrows pointing in opposite directions to show that the line continues on forever in both directions. One thing to remember is that even though we only need two points to name a line, a line is made of an infinite series of points, one right after the other.

This drawing can help you visualize how lines are made up of many points. Start with the two points across the top. Visualize adding more and more points between them until you can't see any space between them. That is a line.





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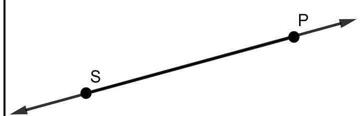
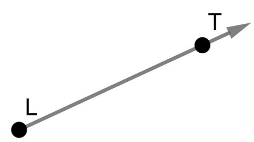
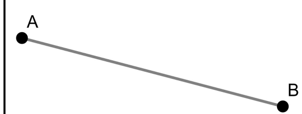
# Lines, Rays, and Segments

In the chart below, there are three figures that each look like lines. They are all straight. In geometry we have a specific name for each of them. It is helpful to understand the ways that they are different from each other.

1. Fill in the following chart

| **Name** | Line | Ray | Line Segment |
|---|---|---|---|
| **Figure** |  |  |  |
| **Notation**[2] | $\overleftrightarrow{SP}$ **or** $\overleftrightarrow{PS}$ | $\overrightarrow{LT}$ | $\overline{AB}$ **or** $\overline{BA}$ |
| **Characteristics** *(Briefly describe each figure. What makes it different from the other two?)* | | | |

[2] *Notation* is the way we write something in mathematics, using symbols, letters, or numbers.



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Of these three figures, the most basic is the **line**, which you already read about. In the chart above, *Line SP* has arrows pointing in opposite directions. Lines in geometry are drawn this way to show that they continue in both directions forever.

Most geometric figures, like shapes and angles, are made up of *parts* of a line.

A **segment** (or line segment) is part of a line, defined by two endpoints and all the points between them. The rungs of a ladder are physical examples of line segments.


The name of this line segment is $\overline{AB}$ or $\overline{BA}$. For line segments, the order of the points doesn't matter. However, it is important that you use the end points in the name.

A **ray** is a part of a line, starting with one endpoint and made up of all the points on one side of that endpoint. A beam of light from a flashlight is a physical example of a ray.

The name of this ray is $\overrightarrow{CE}$ **.** When we name rays, the name must start with the endpoint (C, in this case). The first letter is the endpoint and the second letter is any point the ray goes through.


2. Write all possible notations for each drawing.

| a. | b. | c. |
|---|---|---|
|   |   |   |



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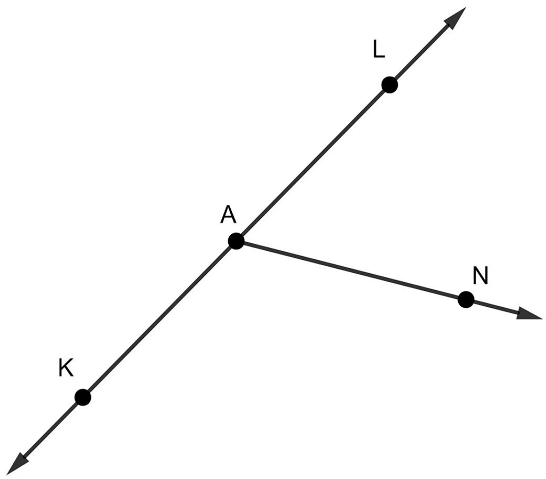
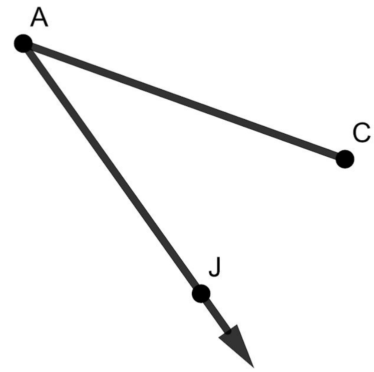
d. For this figure, use the notation that describes the <u>entire</u> ray.


e. *Fill in the blanks.*




___ intersecting with ___ at Point ___.

f. *Fill in the blanks.*




___ intersecting with ___ at Point ___.

3. Draw the following figures.

a. $\overrightarrow{EK}$

b. $\overline{SR}$

c. $\overleftrightarrow{JL}$

d. $\overleftrightarrow{PR}$ intersecting $\overleftrightarrow{PQ}$ at Point P



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| | |
|---|---|
| e. $\overrightarrow{MJ}$ intersecting $\overrightarrow{JT}$ at Point J | f. $\overline{HR}$ |

Use the line below to answer questions 4 and 5.


4. How many line segments are there? Write the names of any line segments you see.

5. How many rays are there? Write the names of any rays you see.



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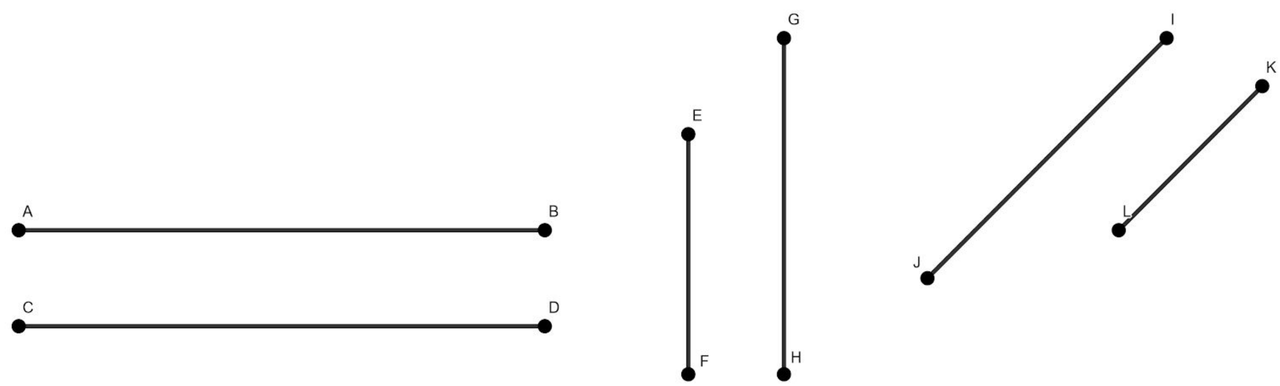
## Parallel and Perpendicular Lines

Look around and chances are you will see parallel and intersecting lines.

**Parallel lines** are lines that lie on the same flat surface and do not intersect.

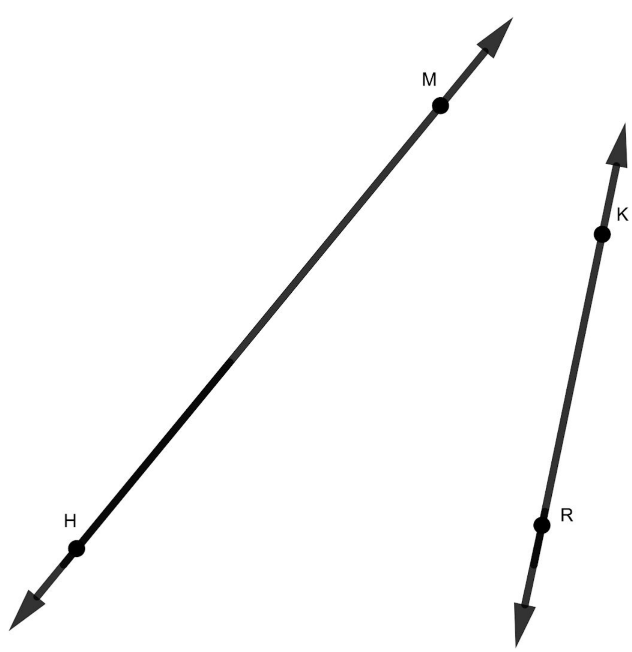
Below are some examples of lines that are parallel.




$\overline{AB}$ is parallel to $\overline{CD}$ which can be written as $\overline{AB} \parallel \overline{CD}$.

$\overline{EF}$ is parallel to $\overline{GH}$ which can be written as $\overline{EF} \parallel \overline{GH}$.

$\overline{JI}$ is parallel to $\overline{LK}$ which can be written as $\overline{JI} \parallel \overline{LK}$.




One important thing to remember is that just because drawings of lines do not intersect, it does not necessarily mean they are parallel.

For example, $\overleftrightarrow{HM}$ and $\overleftrightarrow{RK}$ are considered intersecting lines. If you extend the lines, they will eventually intersect at a single point.



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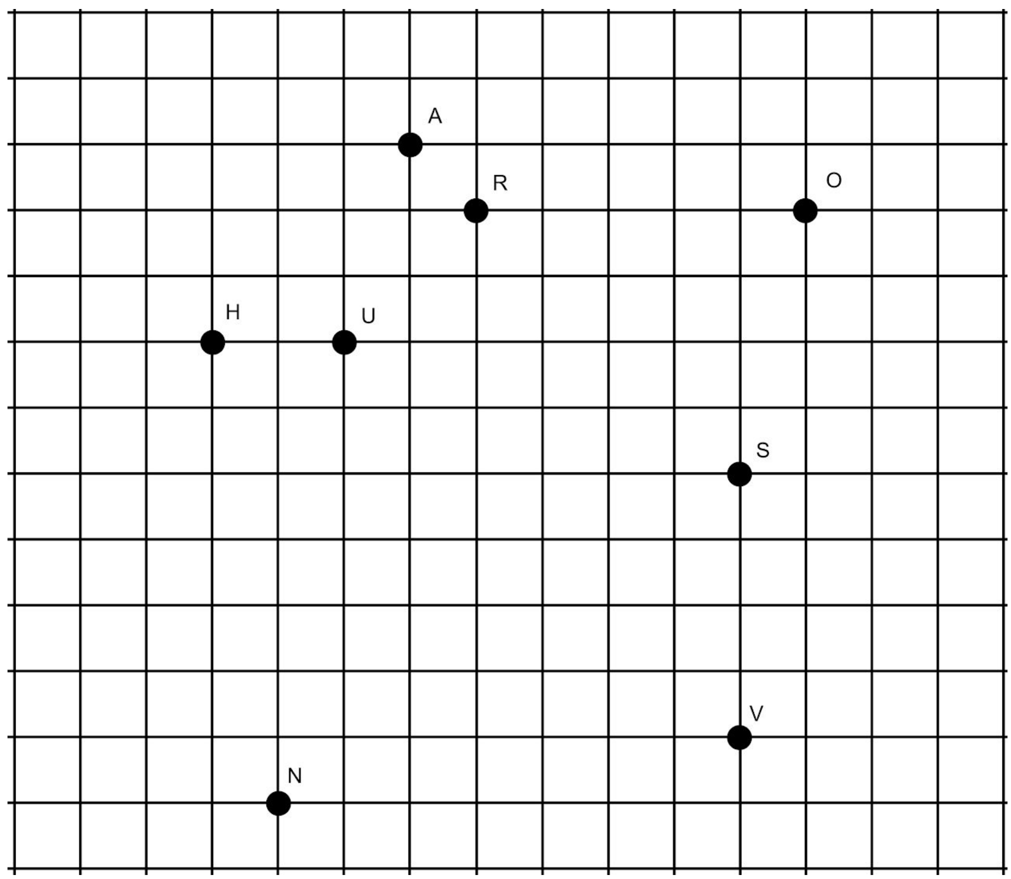
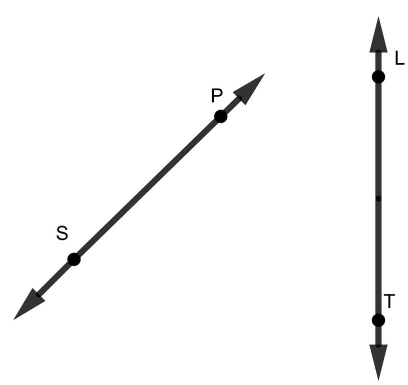
6. Use the points below to draw pairs of parallel line segments. How many parallel lines can you find?




7. Which of the following statements is true about $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$?




a. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are parallel because they do not intersect.

b. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are parallel because if you continue each line, they will intersect.

c. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are not parallel because if you continue each line, they will intersect.

d. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are not parallel because they do not intersect.



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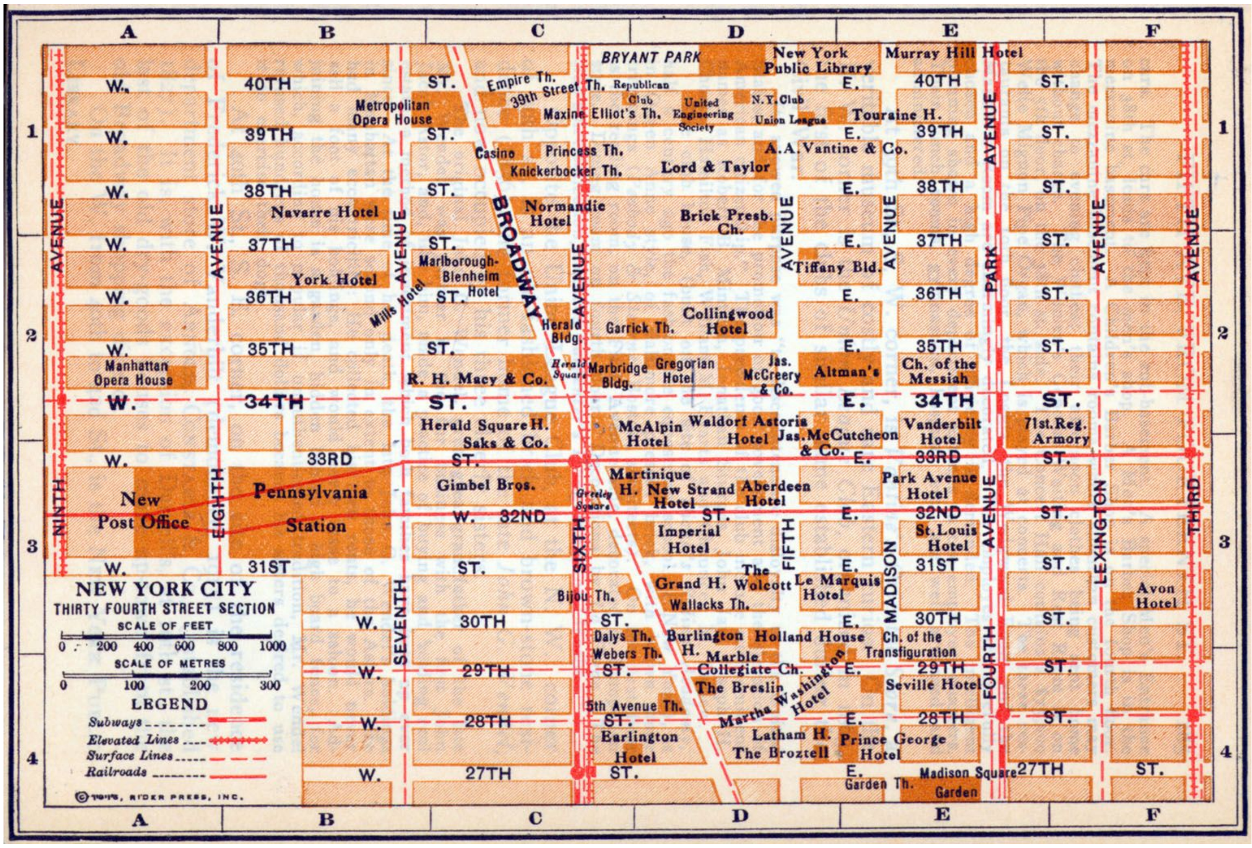
One place where we see parallel and intersecting lines are the streets and roads we use everyday. This map shows a section of Manhattan in New York City.




8. Use the names of the streets on the map to complete the following sentences.

   a. ________________________ is parallel to ________________________.

   b. ________________________ is parallel to ________________________.

   c. _______________ is parallel to _______________ and _______________.

   d. ___________________________ is not parallel to any other street on this map.



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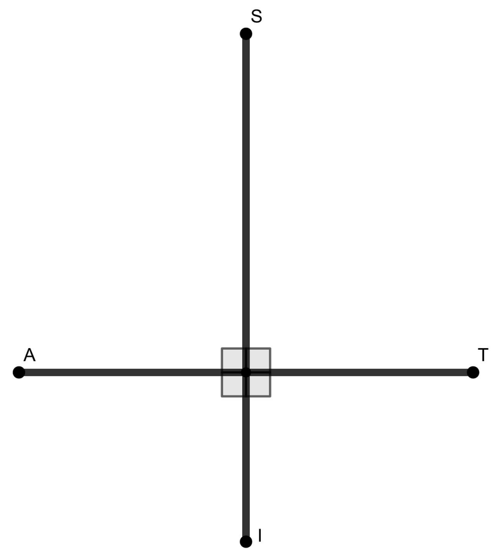
You may have noticed that many of the streets and avenues intersect in such a way that they form an L or a T.

When two lines intersect to form an L or a T, we call them **perpendicular lines.**

Perpendicular lines are lines that intersect to form right angles.

For example, $\overline{AT}$ and $\overline{SI}$ are perpendicular lines.

The mathematical symbol for perpendicular is $\perp$**.** Notation for showing that two lines are pendicular can look like this:




$$\overline{AT} \perp \overline{SI}.$$

This notation can be read as "*Line AT is perpendicular to Line SI*".

9. Use the names of the streets on the map of Manhattan to complete the following sentences.

   a. ______________________ is perpendicular to ________________________.

   b. ______________________ is perpendicular to ________________________.

   c. ______________________ is perpendicular to ________________________.



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There are examples of parallel lines and perpendicular lines all around us.

Tiles, brick walls, sports fields, windows, floors, trees, guitar strings, railroad tracks, telephone wires, tables—these are just a few examples.

10. Look around the room and find 5 examples of parallel lines and 5 examples of perpendicular lines.

| Examples of Parallel Lines | Examples of Perpendicular Lines |
|---|---|
| | |
| | |
| | |
| | |
| | |

11. Using a ruler or other straight edge (the side of a book will work), connect the dots to create the following line segments:



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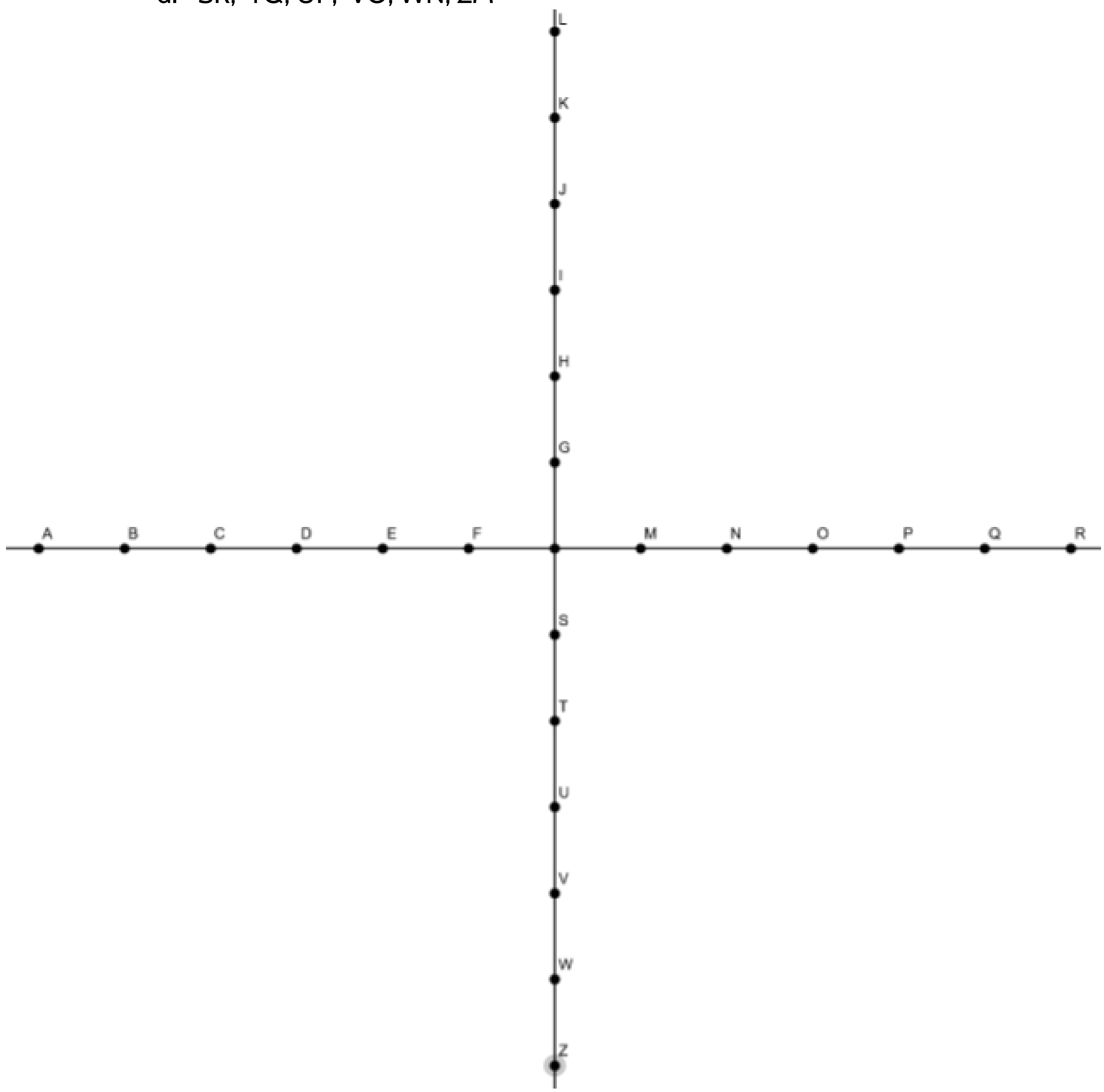
a. $\overline{AG}$, $\overline{BH}$, $\overline{CI}$, $\overline{DJ}$, $\overline{EK}$, $\overline{FL}$

b. $\overline{GR}$, $\overline{HQ}$, $\overline{IP}$, $\overline{JO}$, $\overline{KN}$, $\overline{LM}$

c. $\overline{FZ}$, $\overline{EW}$, $\overline{DV}$, $\overline{CU}$, $\overline{BT}$, $\overline{AS}$

d. $\overline{SR}$, $\overline{TQ}$, $\overline{UP}$, $\overline{VO}$, $\overline{WN}$, $\overline{ZM}$



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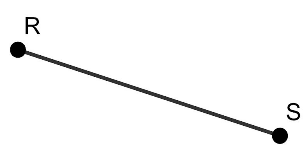
## Lines, Rays and Segments - Answer Key

1. Some things you might have noticed:
    a. Lines have arrows at both ends. Rays have an arrow at one end and a point at one end. Segments have a point at both ends.
    b. There are two ways to use notation for lines and segments, but only one way to use it for a ray.
    c. Lines keep going in both directions and rays keep going in only one direction.

2. Possible notations for each drawing:
    a. $\overline{MT}$ or $\overline{TM}$
    b. $\overleftrightarrow{EF}$ or $\overleftrightarrow{FE}$
    c. $\overrightarrow{GD}$
    d. $\overrightarrow{AC}$
    e. $\overrightarrow{AN}$ intersecting with $\overleftrightarrow{KL}$ (or $\overleftrightarrow{LK}$) at Point A.
    f. $\overline{AC}$ intersecting with $\overrightarrow{AJ}$ at Point A.

3. There are multiple possible ways to draw the given figures. Here are some examples:


a.




b.

c.

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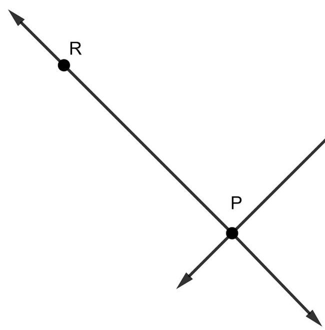
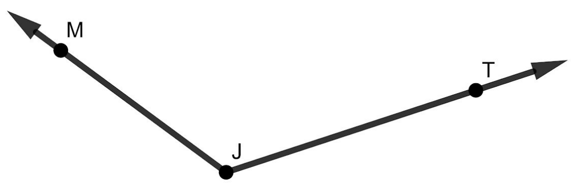



d.



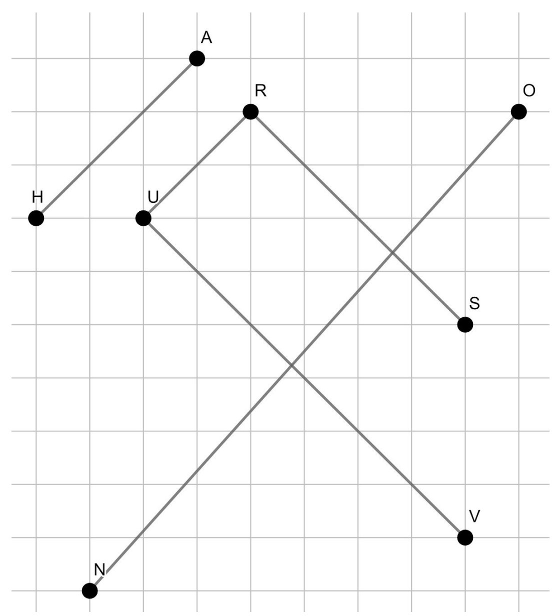

e.


f.

