The Power of Exponents

Fast Track GRASP Math Packet

Part 1

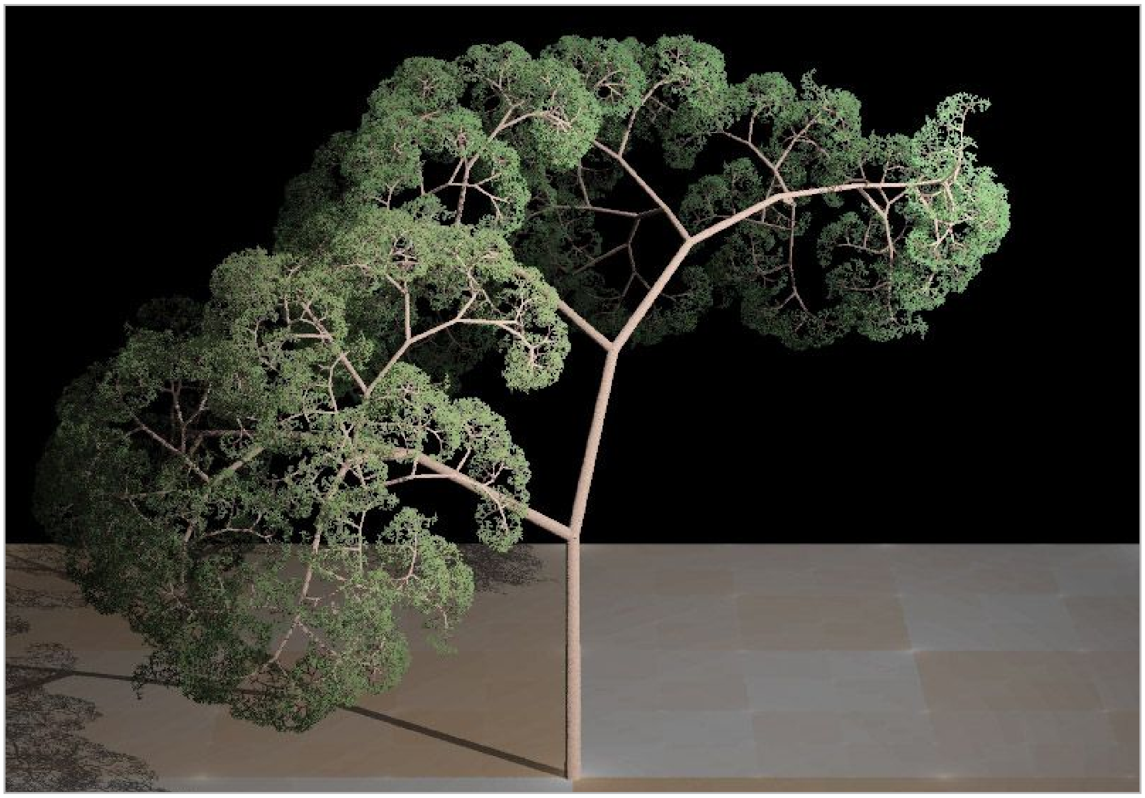

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Table of Contents

Welcome!

Congratulations on deciding to continue your studies! We are happy to share this study packet on exponents and roots. We hope that that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we often see on high school equivalency exams. If you study these topics carefully, while also practicing other basic math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn the math.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanation and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We also hope you will share what you learn with your friends and family. If you find something interesting in here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **http://www.acces.nysed.gov/hse/hse-prep-programs-maps**

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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Language and Skills Practice

Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

base · evaluate · exponent · factor · multiple · power · square root · variable

In this first activity, you will think about each word and decide how familiar you are with it. For example, think about the word "cube." Which of these statements is true for you?

- I know the word "cube" and use it in conversation or writing.
- I know the word "cube," but I don't use it.
- I have heard the word "cube," but I'm not sure what it means.
- I have never heard the word "cube" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "cube" might look when you're done:

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
cube	✓				like a box, all the sides are the same length

Complete the table on the next page.

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
base					
evaluate					
exponent					
factor					
multiple					
power					
square root					
variable					

Using Graphic Organizers to Learn Vocabulary

In order to learn math vocabulary, we need practice using words in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.

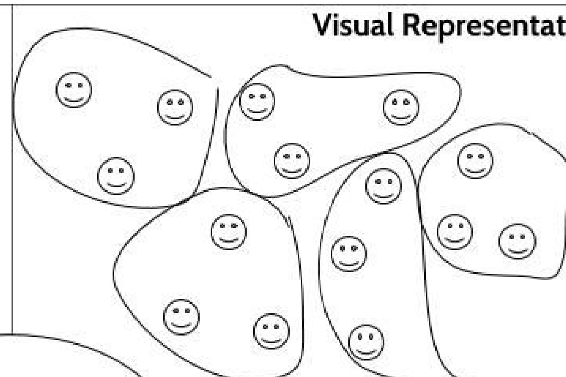
To start, choose a word from the packet and complete the graphic organizer:

- **What is the definition of the word?** You can look at the vocabulary review on page 87 for help. Write the definition in your own words to really make the word yours.
- **Make a visual representation.** You can make a drawing or diagram that will help you remember what the word means.
- **What are some examples of the word you're studying?** Below you can see that there are examples of *quotients*, which are the answers to division problems.
- **What are some non-examples of this word?** These are things that are **not** the word you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

What is it?

A quotient is the result of dividing one number by another. It is the answer to a division question.

Visual Representation



quotient

What are some examples?

15 divided by 3 equals **5**

66 ÷ 6 = **11**

63/18 = **3.5**

5, 11 and 3.5 are quotients in these calculations.

population ÷ area = **density**

What are some non-examples?

4 times 6 equals 24

18 + 5 = 23

17 – 2.5 = 14.5