4. There are three segments: $\overline{HE}$, $\overline{EK}$, and $\overline{HK}$. ($\overline{EH}$, $\overline{KE}$, and $\overline{KH}$ are also possible names for each of the three segments.)

5. There are four rays: $\overrightarrow{EK}$, $\overrightarrow{EH}$, $\overrightarrow{KH}$ and $\overrightarrow{HK}$. What is important is that the endpoints (point E, point K, and point H) are the first letter in each name.

6. There are several examples of parallel line segments. You can see them in the diagram on the right. $\overline{AH}$, $\overline{UR}$ *and* $\overline{NO}$ are all parallel to each other. $\overline{UV}$ is parallel to $\overline{RS}$.



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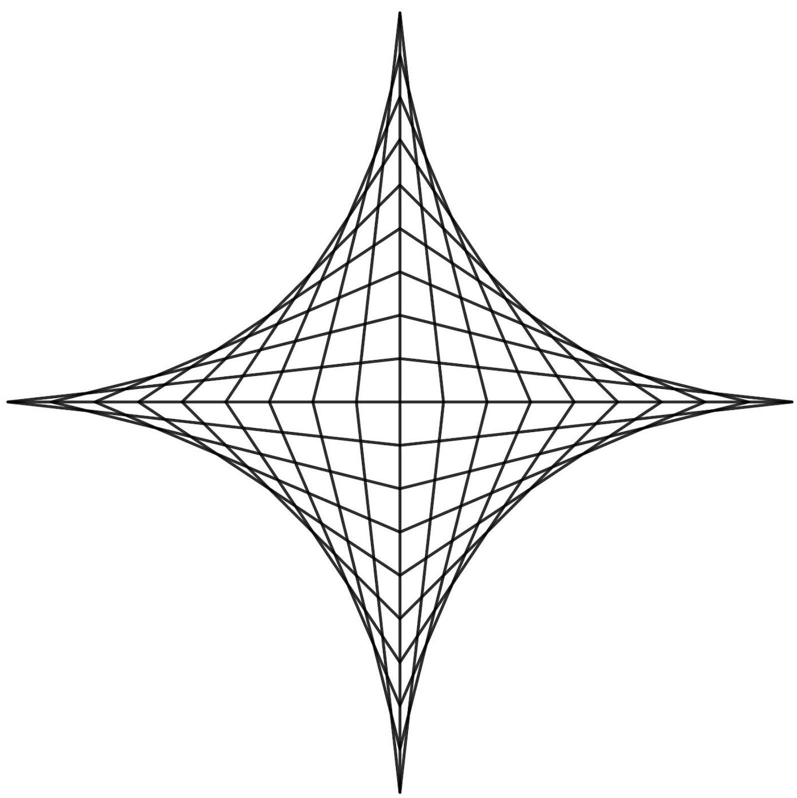
7. Choice C. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are not parallel because if you continue each line, they will intersect.

8. There are many possible correct answers. In general, Manhattan streets are parallel to other streets and avenues are parallel to other avenues. Broadway cuts across Manhattan, intersecting many streets.

9. There are many possible correct answers. In general on this map avenues are perpendicular to streets. So, for example, Sixth Avenue is perpendicular to W. 33rd Street.

10. There are countless examples of parallel and perpendicular lines around us. Be creative!

11. You should get a figure that looks something like the figure below. Notice that even though you drew straight lines, the intersections make them appear to curve.





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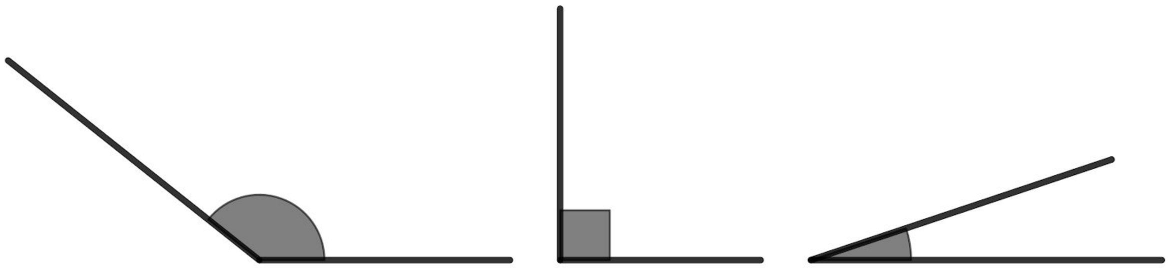
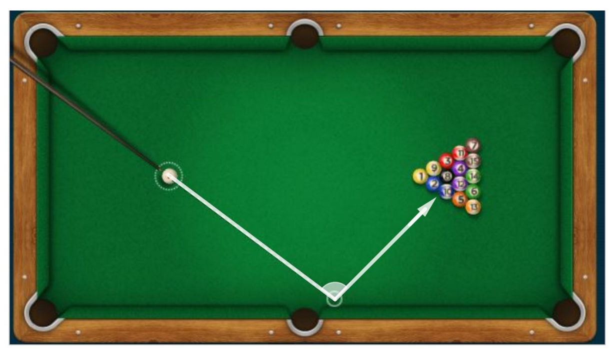
# Angles

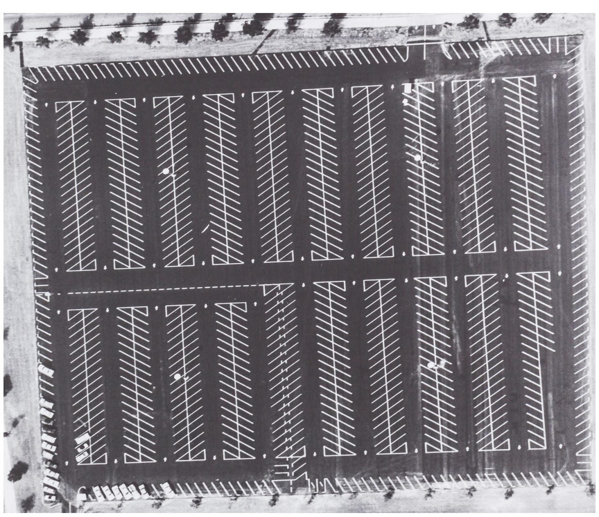
Along with points, lines, segments, and rays, another figure in geometry is the **angle**. Angles also help us describe the shapes in the world.

An angle is an "opening" formed when two segments, lines, or rays intersect.



Here are just a few examples of places you can find angles.







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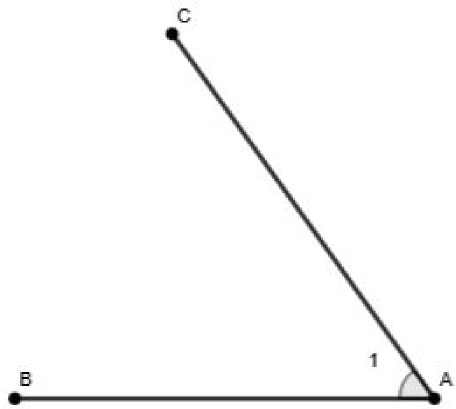
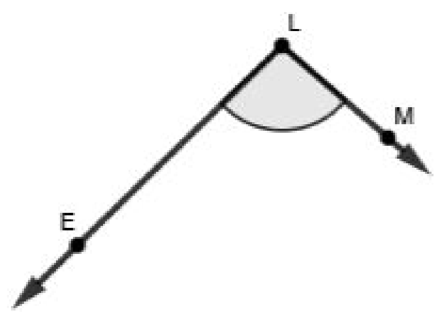
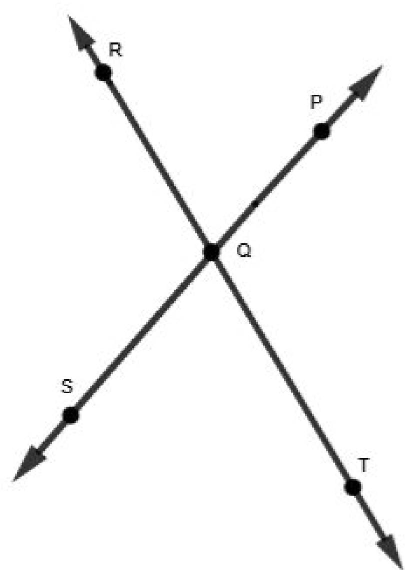
# Naming Angles

| | |
|---|---|
| In this diagram, we see an angle formed by the intersection of two line segment.<br><br>$\overline{AB}$ and $\overline{CA}$ intersect at point A. |  |
| In this diagram, we see an angle formed by the intersection of two rays.<br><br>$\overrightarrow{LE}$ and $\overrightarrow{LM}$ intersect at point L |  |
| In this diagram, we see several angles formed by the intersection of two lines.<br><br>$\overleftrightarrow{SP}$ and $\overleftrightarrow{RT}$ intersect at Point Q. |  |



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There is a special name for the point where the segments, rays, or lines intersect to form an angle. We call this point the **vertex** of the angle. The vertex is important because when we want to name any angle, we need to identify the vertex first.

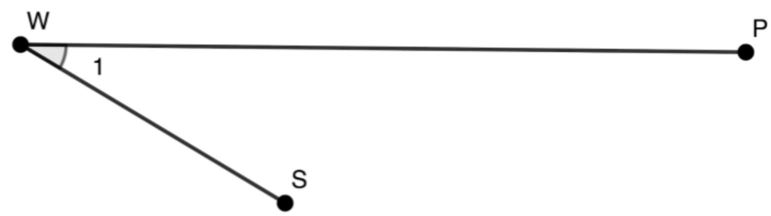
The angle to the right is formed by the intersection of two rays at point A. Point A is the vertex.


We call this angle $\angle CAT$ or $\angle TAC$. Notice that the vertex, point A, is in the middle of the notation. You can start with either of the outside endpoints, but the vertex must always appear in the middle when we name an angle.

We can also refer to this angle as $\angle A$.

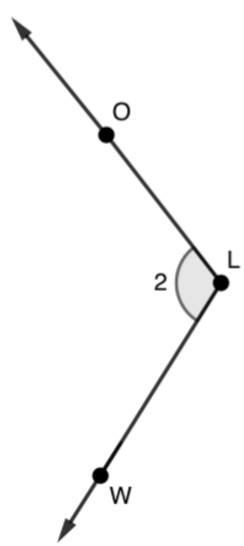
Name the numbered angles using letters.

1.




$\angle 1 =$

2.




$\angle 2 =$



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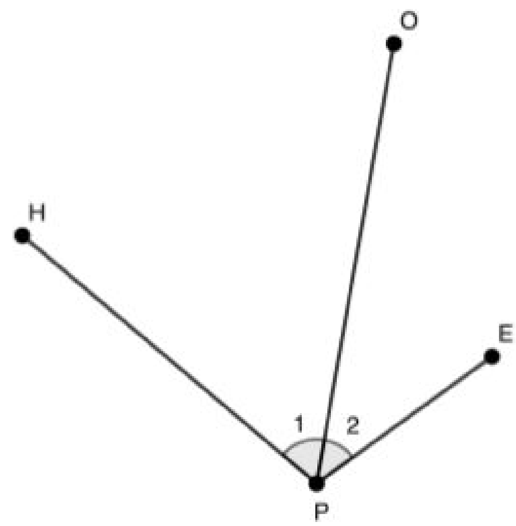
3.



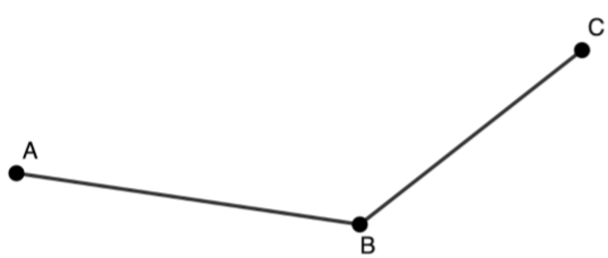

$\angle 1 =$

$\angle 2 =$

In the example above, $\angle 1$ is formed by line segments HP and PO. $\angle 2$ is formed by line segments OP and PE. But there is a third angle in the diagram. Can you see it?

It is the angle formed by line segments HP and PE, or $\angle$HPE.

4. Here is $\angle$ABC.

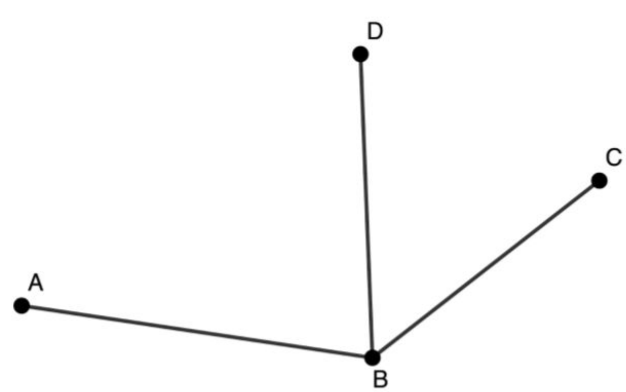



If we add line segment DB, it forms three angles. Try to name them all.

a. $\angle$

b. $\angle$

c. $\angle$



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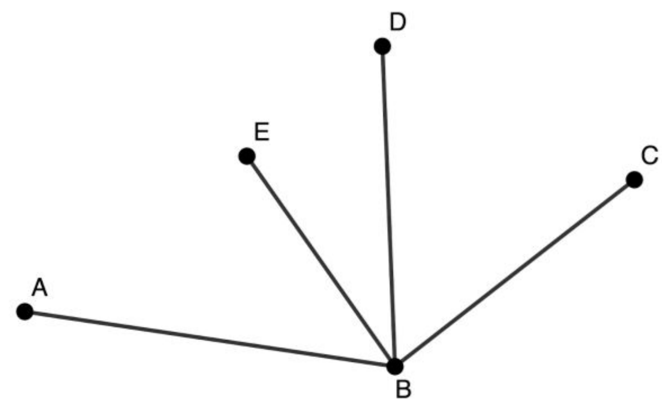
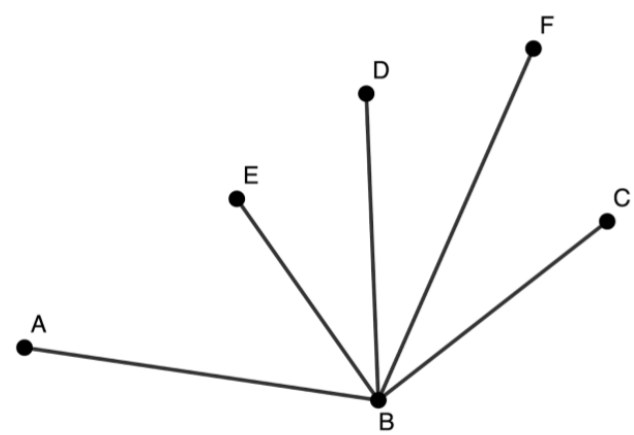
5. If we add another line segment, it forms six angles. Can you name all six?
    a. ∠
    b. ∠
    c. ∠
    d. ∠
    e. ∠
    f. ∠




6. If we add another line segment, how many angles are formed? Write the names of any angles you see.



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7. The chart below shows the numbers of angles formed by different numbers of line segments. What patterns do you notice?

   Can you predict the number of angles made by adding additional line segments?

| Number of Line Segments | Number of Angles |
|---|---|
| 2 | 1 |
| 3 | 3 |
| 4 | 6 |
| 5 | 10 |
| 6 | |
| 7 | |
| 8 | |

   You may want to look back at the intersecting line segments on the previous pages.



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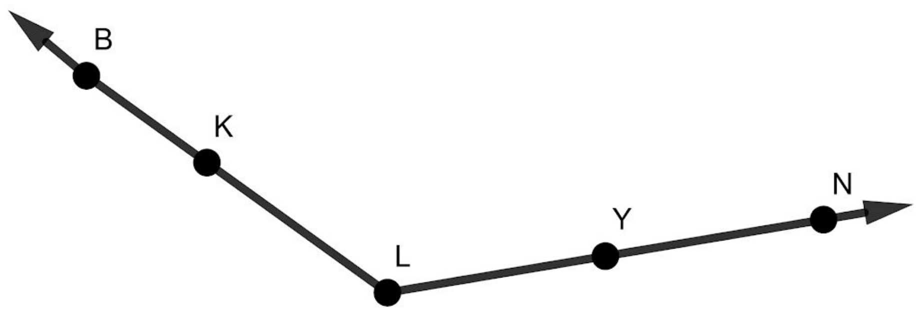
8. Which of the following notations for the angle to the right is <u>not</u> correct?

    a. $\angle$BLY

    b. $\angle$YLK

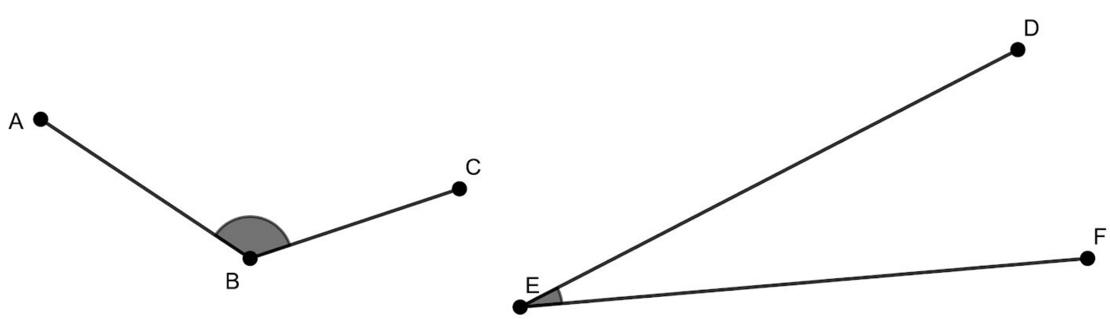
    c. $\angle$BLN

    d. $\angle$BKN




Explain why the notation is incorrect.

9. Compare $\angle$ABC and $\angle$DEF.




Which is the larger angle? Explain how you know.



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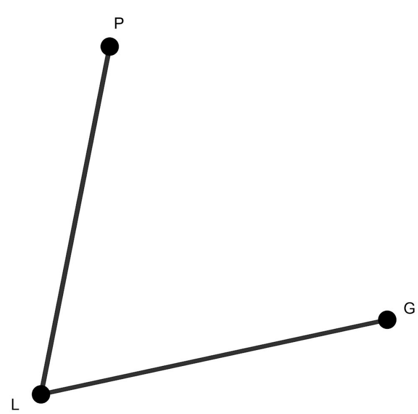
## A Brief History of Angles and Degrees

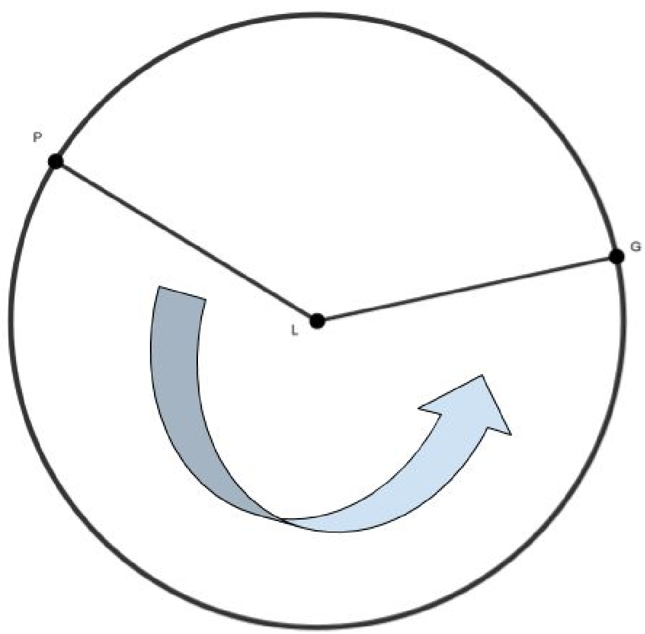
Most people agree that the Babylonians were the first people to measure angles using **degrees**. Babylon was a key city in ancient Mesopotamia. The city was built between the Tigris and Euphrates rivers, just south of Baghdad in what is modern-day Iraq. The Babylonian system for measuring angles is one of the oldest forms of measurement still in use today. Using Fahrenheit to measure temperature is about 300 years old. Measuring temperature with Celsius is about 275 years old. The metric system (millimeters, centimeters, meters, etc) is about 225 years old. The Babylonian system of measuring angles was developed more than 3,500 years ago!


The Babylonians used a circle to describe the entire range of possible angles.

Here is ∠PLG.




Imagine if we made the opening bigger by rotating $\overline{PL}$ counterclockwise.




Eventually, $\overline{PL}$ would come all the way around to meet $\overline{LG}$.



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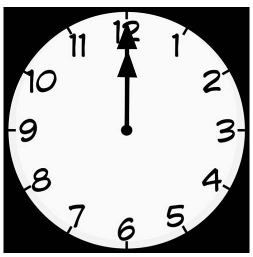
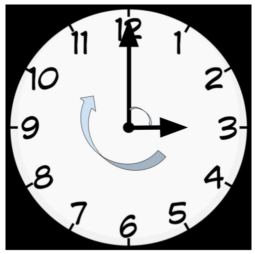
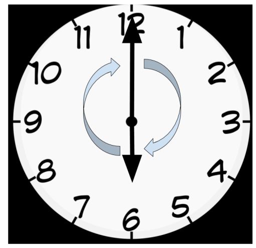
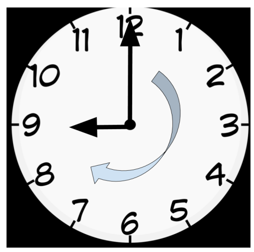
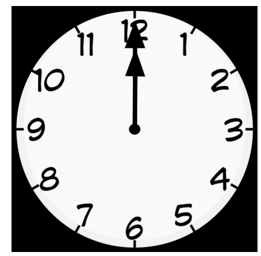
Let’s think about the hands of a clock.

| | | |
|---|---|---|
| The clock below reads twelve o’clock. Let’s imagine it is noon. We can think of the center of the clock as a vertex of the angle formed by the two arms of the clock.  | As the hour passes, the opening between the two arms gets larger.    | Once the hour hand returns to the 12 at midnight, it has moved all the way around the vertex.  |

The ancient Babylonians needed to decide what number of degrees they would use to measure the whole circle. They decided to use 360 degrees (also written as 360°) as the measure of an angle that goes all the way around a vertex.

NOTE: This symbol means “degrees”: °

It is used for degrees in an angle and degrees in temperature. For example, you can write 45° to mean a 45 degree angle and you can also write 45° to refer to temperature (a chilly day in Fahrenheit and a really hot day in Celsius).

In this packet, we will be using the degree symbol to refer to the measurement of angles.



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You might be asking yourself, "Why did the Babylonians choose 360? Why is that the number of degrees in a full turn around a vertex? What is so special about that number?"

Well, for starters, it is very close to the number of days in a year. One year is defined as the amount of time it takes the Earth to complete one full rotation around the sun. It actually takes the Earth 365 days and 6 hours to complete its orbit around the sun, but we usually think of a year as 365 days.

*Babylonian scholars of 4000 years ago knew that there were 365¼* days in a year. *So why didn't they decide that there should be 365 ¼ degrees in a circle?*

The answer to this question is very simple and very human: Who wants to do mathematics with the number *365 ¼*? It's an awkward number! The natural thing to do is to round it to a friendlier value.

*If we round the number 365 ¼ to the nearest five or the nearest ten we get 365 and 370, not the number 360. Why did we humans decide to round all the way* down *to 360?*

The answer here is also very simple and very human. Thousands of years ago there were no calculators and all arithmetic had to be done by hand or in one's head. It is natural to want to work with numbers that are easier to calculate in your head. They also preferred working with numbers that could be divided with no remainders.

In life and in mathematics we often want to divide numbers by two and choosing 365 as the count of degrees in a circle is unfriendly. 365 divided by 2 is 182.5—no thanks!

370 and 360 are even at the least.

We also often want to divide measurements by three as well. In that case, 360 is much better than 370. In fact, 360 is a much friendlier number for arithmetic over 370: you can divide it evenly into two equal pieces, three equal pieces, four, five, six, eight, nine, ten, twelve, fifteen, and many more. In fact there are 24 different ways to divide 360 into equal pieces.

So, it was for two very human reasons—what we experience on this planet and our desire to avoid awkward work—that the Babylonians settled on the number 360 for the count of degrees in a circle.[3]

[3] Adapted from "Two Key - but ignored - Steps to Solving Any Math Problem" by James Tanton



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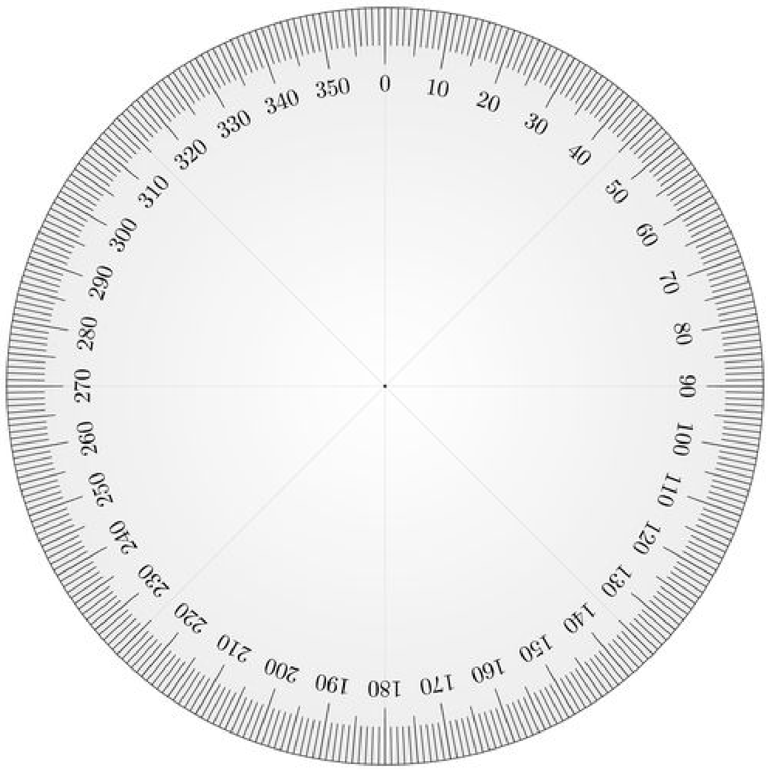
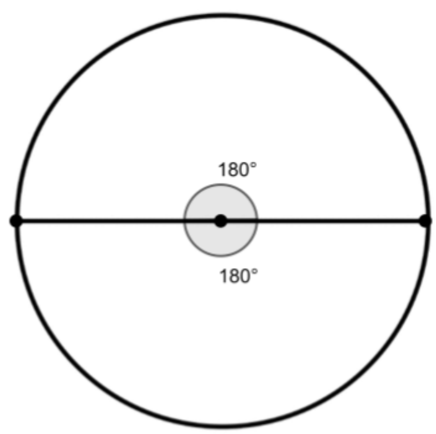
## Dividing 360°

In the last section, you read that one of the reasons the Babylonians choose 360 as the number of degrees in a circle was because there are so many ways to divide 360 into equal groups. For this next activity, you'll identify some of those equal groups.

Remember a complete circle is 360°.




On the next few pages, there are circles. Each circle is divided into a different number of equal angles.




For example, this circle has two equal angles. If we divide 360° into two equal angles, each angle is 180°. Another way to think about this is that an angle of 180° is half of the circle..

What if we divide the circle into three equal angles?



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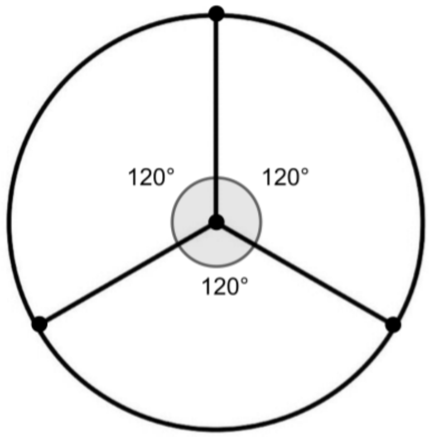
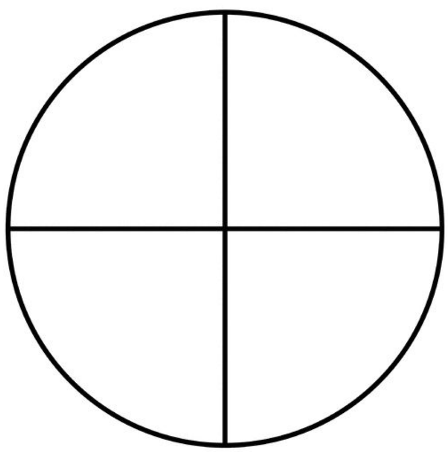
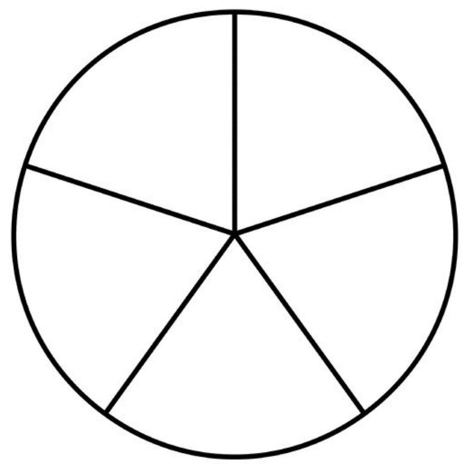
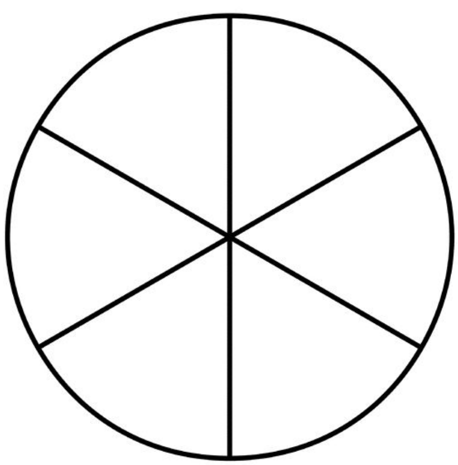


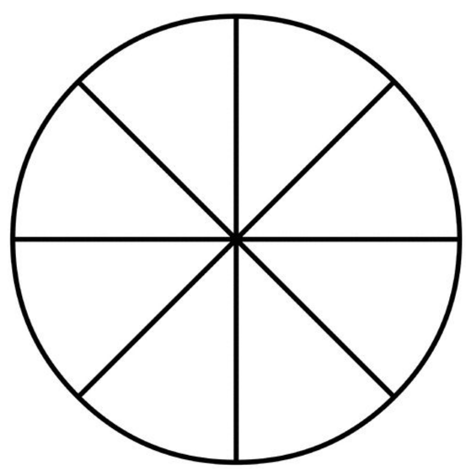

120 + 120 + 120 = 360

10. Find the value of and label each angle. Remember that each circle has to add up to a total of 360°.


a.


b.


c.


d.



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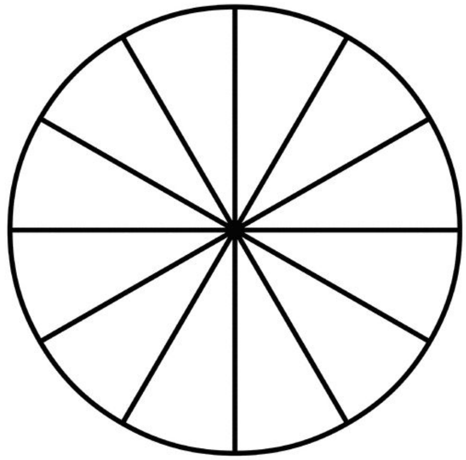
e.



Note: These aren't all the ways to divide up 360° into equal groups, but these are some common angles you might see.



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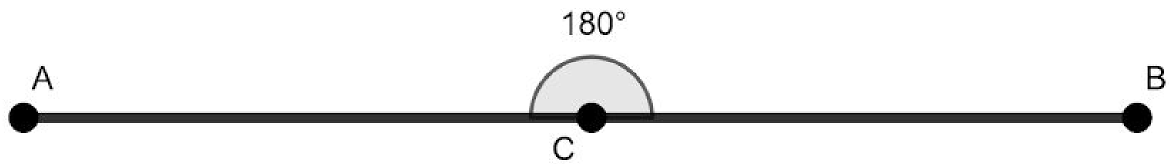
## Measuring Angles

There are tools that we can use to measure angles, but there is a lot we can tell about them just by looking.

One angle that is easy to see is 180°. As you saw in the Dividing 360° activity, a 180° angle is an opening that is half of a full circle, which is 360°. We can also see that an angle that is 180° is a straight line. For this reason,a 180° angle is often referred to as a **straight angle.**




Look at this window. Do you see a straight angle? Do you see any other angles?

The bottom of the window is a straight line, so we can say it is a straight angle, and measures 180°. You might also notice that the window is divided up into 4 equal angles. If the whole opening is 180°, how much do you think each of those 4 angles would measure?

Take a minute to think about it before turning the page.



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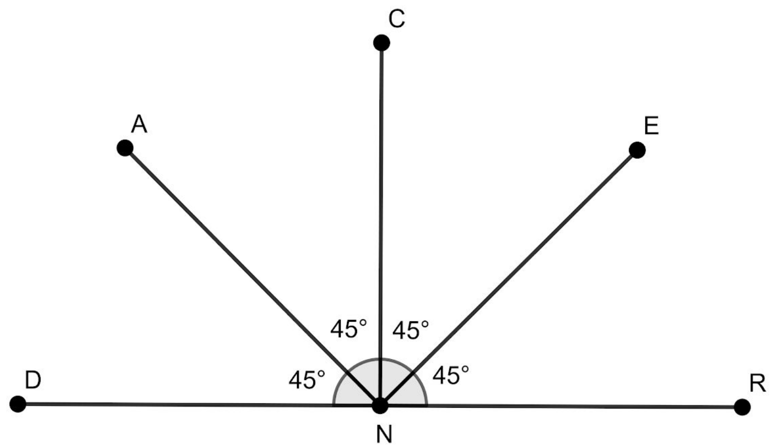
Even without any numbers being given, we can figure out all of the angle measurements. We know that the base of the window is a straight angle measuring 180°. If we divide 180° into 4 equal angles, we end up with four angles that measure 45° each.




Straight Angle DNR can be divided into four 45° angles. 45° + 45° + 45° + 45° = 180°

11. There are three 90° angles in the diagram above. Try to name as many of them as you can below.

12. There are two 135° angles in the diagram as well. Name them.



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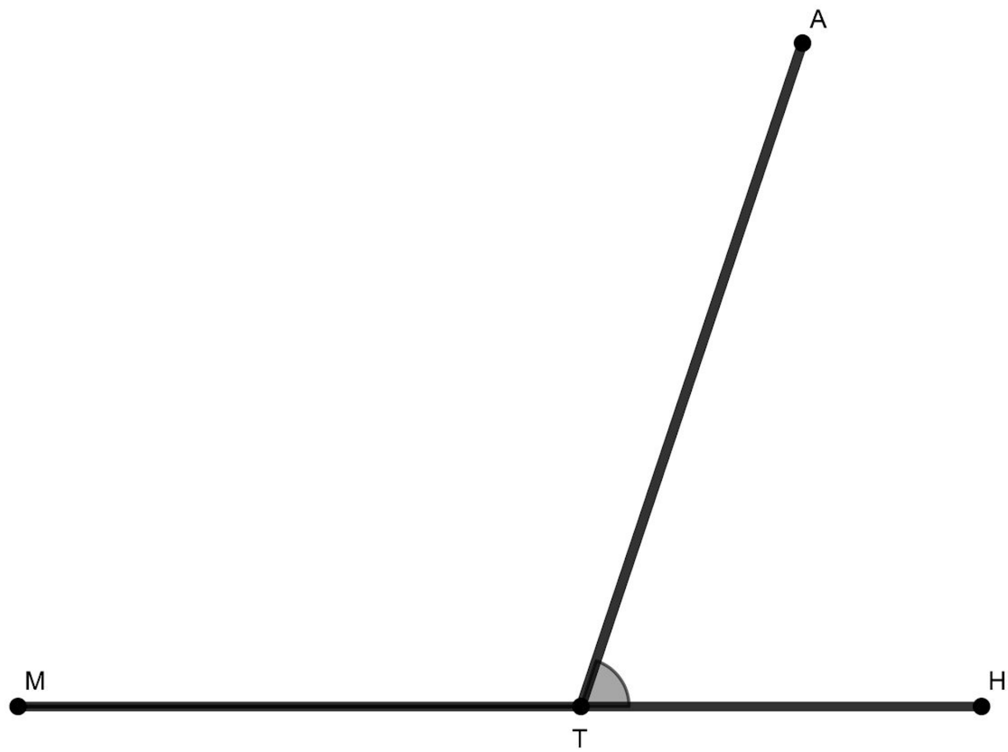
In this diagram, straight angle MTH is divided into two angles by $\overline{AT}$.




The straight angle is divided into ∠MTA and ∠ATH.

Even without numbers, there is a relationship between those two angles. Whenever a straight angle is divided into smaller angles, we know all of those angles need to add up to 180°.

When a straight angle is divided into two angles, there is a special name for those two angles. We say they are supplementary angles.

**Supplementary angles** are any pair of angles that add up to 180°.

In the diagram above ∠MTA and ∠ATH are supplementary angles.



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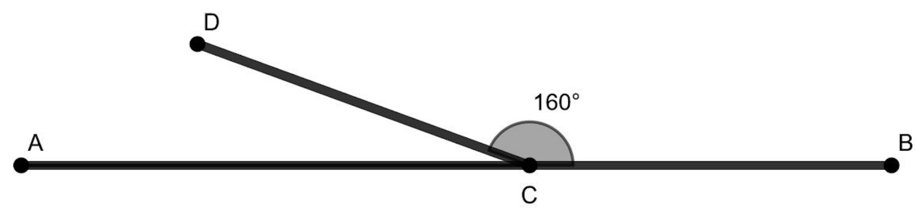
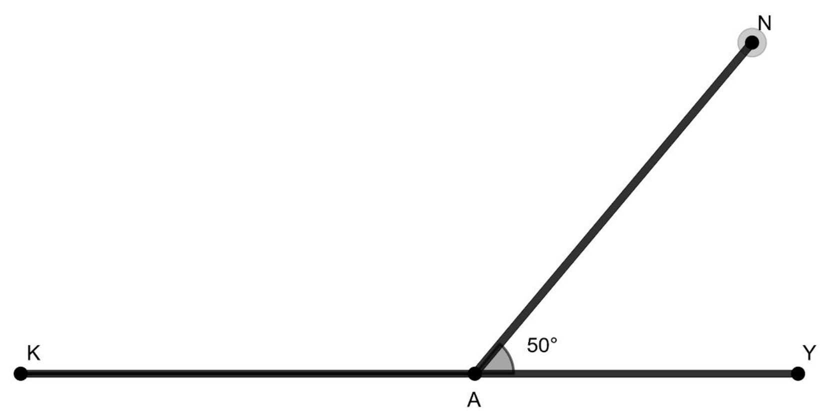
13. Use what you know about supplementary angles to find the measure of the missing angles in each of the examples below:

    a.



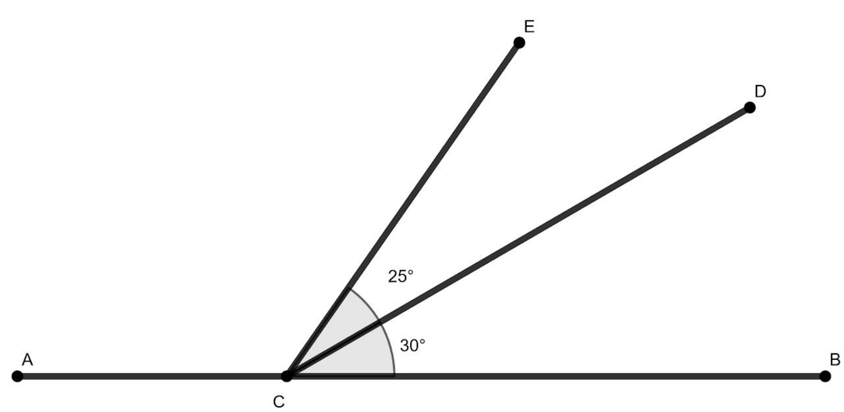

    b.




14. Which of the following pairs of numbers could **not** represent supplementary angles?

    a. 155° and 25°
    b. 90° and 90°
    c. 178° and 12°
    d. 70° and 110°

15. ∠ACE measures __



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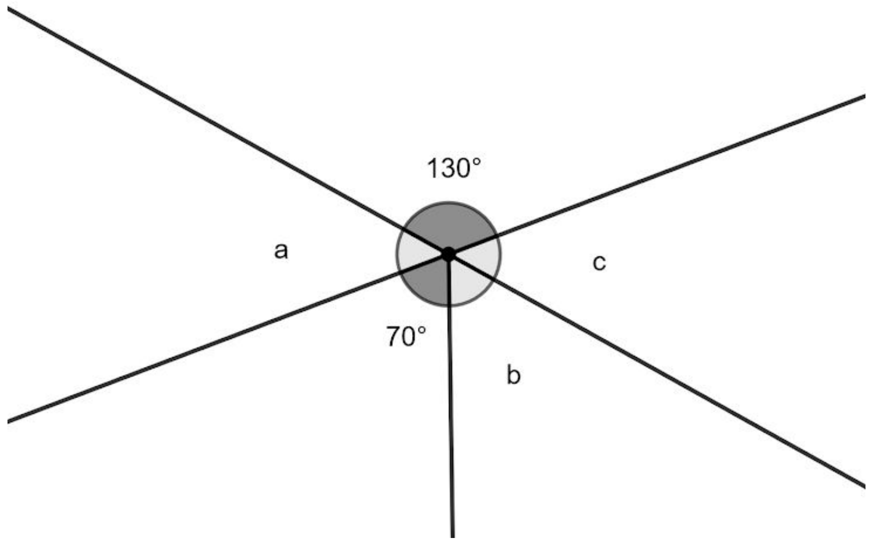
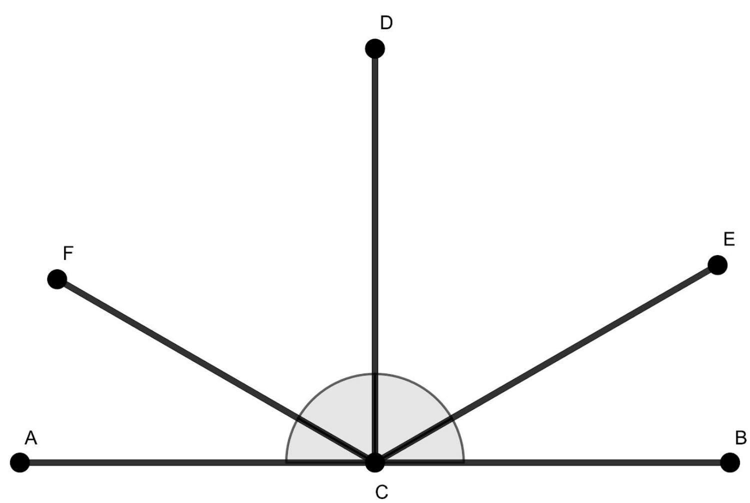
16. Find the measures of ∠a, ∠b, and ∠c.




17. ∠DCF is twice the size of ∠FCA.
∠DCE is twice the size of ∠ECB.
∠DCA is equal to ∠DCB.

What is the measure of ∠FCE?



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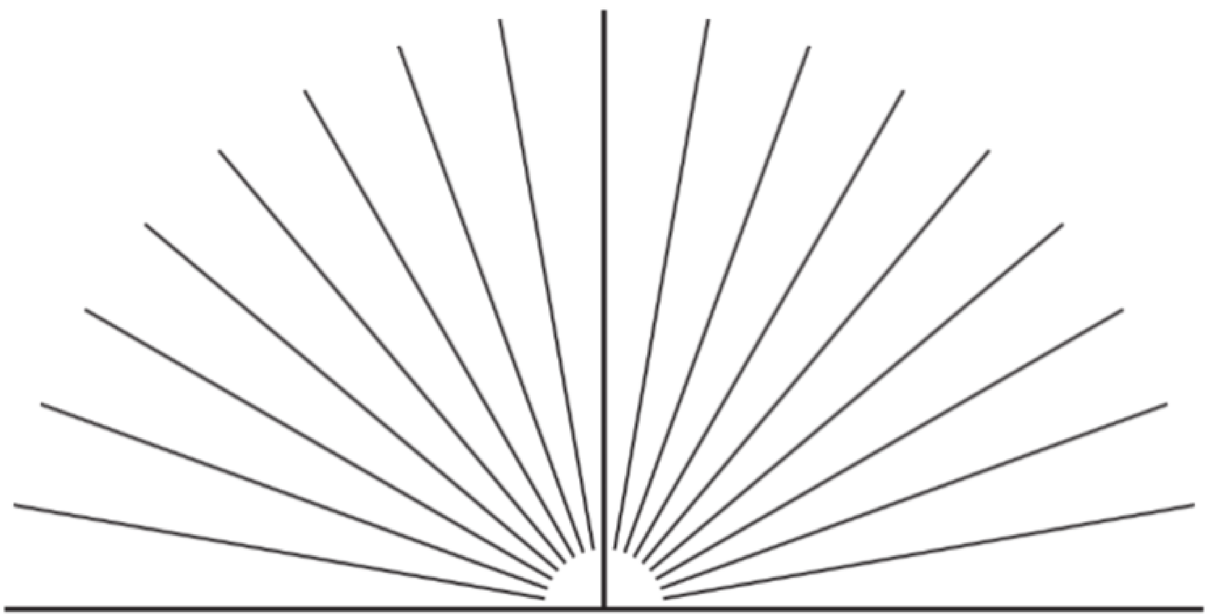
**Numberless Protractor**

There are different tools to measure angles depending on how precise or exact you need your measurements to be. One commonly used tool for measuring angles is called a protractor.

Many protractors have numbers, but this is a numberless protractor.



Write three things you notice about this numberless protractor.

| |
|---|
| |
| |



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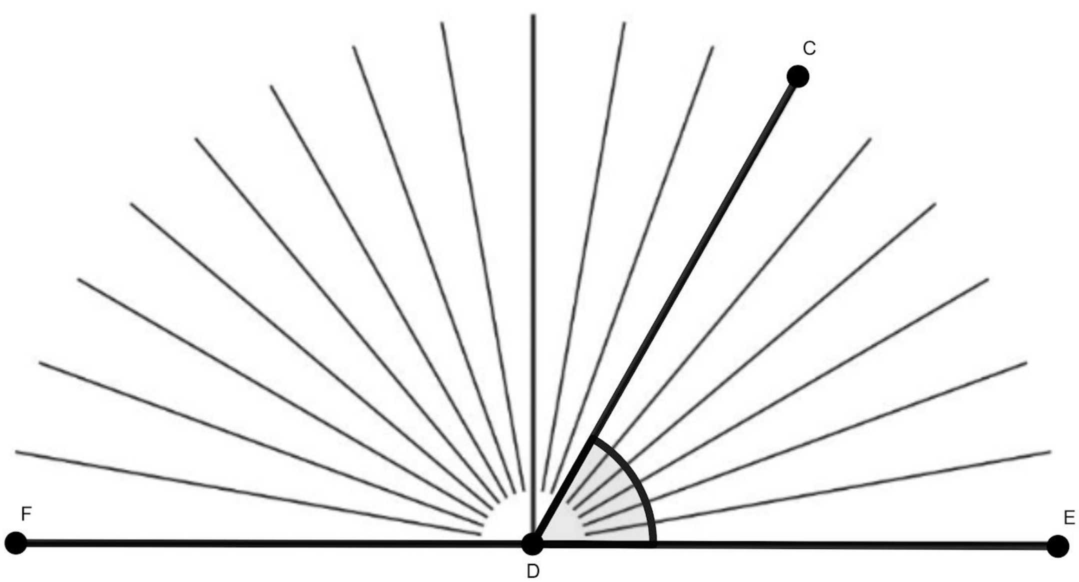
One thing you may have noticed is that there is a straight angle formed across the bottom of the numberless protractor. You may also have noticed that the straight angle is divided up into 18 equal angles.

18. How much does each of those smaller angles measure? How do you know?

We can use a numberless protractor the same way we were able to look at the window on page 46 and figure out the measurements of the smaller angles.

19. Use what you know about the numberless protractor to determine the measurements of ∠CDE and ∠FDC.

    ∠CDE is
    ∠FDC is



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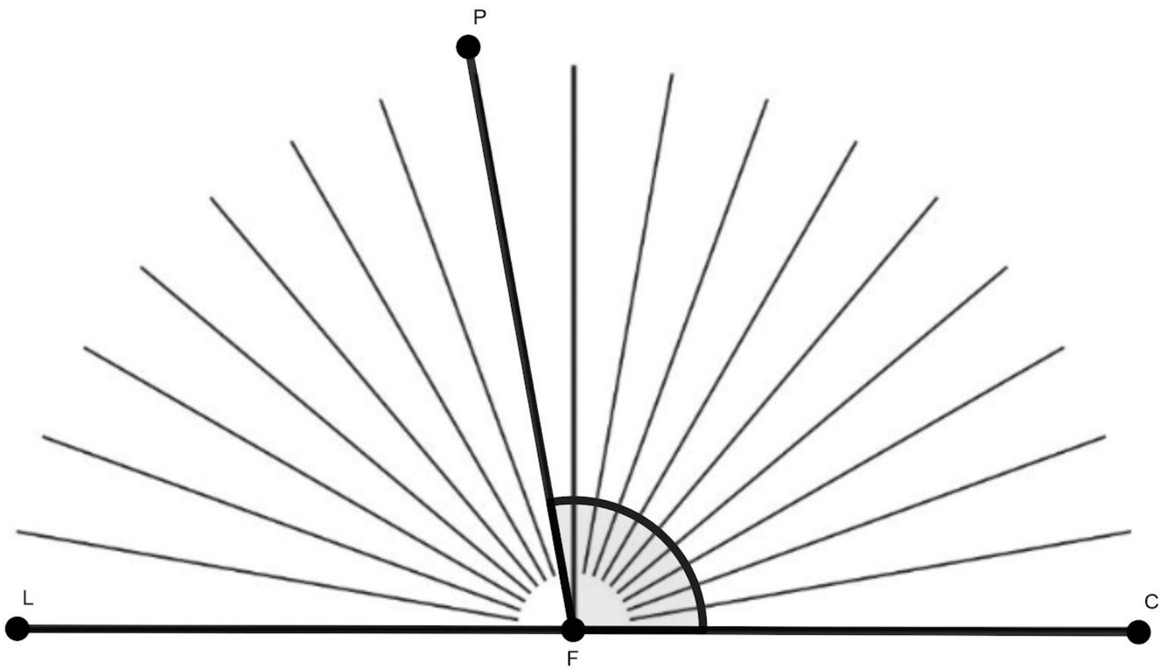
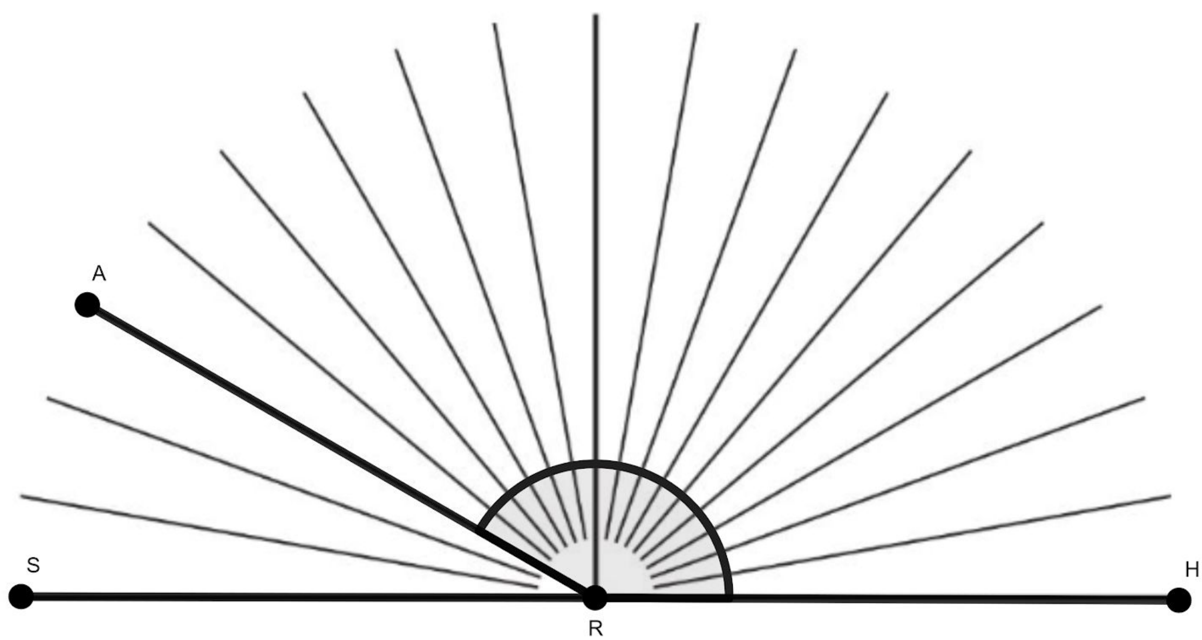
20. What is the measure of ∠LFP?
What is the measure of ∠PFC?




21. What is the measure of ∠SRA?
What is the measure of ∠HRA?



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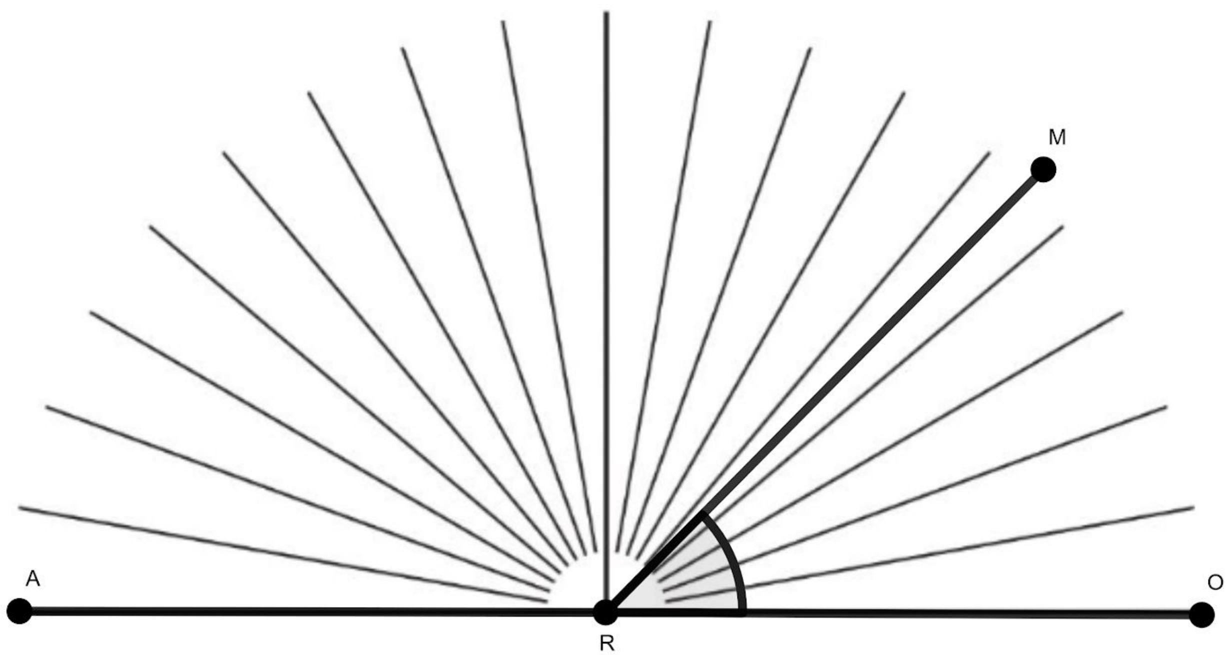
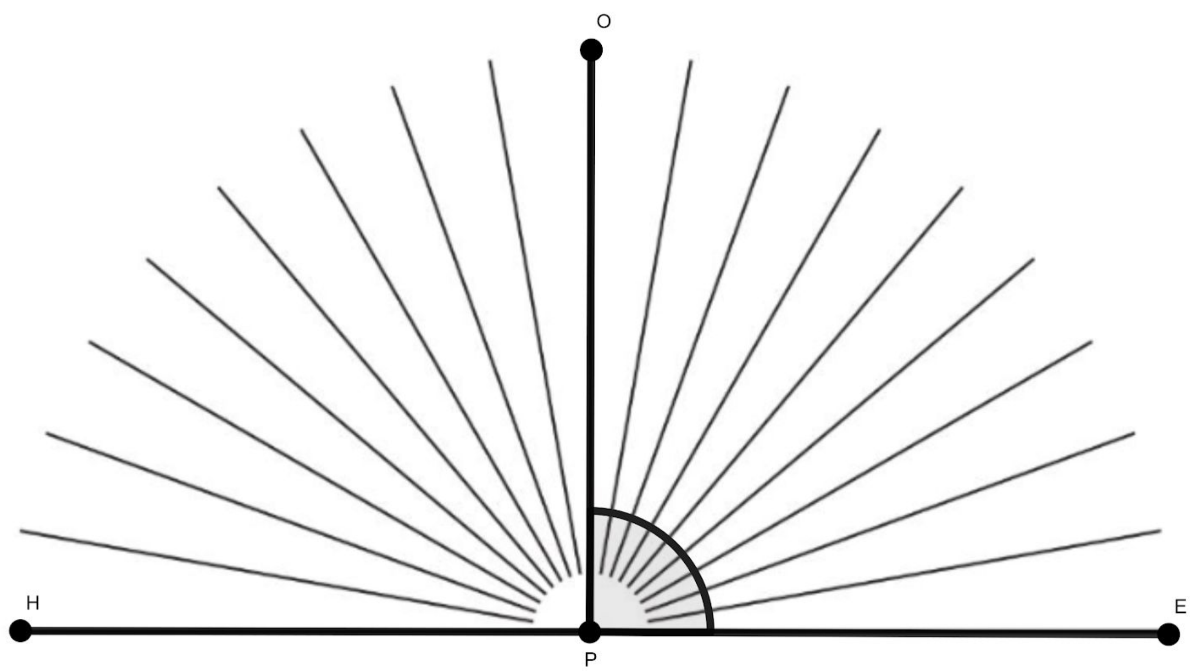
Here's one that is a little different from the others we've seen so far.

22. What is the measure of ∠MRO?
    What is the measure of ∠ARM?




23. What is the measure of ∠HPO and ∠OPE?



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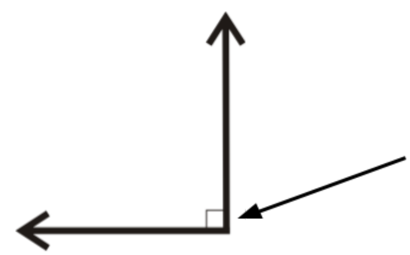
In the last question you saw straight angle HPE divided in half by $\overline{OP}$, forming two 90° angles.

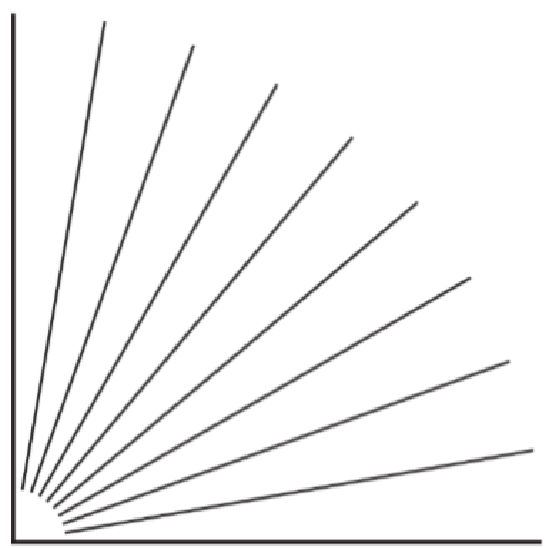
Up until now, our focus has been on 180° angles. We will now spend some time learning about 90° angles.

A 90° angle is called a **right angle**. Right angles are important, especially in construction. Right angles allow us to build perfect squares. You might see an angle with a small square in the angle. That means it is a right angle and measures 90°.



Remember, any lines that intersect to form right angles are called perpendicular lines.

Here is a numberless protractor showing a right angle.



24. If you divide a right angle into <u>three</u> equal angles, what is the measure of each angle?

25. If you divide a right angle into <u>two</u> equal angles, what is the measure of each angle?



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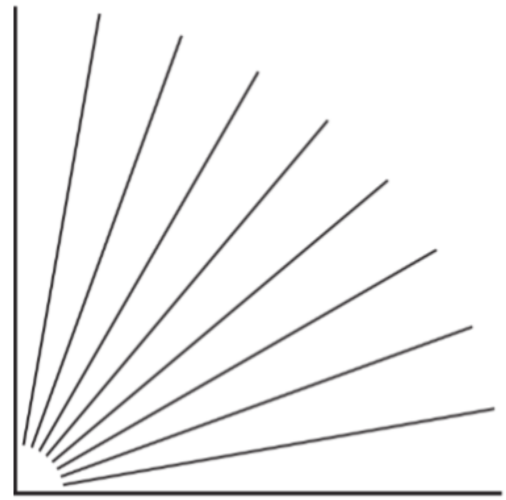
26. Use the numberless protractor on the right to draw a 60° angle.



27. What is the measure of the angle remaining after you drew your 60° angle?

When a right angle is divided into two angles, there is a relationship between those two angles. The two angles together need to add up to 90°. When a right angle is divided into two angles, there is a special name for those two angles. We say they are complementary angles.

**Complementary angles** are any pair of angles that add up to 90°.

The two angles you created above are complementary angles.

28. Which of the following pairs of numbers are complementary angles?

    a. 70° and 30°
    b. 65° and 25°
    c. 120° and 60°
    d. 110° and 70°

29. Which of the following pairs of numbers could **not** represent complementary angles?

    a. 1° and 89°
    b. 14° and 76°
    c. 28° and 62°
    d. 90° and 1°



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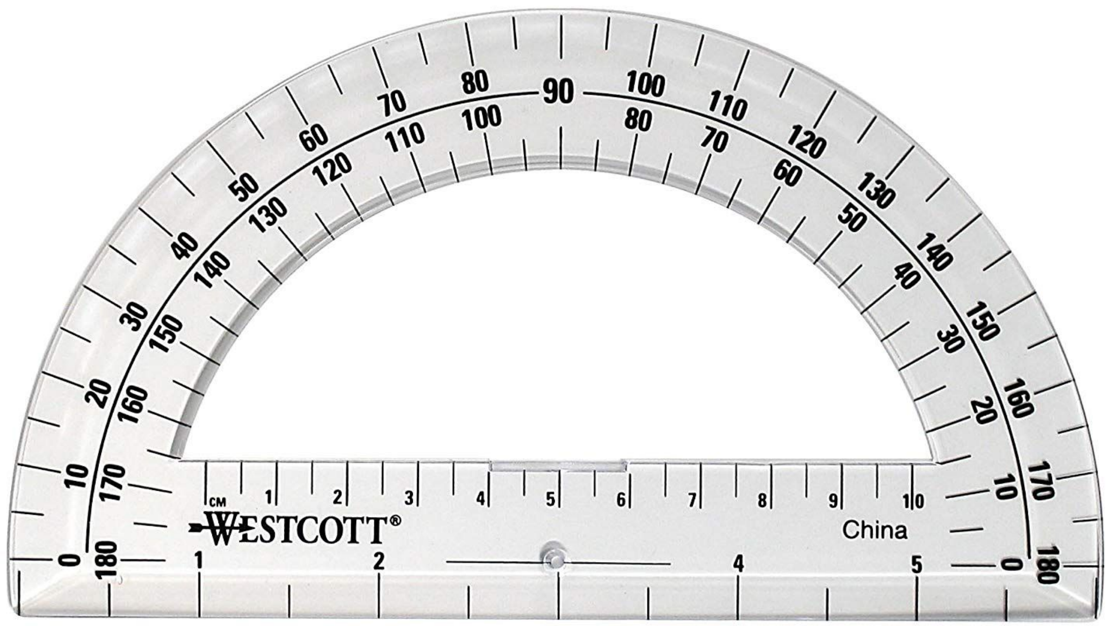
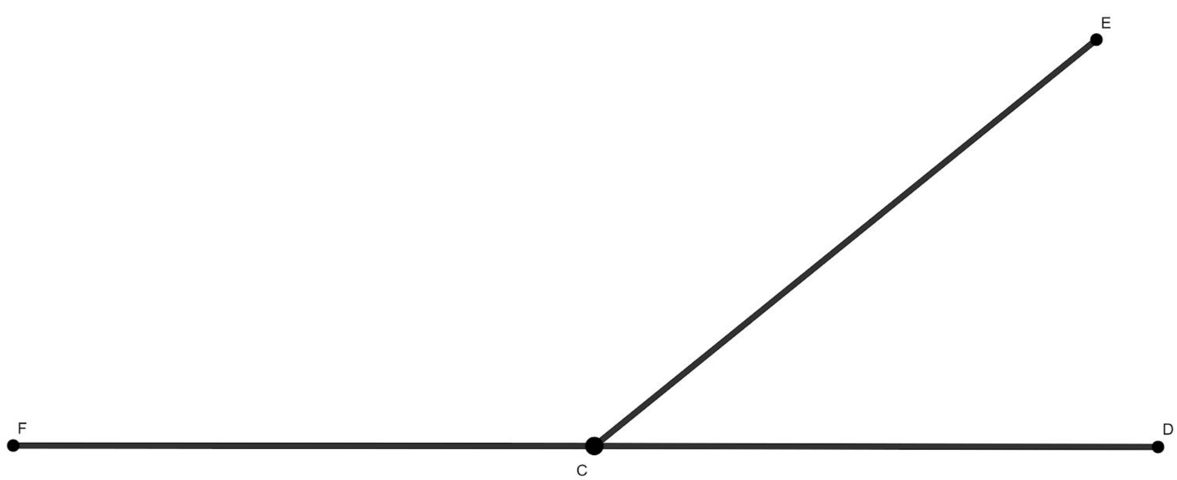
**Standard Protractor**

Here is a protractor with numbers.




Here are two supplementary angles.



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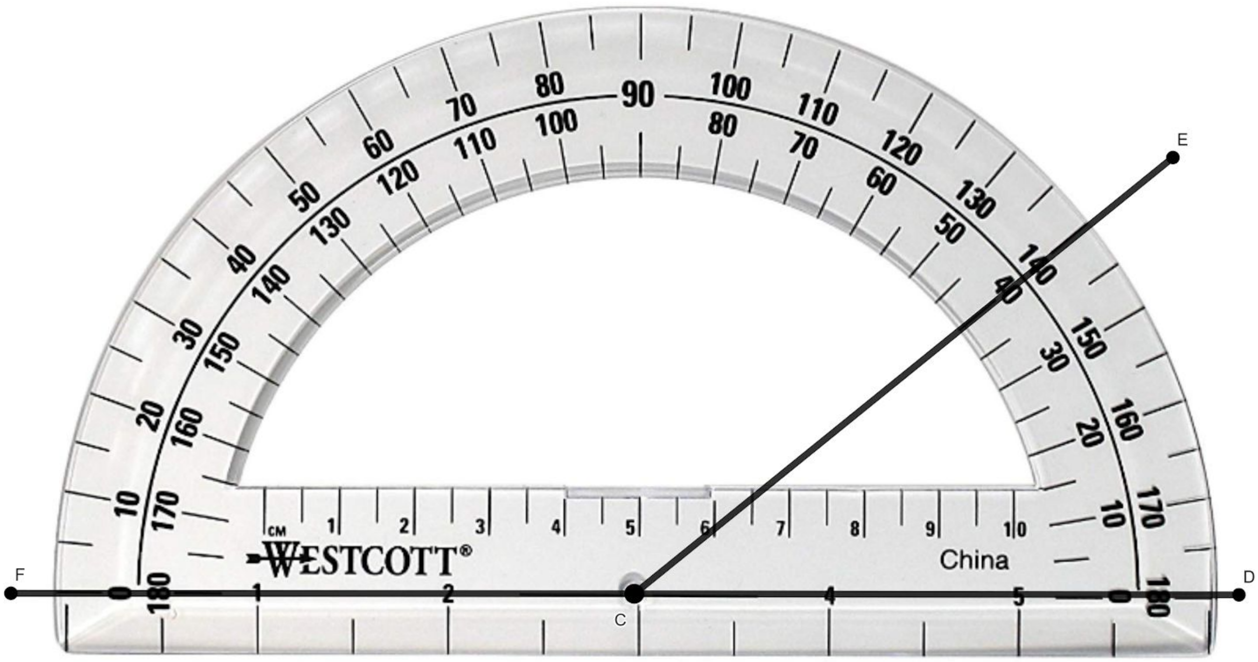
Here are the angles and the protractor together.




Write 4 things you notice.

| |
|---|
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| |



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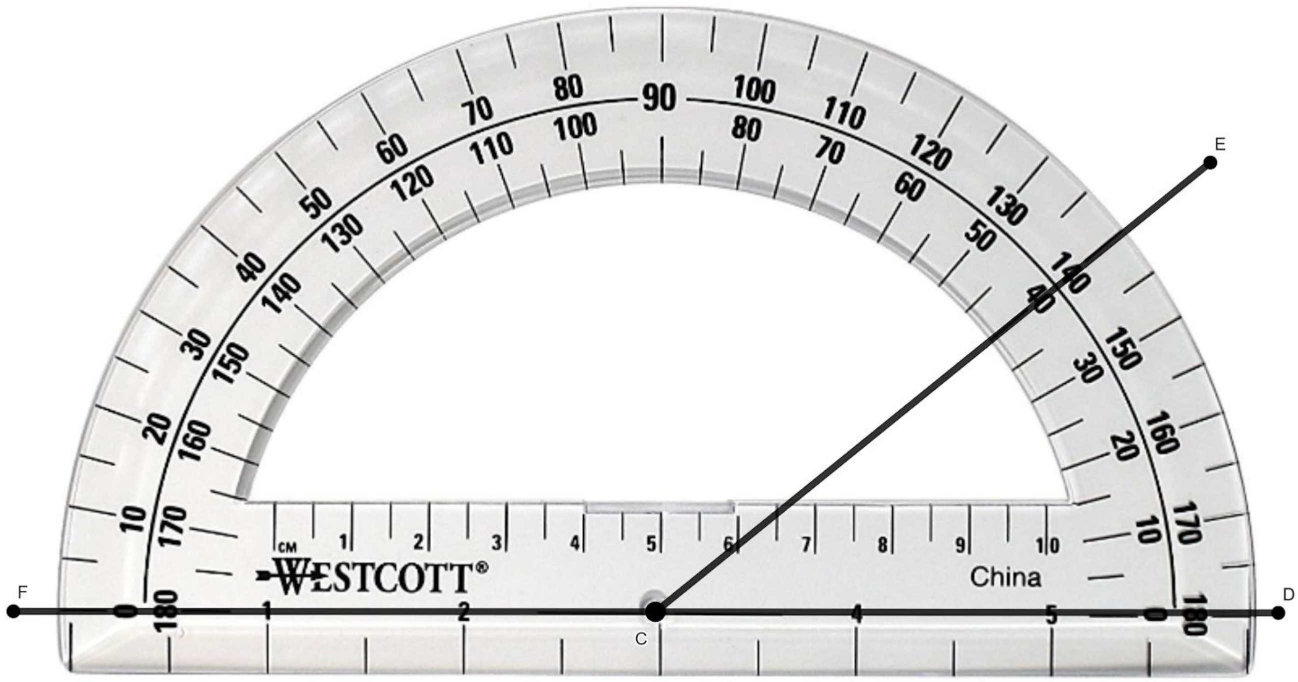
We can use protractors to measure angles the same way we use a ruler to measure length.

If you were using a tape measure to figure out how tall you are, you would not start measuring at your knees. You would line up the tape measure with the bottom of your foot and measure all the way up to the top of your head.

When we are measuring angles with protractors, we need to line them up properly. We do this by placing the small dot at the bottom of the protractor over the vertex of the angle.




You may have noticed that there are pairs of numbers all the way around the protractor. You may have also noticed that each pair adds up to 180°. In the angle above, $\overline{CE}$ lines up with the measure for 140° and 40°. We can read this as the smaller angle (∠ECD) as 40° and the larger angle (∠FCE) as 140°.



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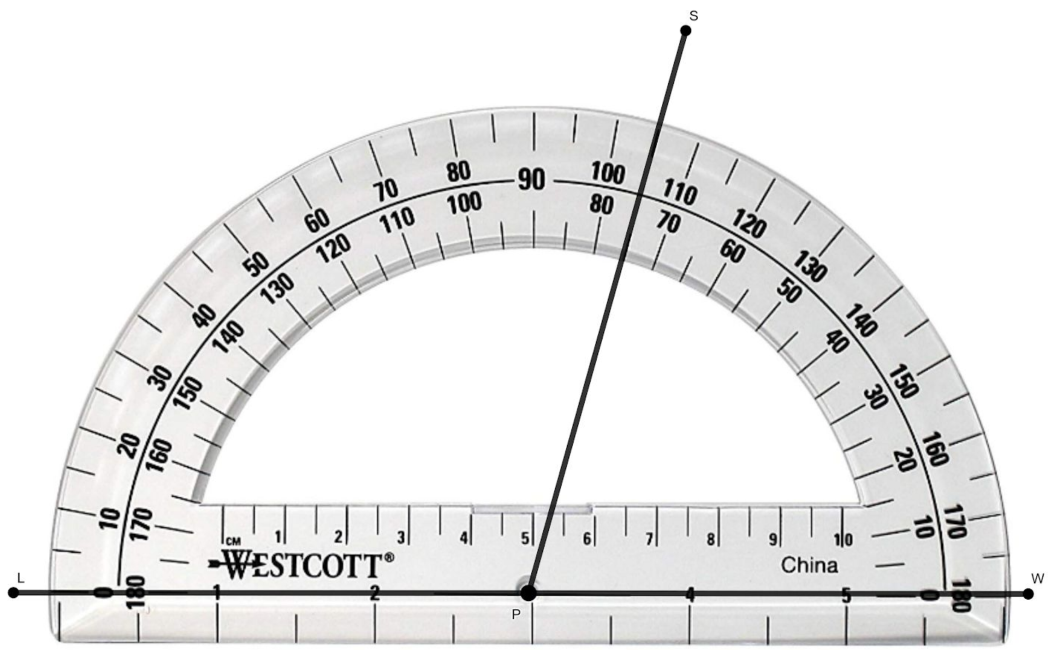
For problems 27-30, use the protractor provided to find the measure of the angles.

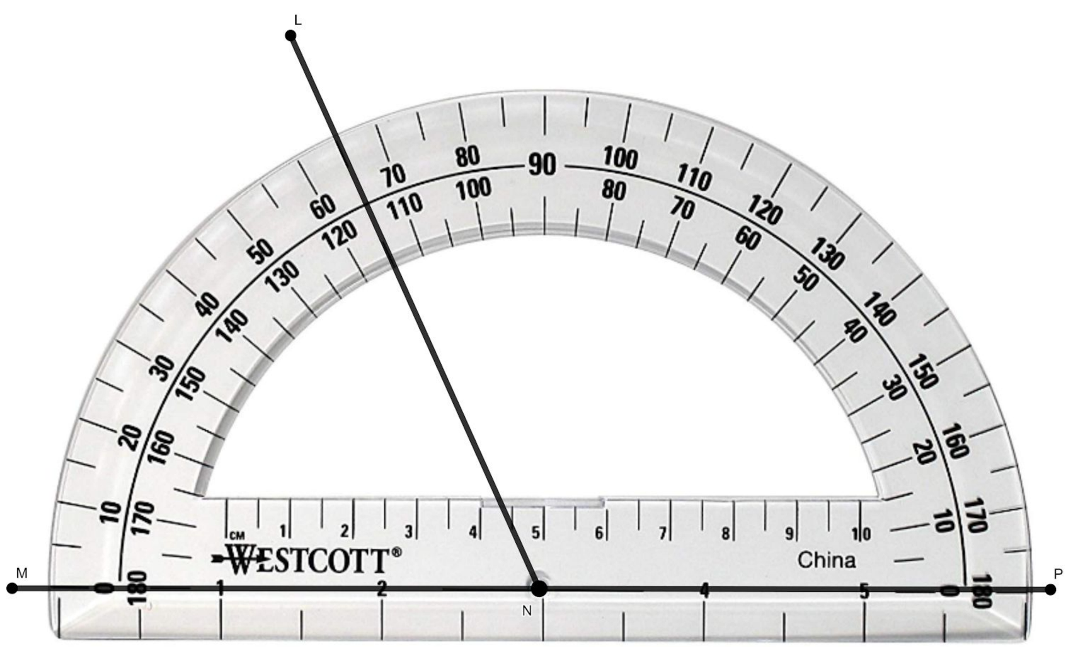
30. ∠SPW measures ___

∠LPS measures ___




31. ∠MNL measures ___

∠LNP measures ___



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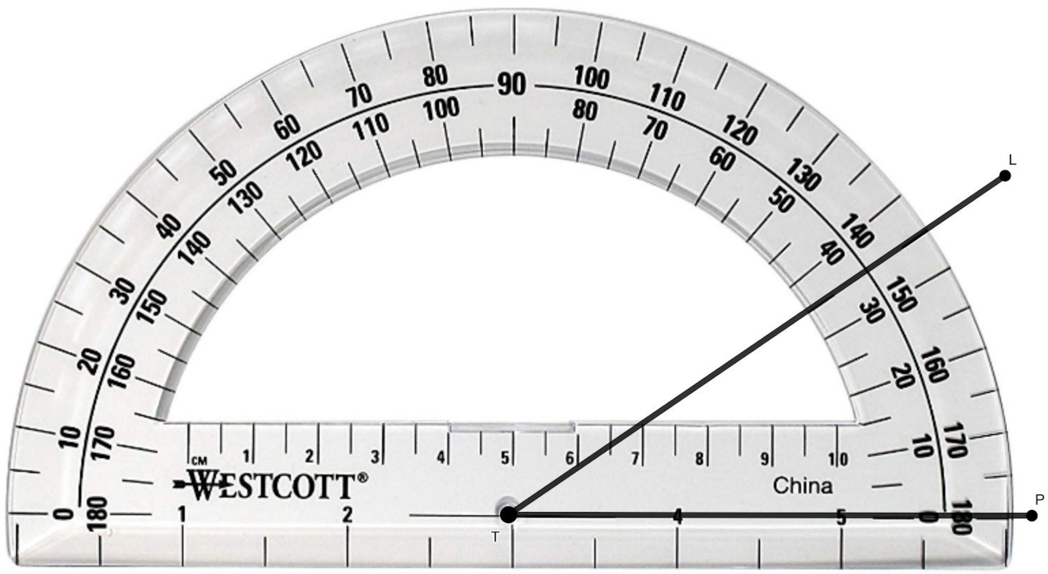
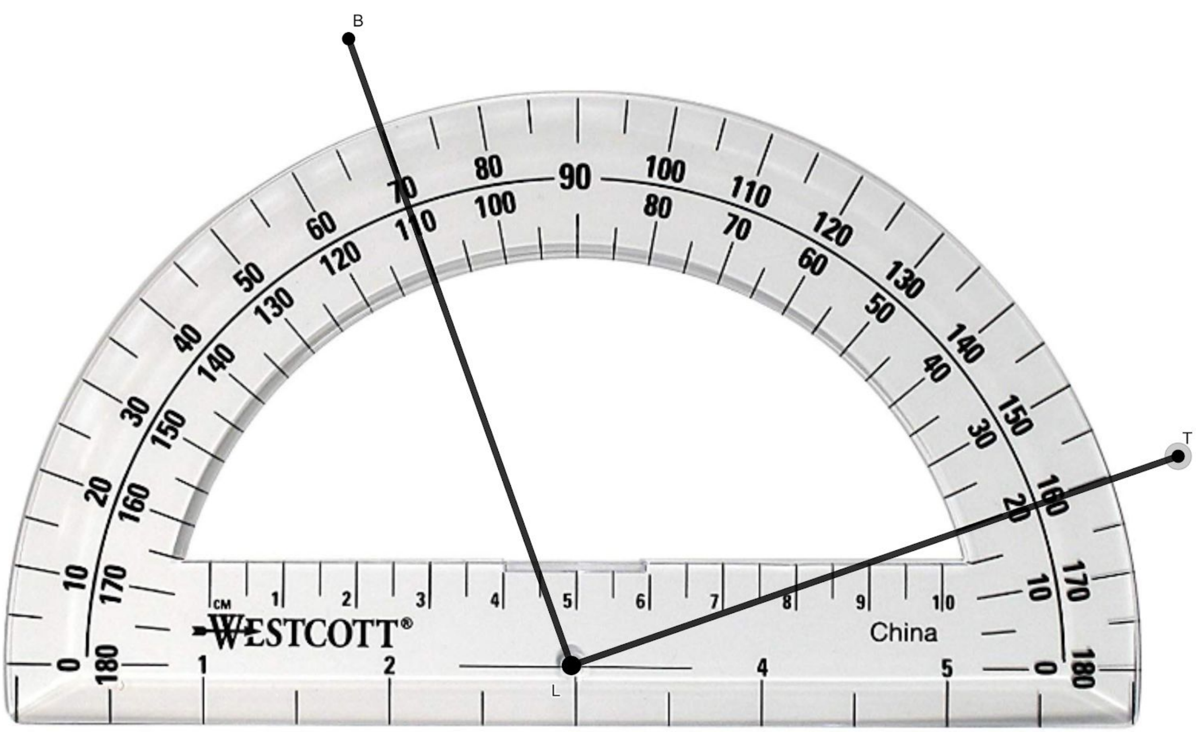
32. ∠LTP measures ___




33. ∠BLT measures ___



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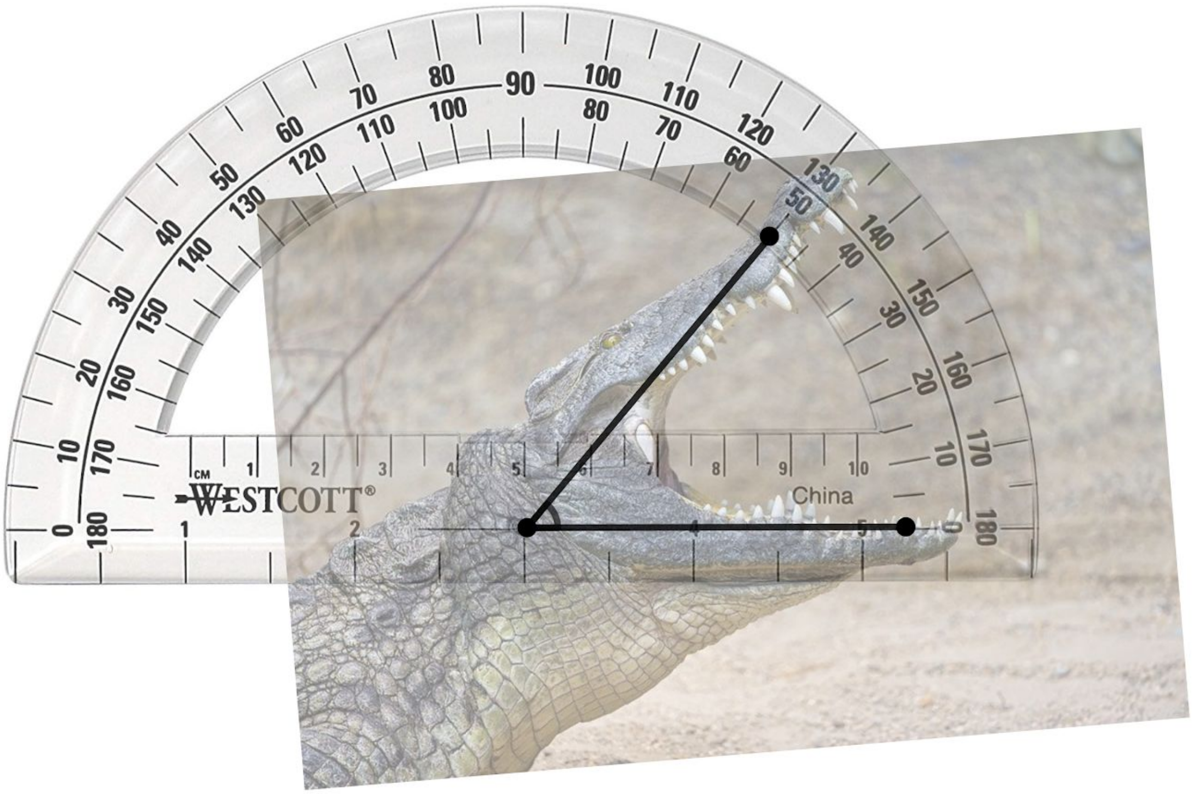
34. What is the angle of this alligator's open jaw?




35. The Tower of Pisa is a bell tower located outside the cathedral of the Italian city of Pisa. It is famous because the building seems to lean due to an unstable foundation.

    About how many degrees off-center is the Tower of Pisa?



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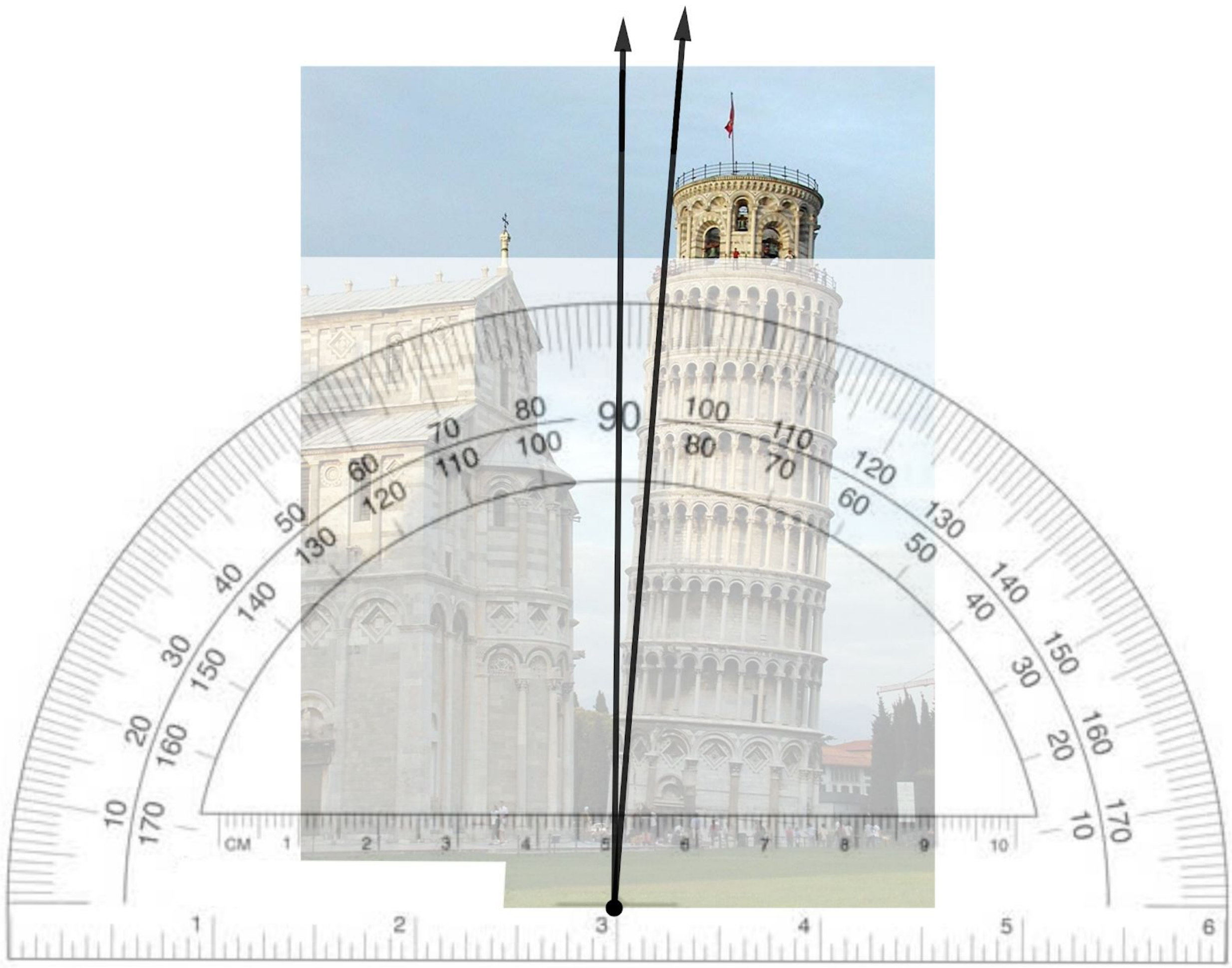
We can use an image of the tower together with a protractor to get a pretty good idea of how many degrees the tower is off-center.



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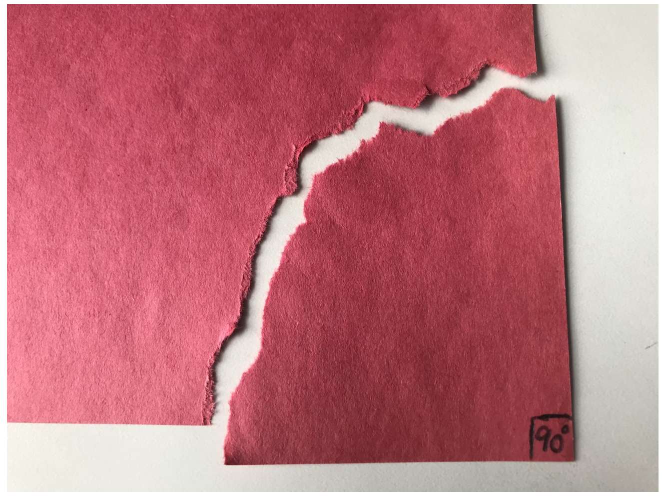
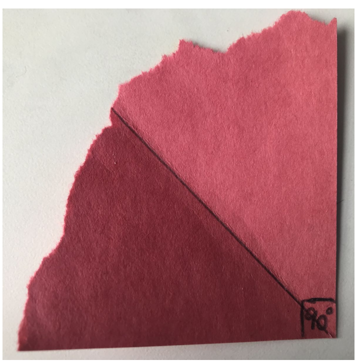
**Making Your Own Tool to Measure Angles**[4]

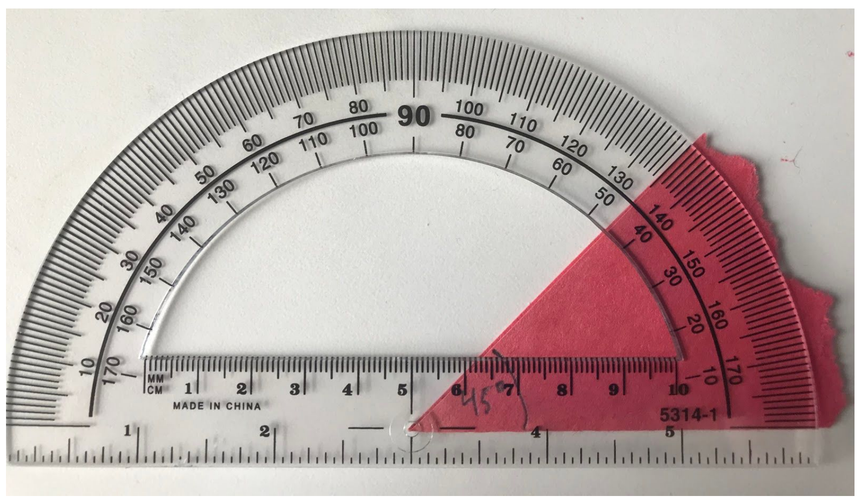
If you are like me, you might not carry a protractor with you at all times. So what can you do if you need to measure an angle?

Let's start with an angle that is easy to find. The corner of every piece of paper forms a right angle. Find a piece of paper and tear off a corner.



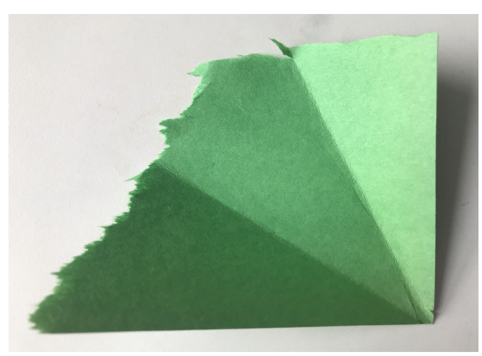

What happens if we fold that 90° angle in half?






Now, tear off another corner and fold it into three equal angles.

What is the measure of each of those angles?



[4] This activity was inspired by the work of Vi Hart, viewable on YouTube.



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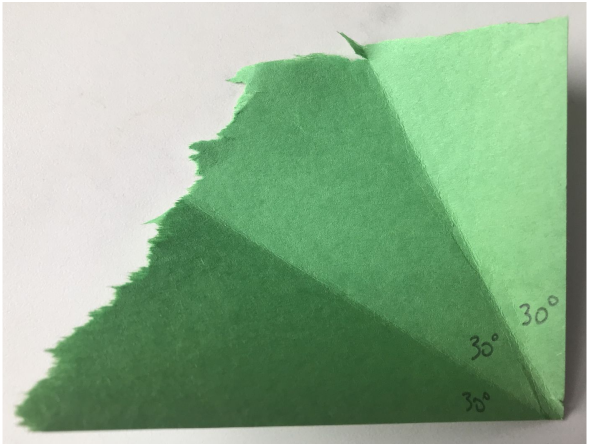
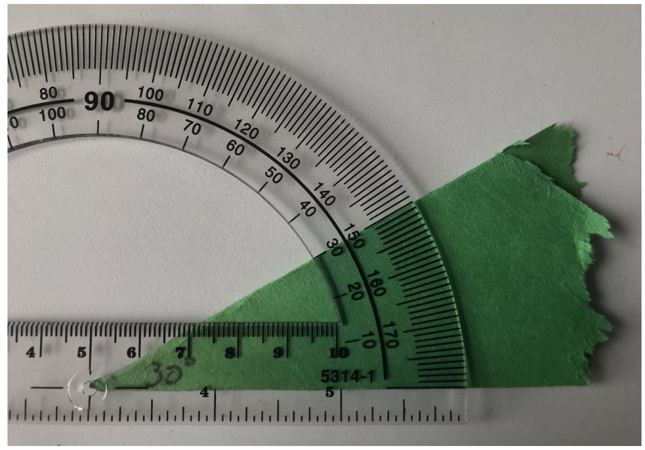
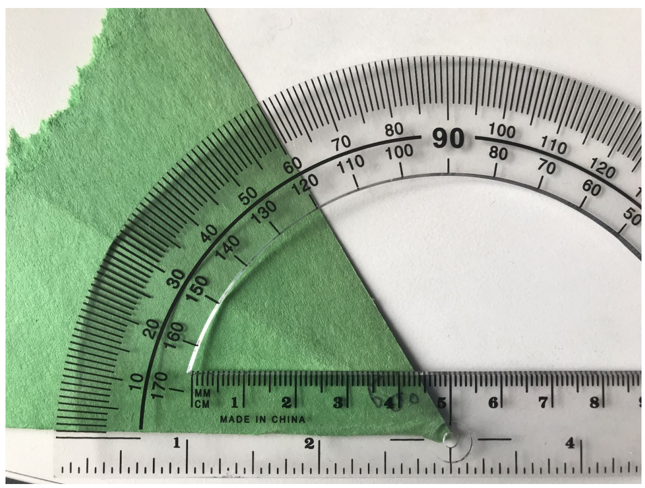
If you divide 90° into three equal angles, each angle will equal 30°.



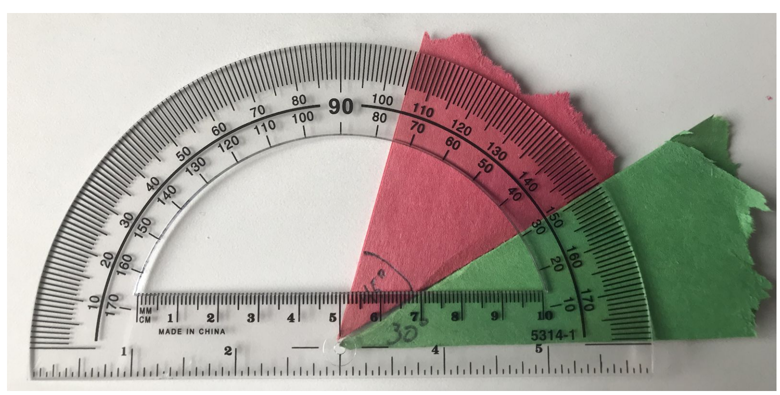




If you fold over one of those three sections, you can make a 60° angle.




With just two pieces of paper, we can now make angles that measure 90°, 45°, 30°, and 60°.

And if we combine those angles, we can make even more. Here's a combination of 45° and a 30° to make a 75° angle.




What other angles can we make using combinations of 90°, 45°, 30°, and 60°?



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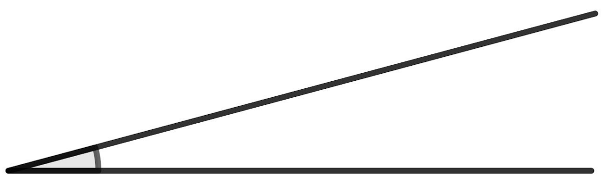
Use your paper tools and what you know about angles draw a line from each angle to their measurement in degrees.

36.

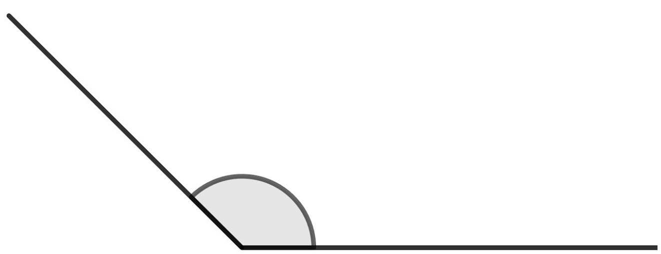
60°

37.



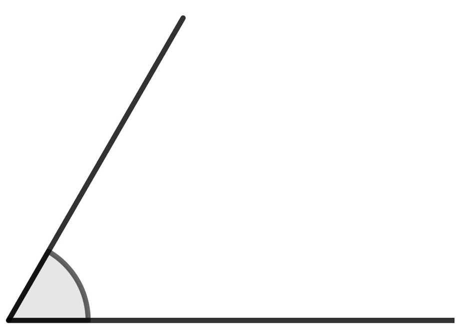
135°

38.



105°

39.



15°



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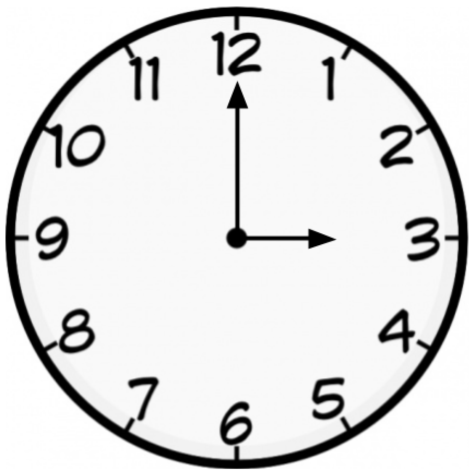
**Clock Angles**

Find the measure of the angle formed by the hands of each clock.

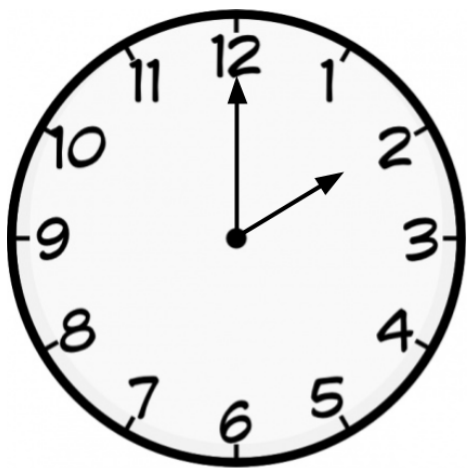
40.




When it is 3 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

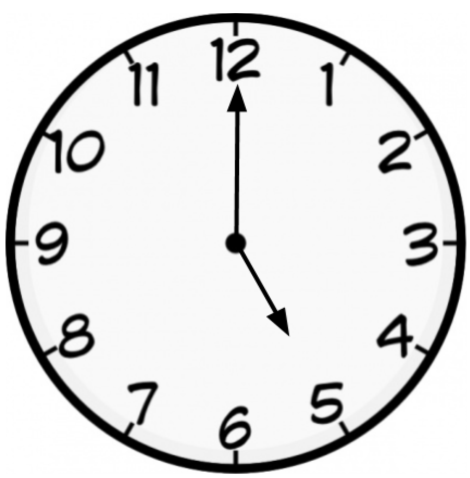
41.




When it is 2 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

42.




When it is 5 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:



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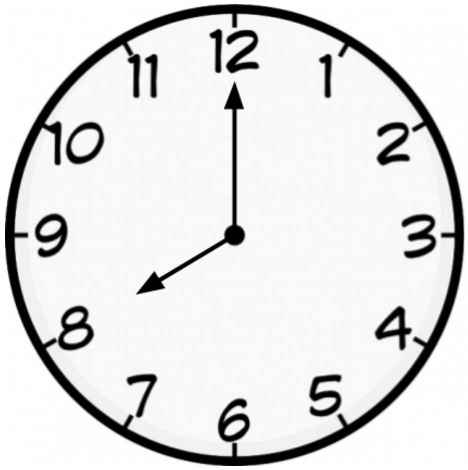
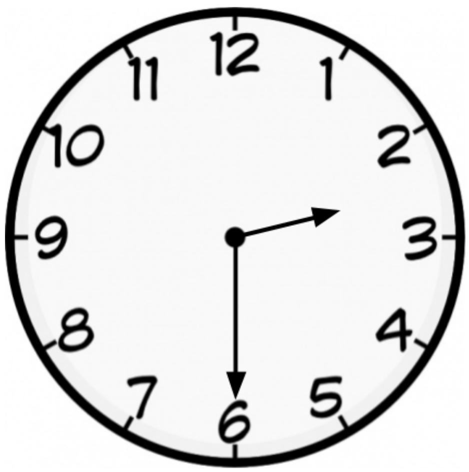
43.




When it is 8 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

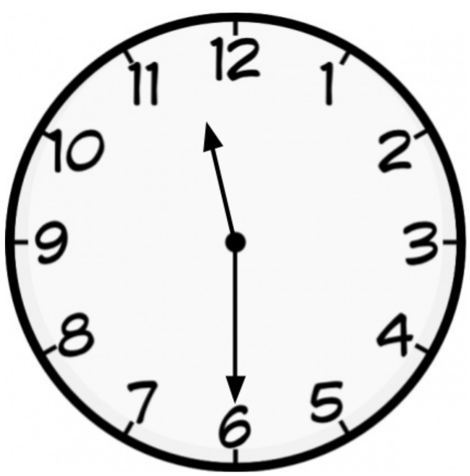
44.




When it is 2:30, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

45.




When it is 11:30, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:



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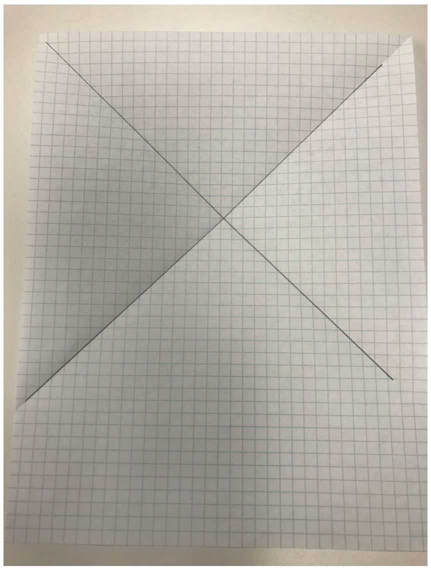
## Special Angle Relationships[5]

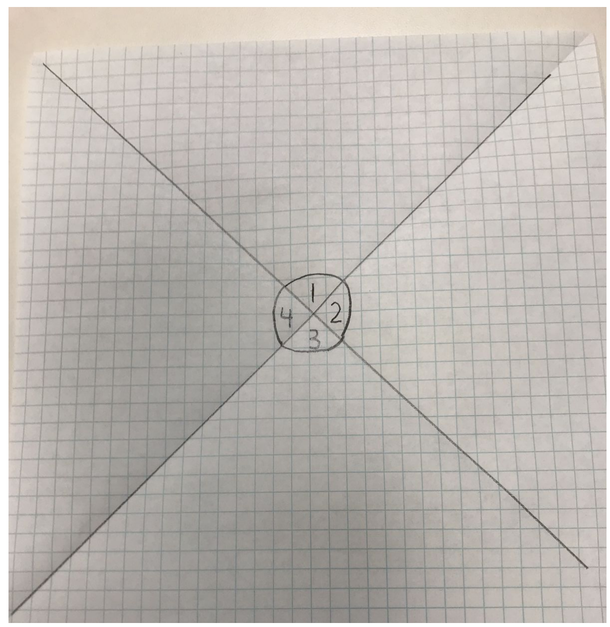
In the last section we looked at how to measure angles. In this section, we are going to discover some special relationships between angles.

For the activities in this section you will need a few sheets of blank paper and a pair of scissors.



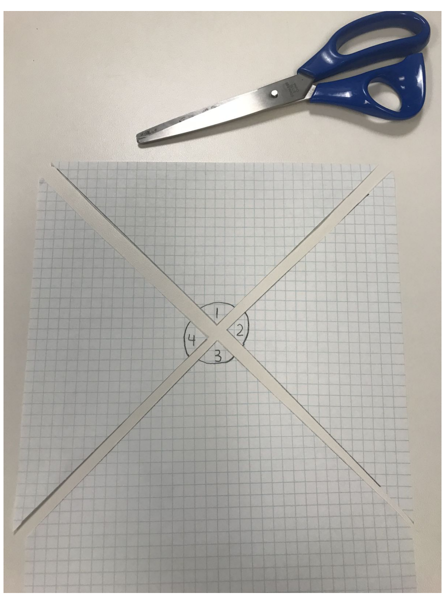
**Step One:**

Take one sheet of blank paper and fold it twice so that the folds create an "X". It will form 4 angles. It does not matter if the angles look exactly the same as the one in the example to the left.




**Step Two:**

Number the angles 1, 2, 3, and 4. Start with the angle at the top and number them clockwise.




**Step Three:**

Cut out the four angles. You might want to trace the folds with a pencil to help you cut in a straight line.

**Step Four:** Take a few minutes and move the angles around. What relationships do you notice?

[5] This activity was inspired by the work of Kyle Pearce.



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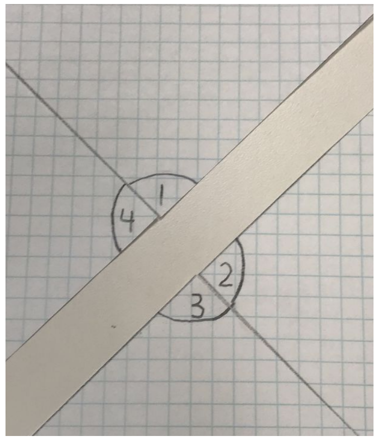
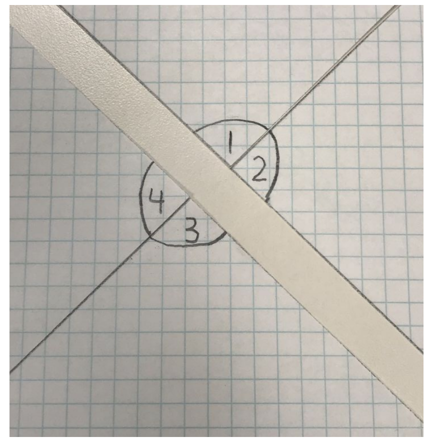
Some relationships you may have noticed:

- When you put some of the angles next to each other, they form straight angles of 180°.

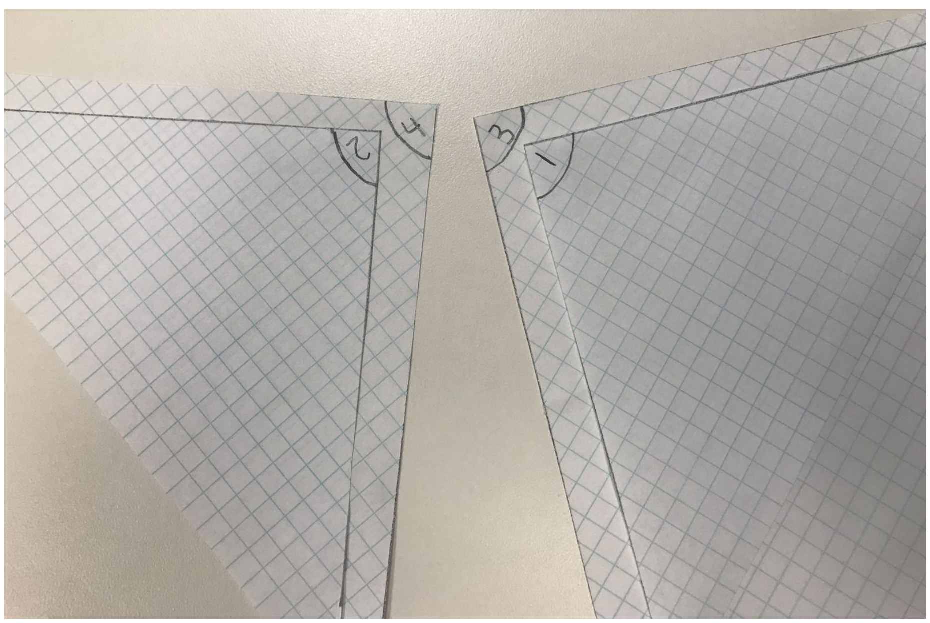
  In the example below, the following pairs of angles are supplementary:

  - ∠1 and ∠4
  - ∠2 and ∠3
  - ∠1 and ∠2
  - ∠3 and ∠4




- The angles opposite each other are the same. We call these vertical angles. You can take ∠2 and it fits exactly over ∠4. And you can take ∠1 and exactly cover ∠3.




When two lines intersect to form four angles, the opposite angles are equal.



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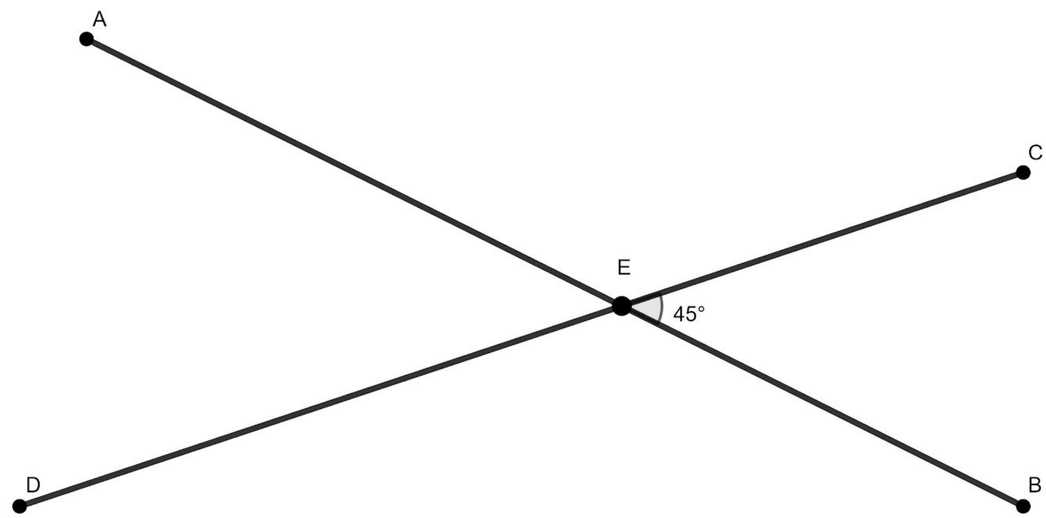
46. ∠BEC measures 45°

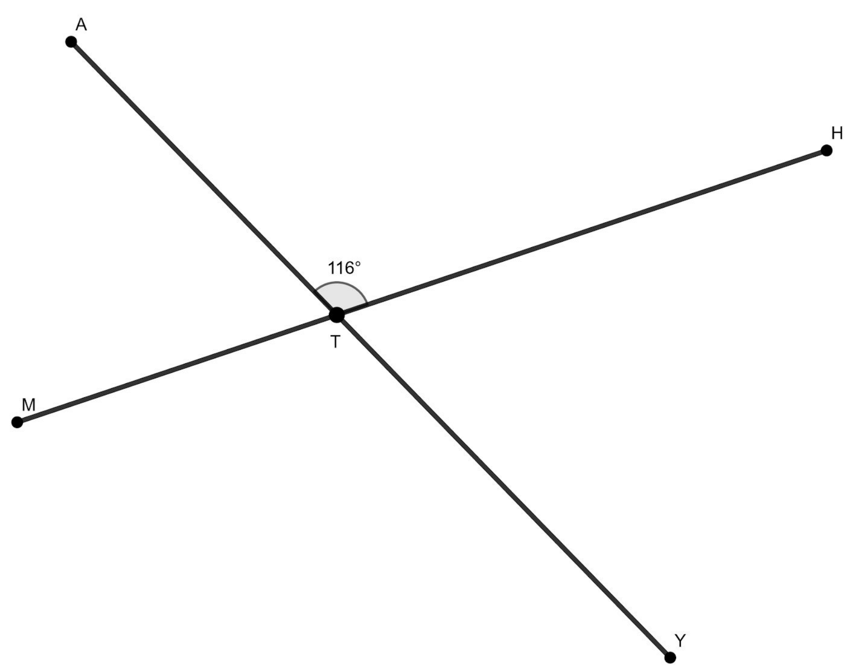



∠CEA measures _____

∠AED measures _____

∠DEB measures _____

47. ∠ATH measures 166°




∠MTA measures ____

∠HTY measures ____

∠MTY measures ____



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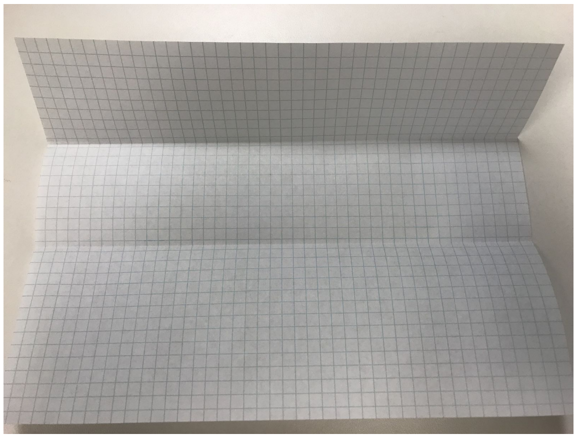
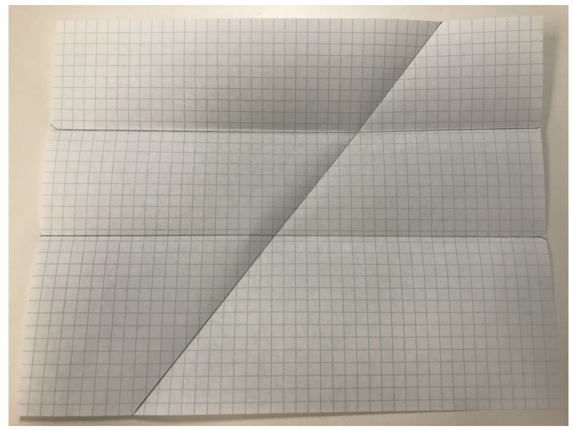
**Angles and Parallel Lines**



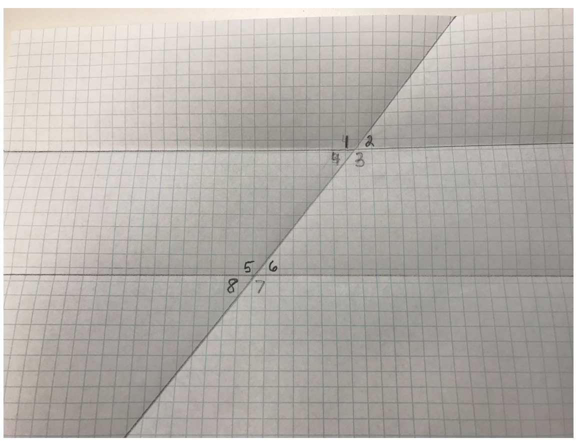
**Step One:**

Take another blank sheet of paper. Fold it twice to create two parallel lines with the folds.



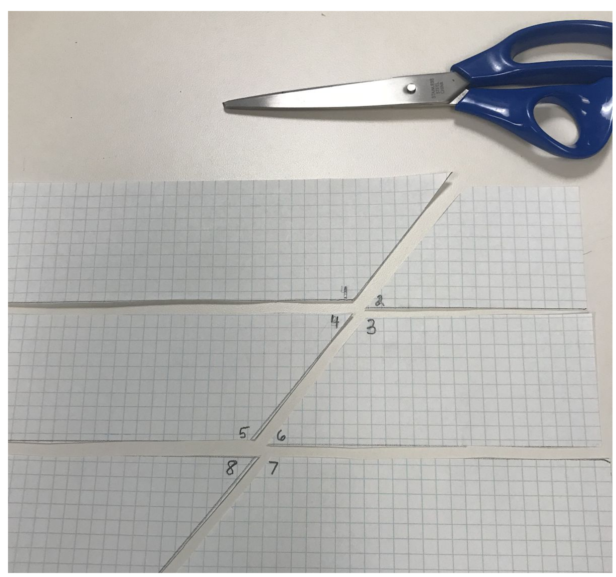
**Step Two:**

Fold the paper again so that the fold cuts through both of the parallel lines.




**Step Three:**

Number the angles in order, going clockwise.




**Step Four:**

Cut out the angles. You might also want to trace the folds with a pencil to help you cut straight.

**Step Five:**

Take a few minutes and move the angles around. What relationships do you notice?



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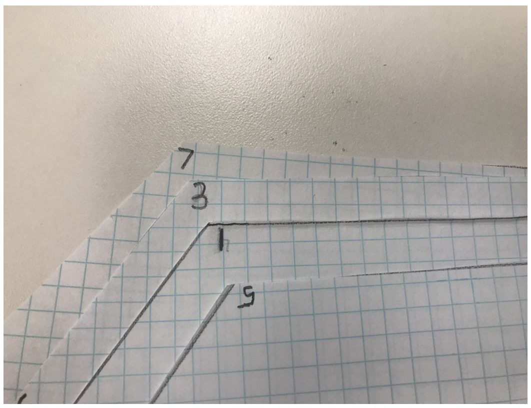
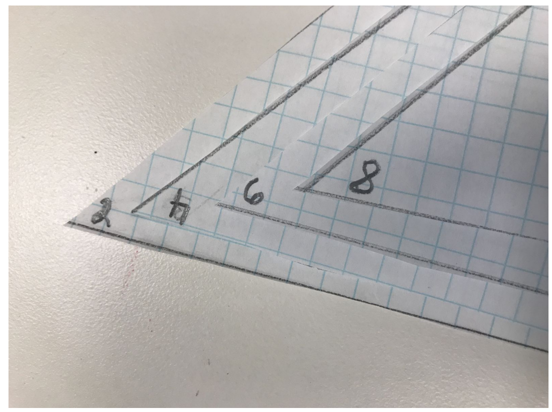
When you have two parallel lines that are both intersected by a straight line, it will make two groups of opposite angles.

In the example above:

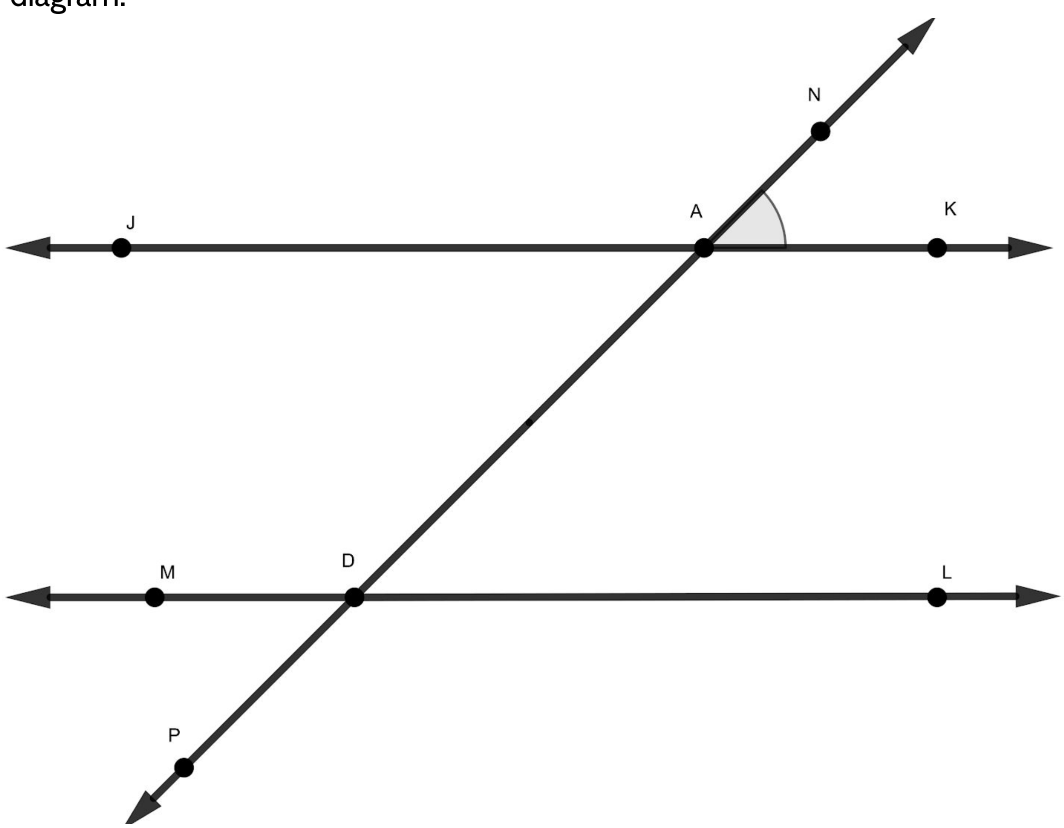
∠1, ∠3, ∠5, and ∠7 are all the same.

∠2, ∠4, ∠6, and ∠8 are all the same.




48. $\overleftrightarrow{JK}$ is parallel to $\overleftrightarrow{ML}$. If ∠NAK measures 52°, fill in the missing angles in the diagram.



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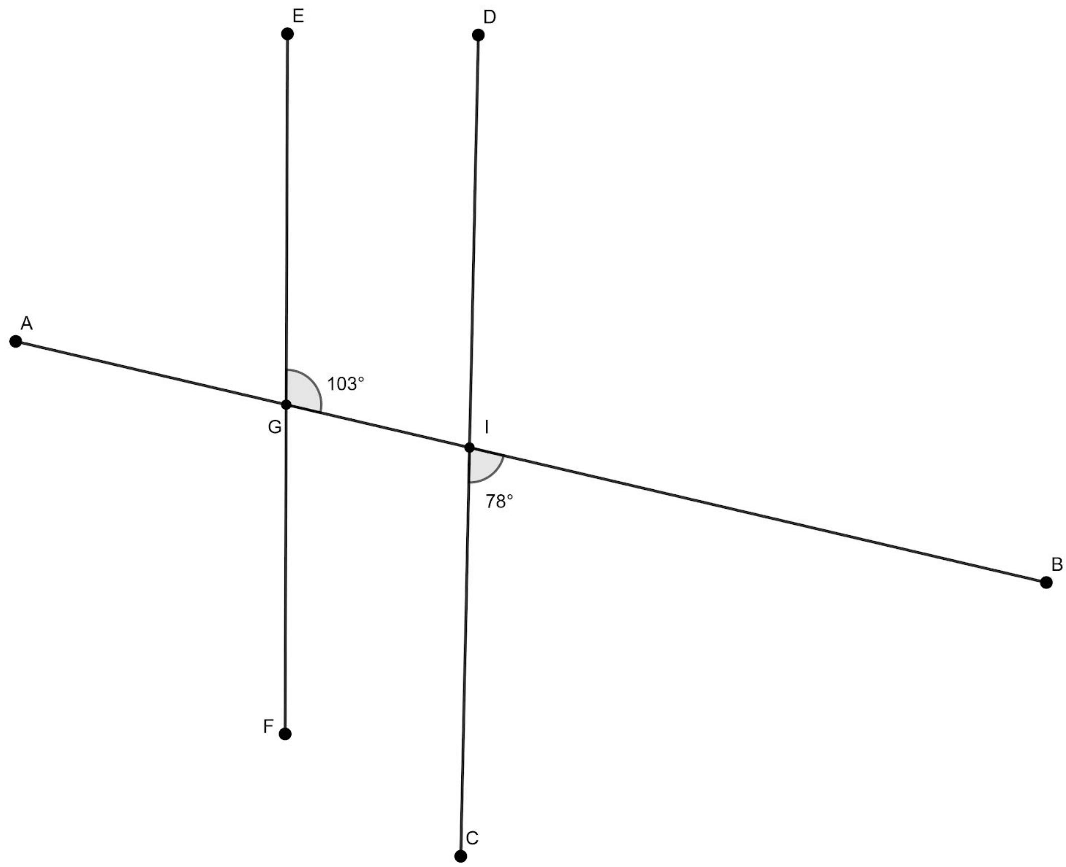
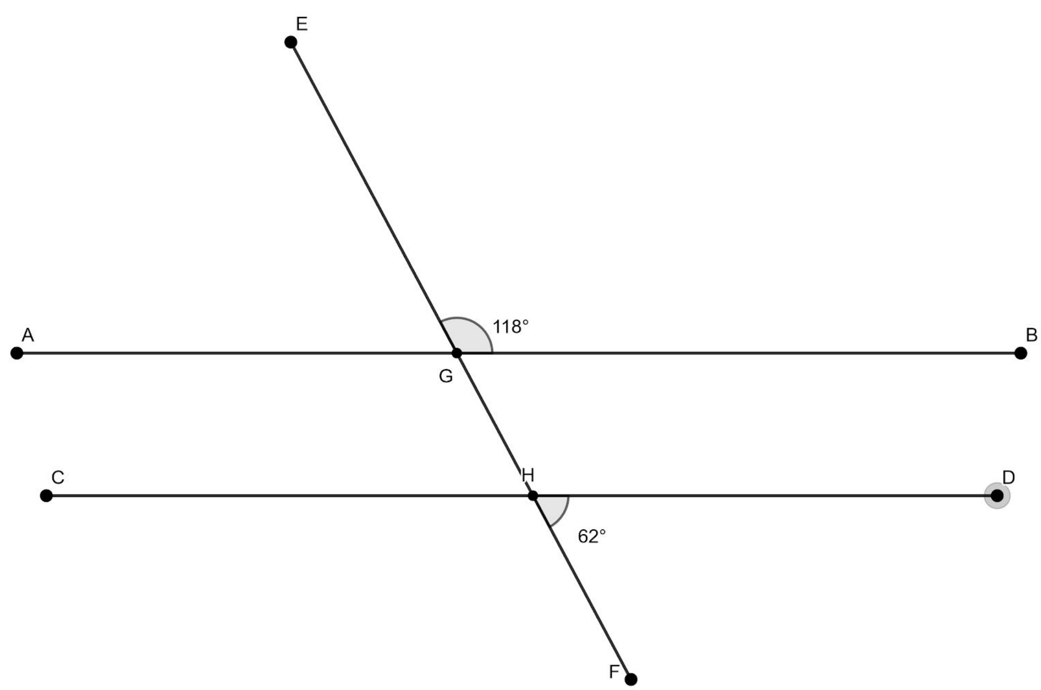
49. Which diagram has a pair of lines that are parallel?

a.




b.



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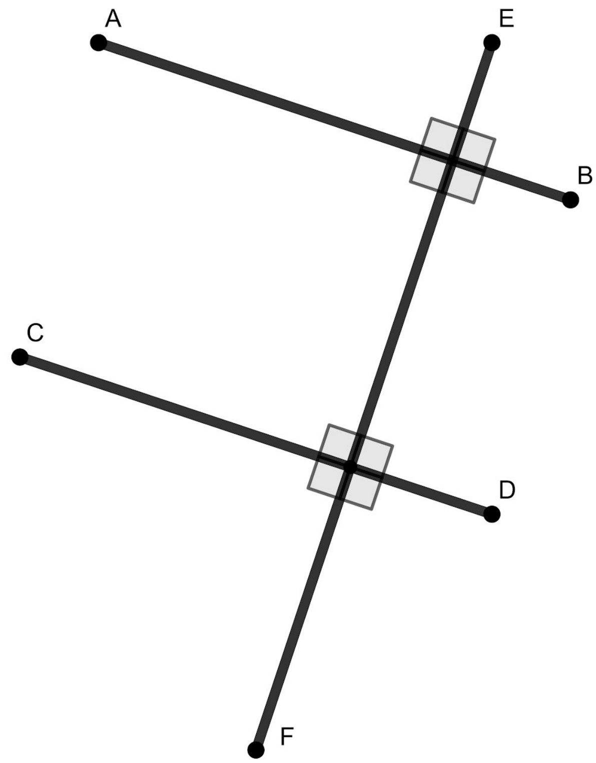
50. Which two statements are true?

a. $\overline{CD}$ is perpendicular to $\overline{AB}$

b. $\overline{FE}$ is perpendicular to $\overline{CD}$

c. $\overline{CD}$ is parallel to $\overline{FE}$

d. $\overline{AB}$ is parallel to $\overline{CD}$



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# Angles - Answer Key

1. ∠SWP, ∠PWS, or ∠W
2. ∠OLW, ∠WLO, or ∠L
3. ∠1 is ∠HPO or ∠OPH. ∠2 is ∠OPE or ∠EPO. Note that because both ∠1 and ∠2 share P as a vertex, ∠P ***is not*** an acceptable name for either angle.
4. The three angles formed are ∠ABD (or ∠DBA) and ∠DBC (or ∠CBD) and ∠ABC (or ∠CBA).
5. ∠ABE, ∠ABD, ∠ABC, ∠EBD, ∠EBC, ∠DBC
6. There are ten angles formed when 5 line segments intersect at the same vertex. ∠ABE, ∠ABD, ∠ABF, ∠ABC, ∠EBD, ∠EBF, ∠EBC, ∠DBF, ∠DBC, ∠FBC
7.

| **Number of Line Segments** | **Number of Angles** | There are different patterns you might notice. Here's one pattern you might have seen:<br><br>With each new line segment, the number of angles formed increases.<br><br>It increases by 2, then 3, then 4, then 5, then 6, then 7... |
|---|---|---|
| 2 | 1 (+2) | |
| 3 | 3 (+3) | |
| 4 | 6 (+4) | |
| 5 | 10 (+5) | |
| 6 | 15 (+6) | |
| 7 | 21 (+7) | |
| 8 | 28 (+8) | |

8. Choice D is the incorrect name. It does not have the vertex in the middle of the name.
9. Angle ABC is the larger angle. Even though the line segments that form angle DEF are longer, angles are the openings between
10.
    a. There are four 90° angles in 360°.
    b. There are five 72° angles in 360°.
    c. There are six 60° angles in 360°.
    d. There are eight 45° angles in 360°.
    e. There are twelve 30° angles in 360°.



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11. $\angle$DNC, $\angle$CNR, and $\angle$ANE are all 90° angles. They each contain two 45° angles.

12. $\angle$DNE, and $\angle$ANR are both 135° angles. They each contain three 45° angles.

13.
    a. We know straight angle ACB is 180°. If $\angle$BCD is 160° than we know $\angle$DCA measures 20° because together they need to add up to 180°.

    b. Similarly, if $\angle$KAY measures 180° and $\angle$NAY measures 50°, than we know $\angle$KAN measures 130°.

14. Choice C. 178° and 12° adds up to 190°. All of the other answer choices add up to 180°.

15. $\angle$ACE measures 125°.

16. $\angle$a measures 50° because it is supplemental to the 130° angle.
    $\angle$b measures 60°, because the 50° angle and the 70° angle form a straight angle with $\angle$b. 50° + 70° is 120°. Angle b has to be 60° because 120° + 60° is 180°.
    $\angle$c measures 50° because it is supplemental to the 130° angle.

17. There are many different ways to approach this problem. If $\angle$DCA is equal to $\angle$DCB than each of those angles has to measure 90° . If $\angle$DCA is 90° , we can ask ourselves, what number can we double and add to itself to get 90°. If you try different combinations, eventually you will find that 60° is twice as big as 30° and 30°+60° equals 90° . If $\angle$FCD is 60°, then $\angle$DCE is 60°, than $\angle$FCE is 120°.

18. Each of the smaller angles is a 10° angle. If you divide 180 into 18 equal sections, each section is 10. Another way to think about it is
    10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°=180°

19. $\angle$CDE measures 60°. $\angle$FDC measures 120°.

20. $\angle$PFC measures 100°. $\angle$LFP measures 80°.

21. $\angle$SRA measures 30°. $\angle$HRA measures 150°.

22. $\angle$MRO measures 45°. Line segment RM divides the 10° in half. So the opening is 10°+10°+10°+10°+5°. $\angle$ARM measures 135°.

23. $\angle$HPO and $\angle$OPE each measure 90°.

24. 30°

25. 45°



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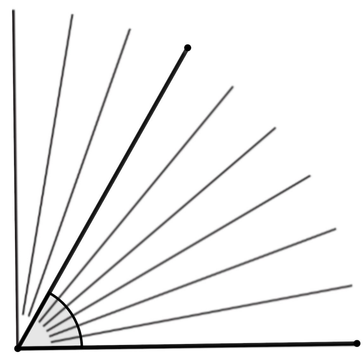
26. 

27. 30°

28. Choice B. 65° and 25° add up to 90°. Choices C and D are supplementary angles.

29. Choice D.

30. ∠SPW measures 75° and ∠LPS measures 105°.

31. ∠MNL measures 65° and ∠LNP measures 115°.

32. ∠LTP measures 35°.

33. ∠BLT measures 90°.

34. The alligator's jaw is open at a 50° angle.

35. The Leaning Tower of Pisa leans at about a 4° angle.

36. 105°. This angle is just a little bit wider than a right angle. It can be made by combining a 45° and a 60° angle.

37. 15°. This is the smallest angle. It is half of a 30° angle.

38. 135°. This is the largest angle on the page. It is a combination of a 90° and 45°.

39. 60°.

40. 90°. When the hands are at 12 and 3, they form a right angle.

41. 60°. One way you might answer this one is to use the measurement in the last question. If 3 o'clock is 90°, you can think about dividing that 90° into three equal angles. So at 1 o'clock, the hands would form an opening that was 30° and at 2 o'clock, the opening would be open 30° more. 30° + 30° is 60°.

42. 150°. There are a few ways you can answer this one. If you think of each hour as 30°, to form the opening for 5 o'clock you would need five 30° angles, which adds up to 150°. Another way is to start with what we know about 3 o'clock. If the angle of the



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hands at 3 o'clock is 90°, than opening it up two more hours would be another 60°. 90° + 60° is 150°.

43. 120°. This one might feel a little different because the angle is facing the other direction from the previous examples. One way to look at it is to say that when the hands are at 9 o'clock, they form a 90° angle. If you open the angle to 8 o'clock, that would add another 30°. 90° + 30° is 120°.

44. 105°. If the hands of the clock were at 3 and 6, they would form a 90° angle. Here the hour hand is halfway between the 2 and the 3, so the angle is open more than 90°. Since the time reads 2:30, we know the hour hand must be exactly halfway between the 2 and 3. Since each hour forms a 30° angle, half if that would form a 15° angle. 90° + 15° is 105°.

45. 165°. There are several different ways to answer this one. If the hands of the clock were at 12 and 6, they would form a 180° straight angle. Since the clock reads 11:30, we know the hour hand is halfway between the 11 and the 12. That means the opening of the angle is 15° less than 180°. 180° - 15° is 165°.

46. ∠CEA measures 135°. ∠AED measures 45°. ∠DEB measures 135°.

47. ∠MTA measures 64°. ∠HTY measures 64°. ∠MTY measures 116°.

48. ∠ADL, ∠MDP, ∠JAD all measure 52°. ∠JAN, ∠KAD, ∠MDA, ∠PDL all measure 128°.

49. Choice B is the only answer with parallel lines. If you fill in all the missing angles, choice B is the only one that has two groups of opposite angles.

50. Choices B and D are both true statements.



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## Area and Perimeter

To measure length the Ancient Egyptians used units based on parts of the human body.

Here are some of the units they used to measure length:

- A *digit* = the width of a finger
- A *palm* = the width of your pinky finger to your pointer finger
- A *cubit* = the length from the tip of your finger to your elbow. There are 7 palms in a cubit. A person who is 6 feet tall would be about 4 cubits tall.
- A *khet* = 100 cubits.

Priests served a very important role in Ancient Egyptian civilization. One of the things they were tasked with was to collect taxes from the farmers during the harvest. The amount of tax depended on the size of the farm. The bigger the farm, the more taxes they had to pay.

But what do you do if you have two farms that we shaped like the ones to the right. The top one looks wider, but the bottom one looks longer.

Which one is bigger?

Which farm should pay more in taxes?

And how much taxes should they each pay?

To answer these questions, the priests of Egypt came up with an idea. Some people say they were inspired by the square tiles that covered the floors of the temple. They imagined covering the farms with squares and counting the squares.



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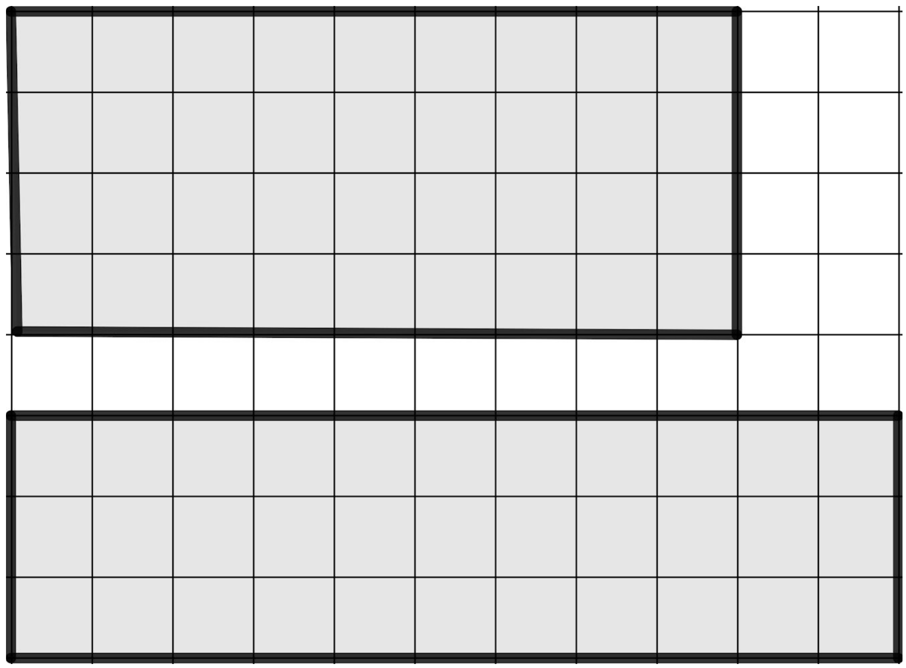

We would need to cover the surface with the same sized squares so we could make a fair comparison. To do this, we use something called a "unit square". A **unit square** is a square whose sides are 1 unit in length. The sides of a unit square can be any unit of length. For example, a square inch is a square that measures 1 inch on each side. A square mile is a square that measures 1 mile on each side.

Let's imagine the rectangle at the top of the this page represents a farm that is 9 miles by 4 miles and the rectangle below that one represents a farm that is 11 miles by 3 miles. It takes 36 squares to cover the surface of the farm on the top and 33 squares to cover the surface of the farm on the bottom. So the farm on the top has a larger area.

We said earlier that when we are talking about **area,** we are talking about the size of a surface. The Ancient Egyptians gave us a way to measure surfaces by asking how many squares it takes to cover that entire surface. That is the key question of area.

**We measure area based on how many <u>squares</u> it takes to completely cover a surface.**



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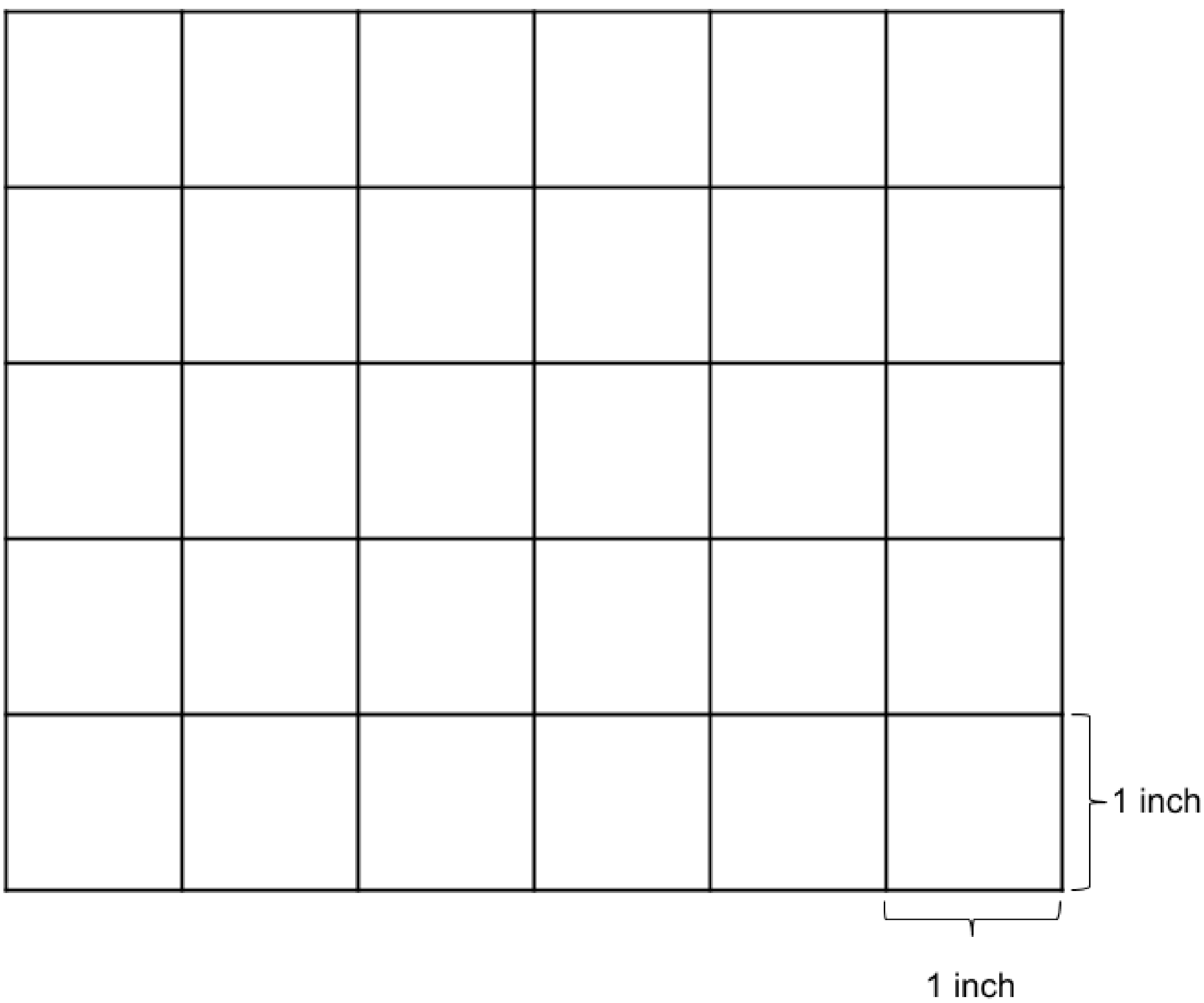
What are some surfaces we can measure? Examples of surfaces include floors, walls, fields, There are so many situations when we need to measure the size of a surface. Carpenters need to know the area of floors and walls to buy the materials they need for construction jobs. Farmers and community gardeners need to know the area of their growing space so they can make decisions about how much they can grow. Other examples of surfaces include a lawn, a cattle have enough space to graze, laying down a carpets—these are all situations when we need to understand area.




**The area of the rectangle above is <u>30 square inches</u>.**



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1. What do you think the 30 stands for?

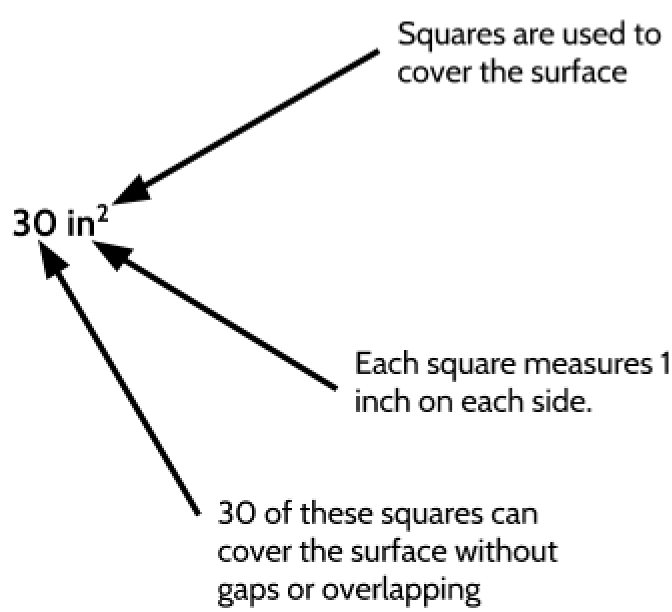
2. Why do you think the word “square” is used?

3. Why do you think it says square “inches”?

There are a few ways we can write 30 square inches:

30 square inches　　30 sq. in.　　30 $in^2$

Any measurement in area will have a number, a unit (in this example the unit is “inches”) and the word square. Each part of that measurement means something.



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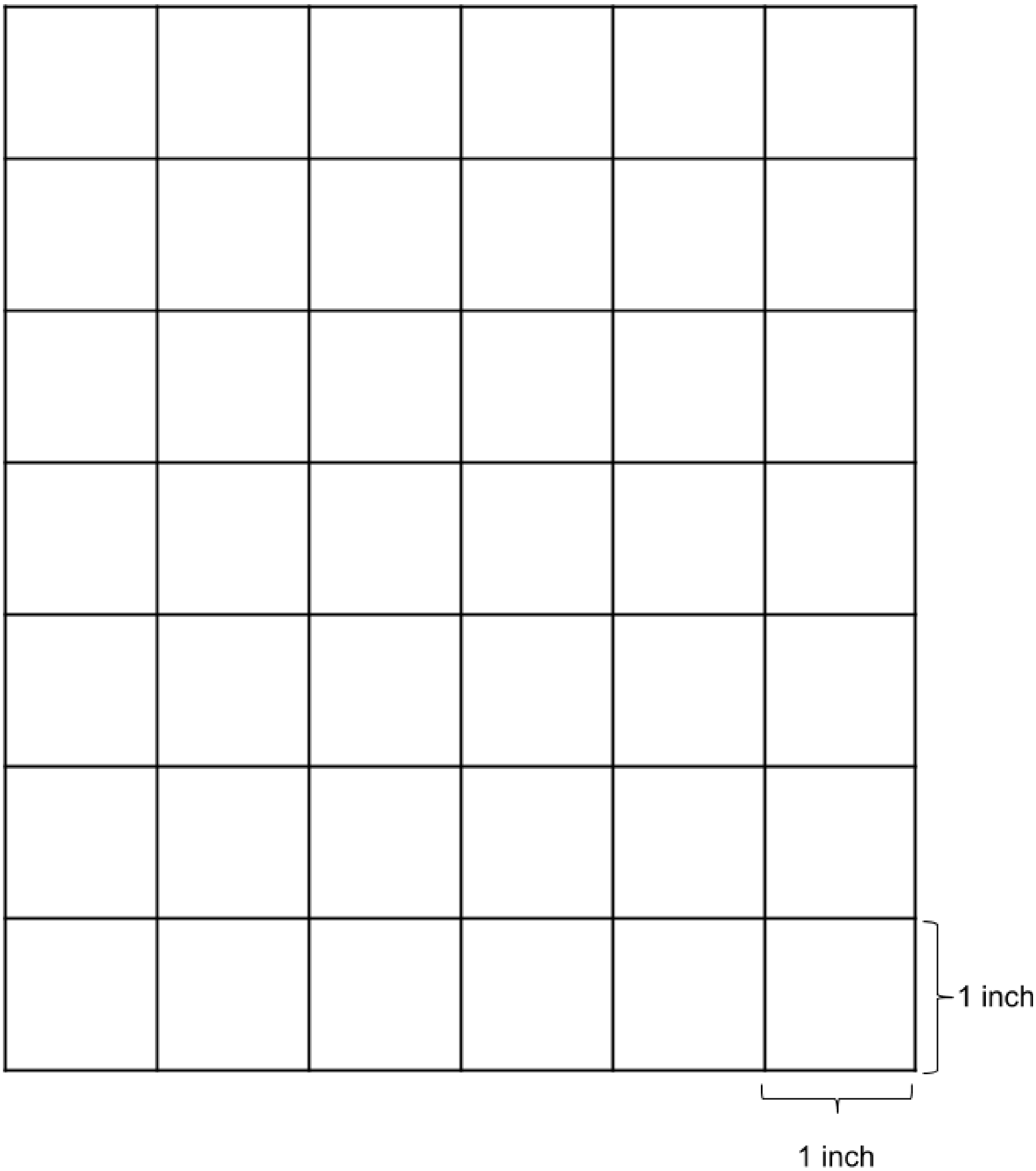
4. What is the area of the rectangle below?



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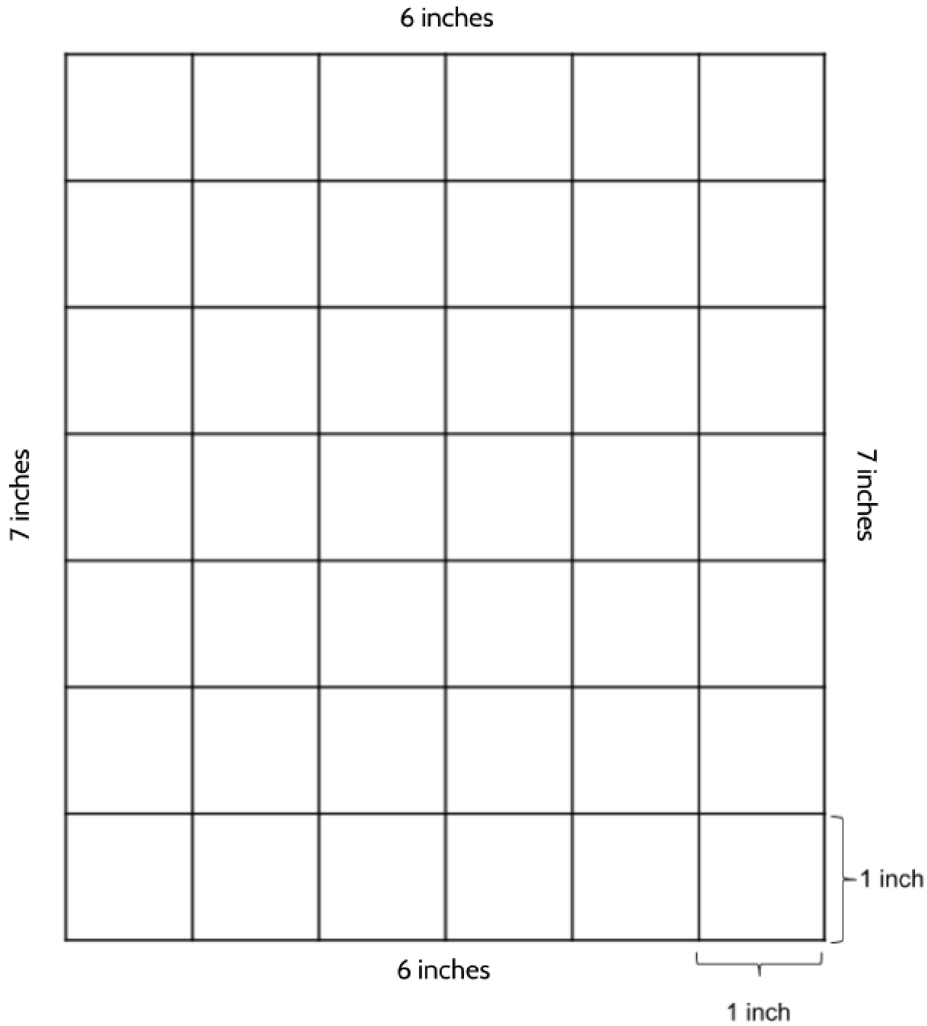
In addition to the area, we can also measure the length of each side of the rectangle. Since we know that the side of each square is 1 inch, we can measure the length all the way around the rectangle by counting the inches.




If we add up the length of each side of this rectangle, we get 26 inches.

7 inches + 7 inches + 6 inches + 6 inches.

In geometry we have a special word for this measurement. **Perimeter** is the whole length of the border around an area or shape. It comes from the Greek word *peri* meaning "around" and *metron* meaning "measure."



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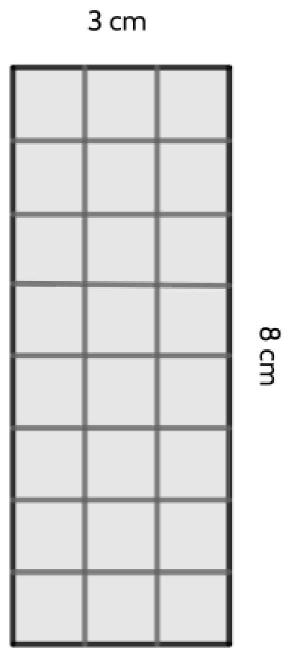
5. Find the area and perimeter of each rectangle.

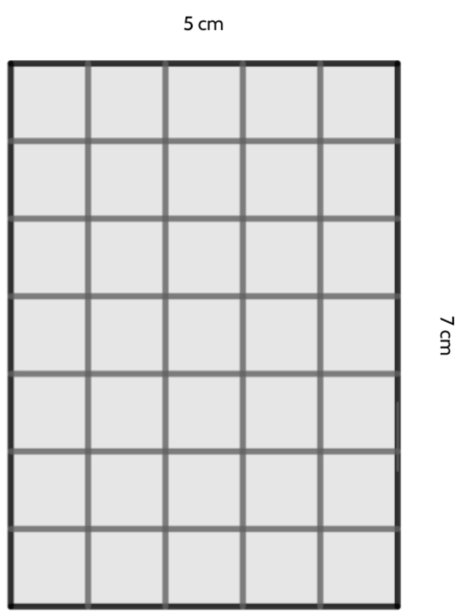
a.




Area = _____ square centimeters

Perimeter = _____ centimeters

b.




Area = _____ square centimeters

Perimeter = _____ centimeters



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6. On the next page, there is a grid made up of square centimeters. Each square measures 1 centimeter on each side.

   Please draw the following figures of that grid and label each shape:

   a. Rectangle A: Create a rectangle with an area of 28 square centimeters.

   b. Rectangle B: Create two rectangles with different perimeters that both have an area of 15 sq. cm.

   c. Shape C: Create a figure that is not a rectangle that has an area of 10 $cm^2$

   d. Rectangle D: Create a rectangle with a perimeter of 26 centimeters that has an area of 36 square centimeters.

   e. Rectangle E: Create a rectangle with two sides that each measure 2 centimeters, and whose area is exactly 13 $cm^2$



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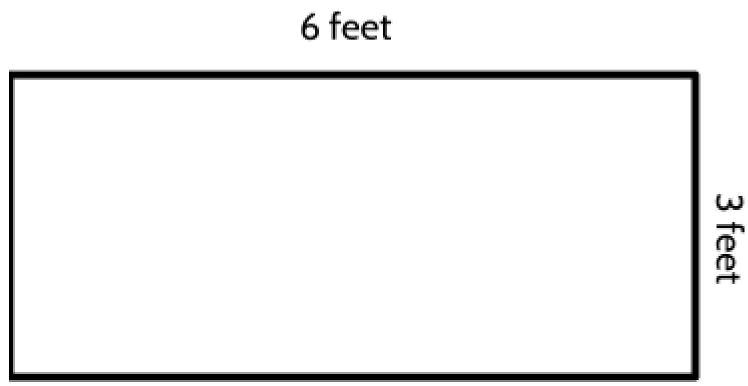
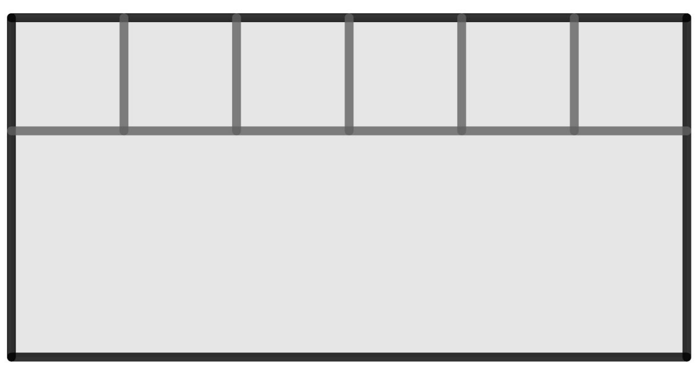
In all the examples we've looked at so far, you have had squares drawn in to cover the surface of each rectangle. But what happens when there are no square inches or square centimeters drawn in?

How could we figure out how many square feet it would take to cover the surface of this rectangle?




Here is one method: We know the rectangle measures 6 feet across.

Can you imagine a row of 6 square feet along the top?



Since the rectangle measures 3 feet down, we can fit a total of three rows of square feet.

- ➔ 6 square feet
- ➔ 6 square feet
- ➔ 6 square feet

For a total of 18 square feet!



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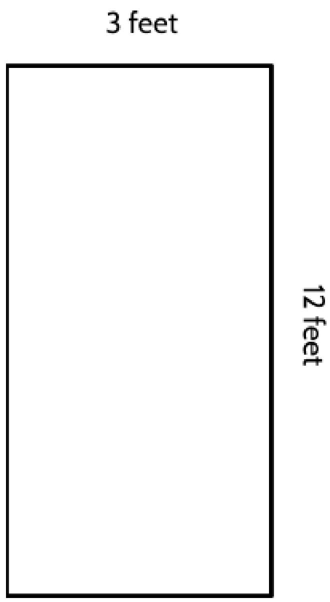
One thing you might notice about 6 and 3 is that when you multiply them you get 18.

Multiplying 6 x 3 means adding 3 groups of 6. When we are working with the area of rectangles, that means 3 rows of 6 square feet for a total of 18 square feet.

7. What is the area and perimeter of this rectangle?



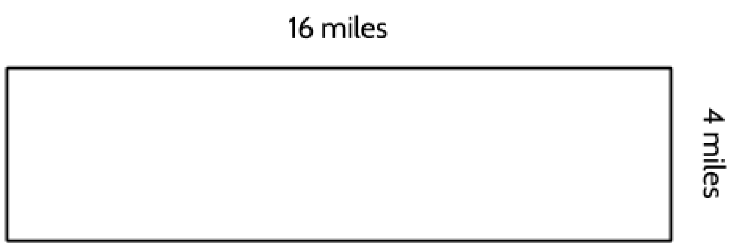

a. Area = 36 feet
Perimeter = 30 feet

b. Area = 30 square feet
Perimeter = 36 feet

c. Area = 36 square feet
Perimeter = 30 feet

d. Area = 36 square feet
Perimeter = 15 feet

8. What is the area of this rectangle? What is the perimeter of this rectangle?



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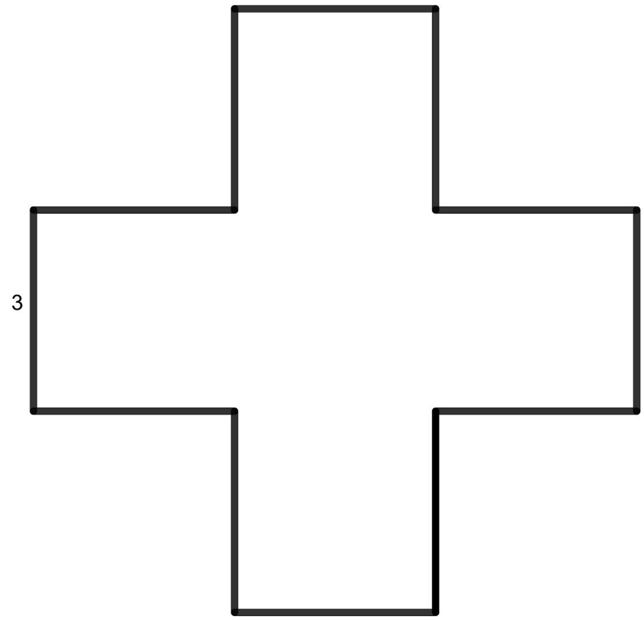
Use the figure below to answer questions 9 and 10.




All segments measure 3 feet in length.

9. What is the area of this figure?
    a. 27 square feet
    b. 36 square feet
    c. 45 square feet
    d. 56 square feet

10. What is the perimeter of the figure to the left? Explain how you know.

11. Imagine the area of a classroom is 240 square feet. Write an explanation of what 240 square feet means for someone who doesn't understand area.

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________



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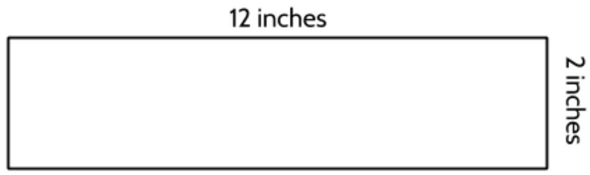
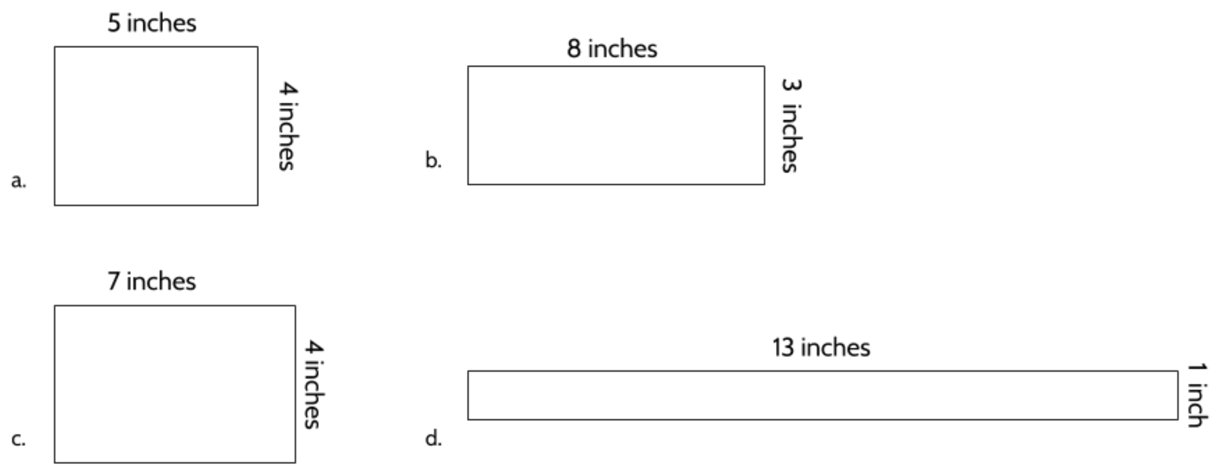
12. Look at the figure below.




Which of these rectangles has the same area but a different perimeter as the figure above?




13. The length of one side of a rectangle is 22 cm and its perimeter is 72 cm. What is the area of the rectangle? *<It might help to sketch a rectangle.>*

14. What is the greatest perimeter you can make with a rectangle that has an area of 24 square feet?



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15. Fill in this chart with the missing measurements.

| Rectangle | Longer Sides | Shorter Sides | Area | Perimeter |
|---|---|---|---|---|
| 1 | 10 cm | 6 cm | | |
| 2 | 12 cm | 5 cm | | |
| 3 | 4 in | | 12 $in^2$ | |
| 4 | 6 ft | | | 20 ft |
| 5 | | | 16 sq in | 20 in |
| 6 | | 2 cm | 30 sq in | |
| 7 | | 20 cm | | 100 cm |
| 8 | 8 ft | | 56 ft2 | |
| 9 | | | 24 sq in | 22 in |
| 10 | | | 100 sq feet | 58 ft |
| 11 | | | 144 sq cm | 48 cm |
| 12 | | | 180 $mi^2$ | 98 mi |
| 13 | 4.5 cm | 4 cm | | |
| 14 | | 2 in | | 21 in |



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## Area and Perimeter - Answer Key

1. The 30 stands for the number of squares it takes to cover the entire rectangle.
2. The word “square” is used because we are covering the entire rectangle with squares.
3. We use the word square inches, because each square measures an inch on each side.
4. 42 square inches.
5. 
    a. Area = 24 square centimeters. Perimeter = 22 centimeters.
    b. Area = 35 square centimeters. Perimeter = 24 centimeters.
6. 
    - Rectangle A - There are several different rectangles you might draw that could have an area of 28 sq cm. Some possible answers include: a rectangle that is 7 cm by 4 cm, a rectangle that is 14 cm by 2 cm.
    - Rectangle B - A rectangle that is 3cm by 5 cm has a perimeter of 16 cm. A rectangle that is 15 cm by 1 centimeters has a perimeter of 32 centimeters. Both have an area of 15 square cm.
    - Rectangle C - There are so many correct answers. The important thing is that whatever shape you have, it takes 10 square centimeters to cover that shape.
    - Rectangle D - A rectangle that is 9 cm by 4 cm has a perimeter of 26 centimeters and an area of 36 sq cm.
    - Rectangle E - This one is tricky. For a rectangle to have two sides that measure 2 cm and an area of 13 $cm^2$ you need the other sides to measure 6 and a half centimeters.
7. Choice C is the correct answer. Area = 36 square feet. Perimeter = 30 feet.

    Choice A has the correct numbers, but the area is given as 36 feet, which is a measure of length and not of area. Choice B has the measurement for area and perimeter reversed. Choice D has the correct area, but only half of the perimeter. Remember when calculating the perimeter, you need to add the length of every side, not just the sides that are given.

8. Area = 64 square miles. Perimeter = 40 miles.



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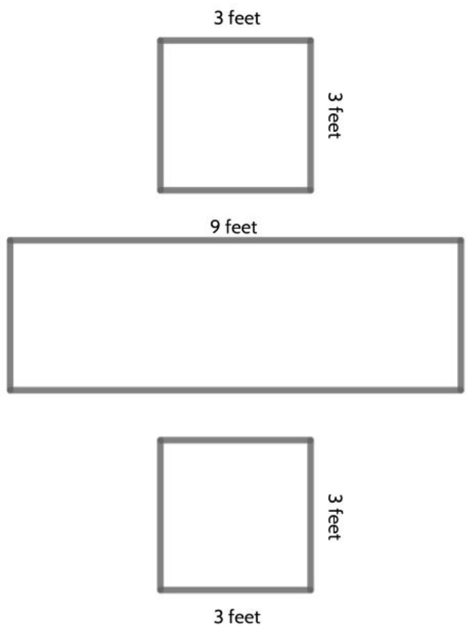
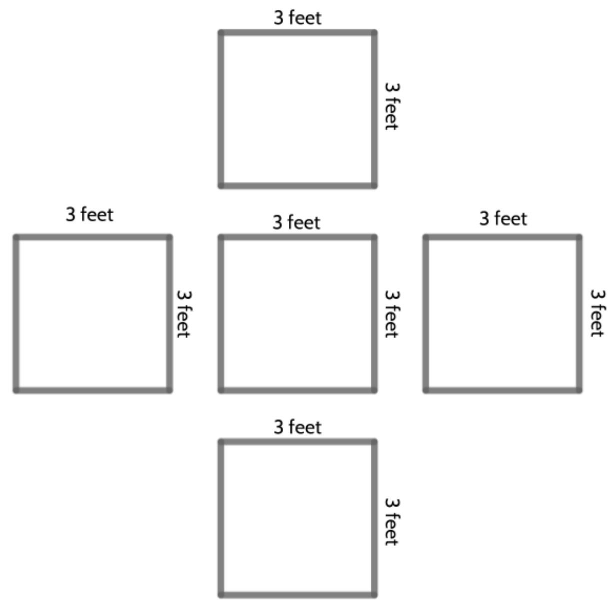
9. Choice C. There are a few different ways you might try to break up this shape. Here are two examples:




9 sq ft + 27 sq ft + 9 sq ft = 45 sq ft

9 sq ft + 9 sq ft + 9 sq ft + 9 sq ft + 9 = 45 sq ft

10. 36 feet. There are 12 sides to this figure and each side measures 3 feet in length.

11. If a room has an area of 240 square feet it means it would take 240 squares to cover the floor of that room, where each of those squares measures 1 foot on each side.

12. Choice B. A 12 x 2 rectangle has an area of 24 square inches. The 8 by 3 rectangle is the only other rectangle with an area of 24 square inches. *Be careful: The rectangle in Choice C has an area of 28 sq. in, and the 12 x 2 rectangle has a perimeter of 28 inches. The rectangle in Choice D has the same perimeter but a different area. But the question is asking for a rectangle with the same area and different perimeter.*

13. The area of the rectangle is 308 sq cm. We know one side measures 22 cm and the perimeter is 72 cm. If we sketch out a rectangle, we have 44 of the 72 cm we need for the perimeter. We need 28 more cm to make a perimeter of 72 cm. Since we have two sides, we need that 28 to be divided between those 2 sides. That means each shorter side is 14 cm. 14 cm *x* 22 cm is 308 sq cm.



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14.

| Possible Rectangle | **Shorter Side** | **Longer Side** | **Perimeter** |
|---|---|---|---|
| **1** | **1 foot** | **24 feet** | **50 ft** |
| 2 | 2 feet | 12 feet | 28 ft |
| 3 | 3 feet | 8 feet | 22 ft |
| 4 | 4 feet | 6 feet | 20 ft |

15.

| **Rectangle** | **Longer Sides** | **Shorter Sides** | **Area** | **Perimeter** |
|---|---|---|---|---|
| **1** | **10 cm** | **6 cm** | 60 sq cm | 32 cm |
| **2** | **12 cm** | **5 cm** | 60 sq cm | 34 cm |
| **3** | **4 in** | 3 in | **12 $in^2$** | 14 in |
| **4** | **6 ft** | 4 ft | 24 $ft^2$ | **20 ft** |
| **5** | 8 in | 2 in | **16 sq in** | **20 in** |
| **6** | 15 in | **2 in** | **30 sq in** | 34 in |
| **7** | 30 cm | **20 cm** | 600 $cm^2$ | **100 cm** |
| **8** | **8 ft** | 7 ft | **56 ft2** | 30 ft |
| **9** | 8 in | 3 in | **24 sq in** | **22 in** |
| **10** | 25 ft | 4 ft | **100 sq feet** | **58 ft** |
| **11** | 12 cm | 12 cm | **144 sq cm** | **48 cm** |
| **12** | 45 miles | 4 miles | **180 $mi^2$** | **98 miles** |
| **13** | **4.5 cm** | **4 cm** | 18 sq cm | 17 cm |
| **14*** | 8.5 in | **2 in** | 17 $in^2$ | **21 in** |



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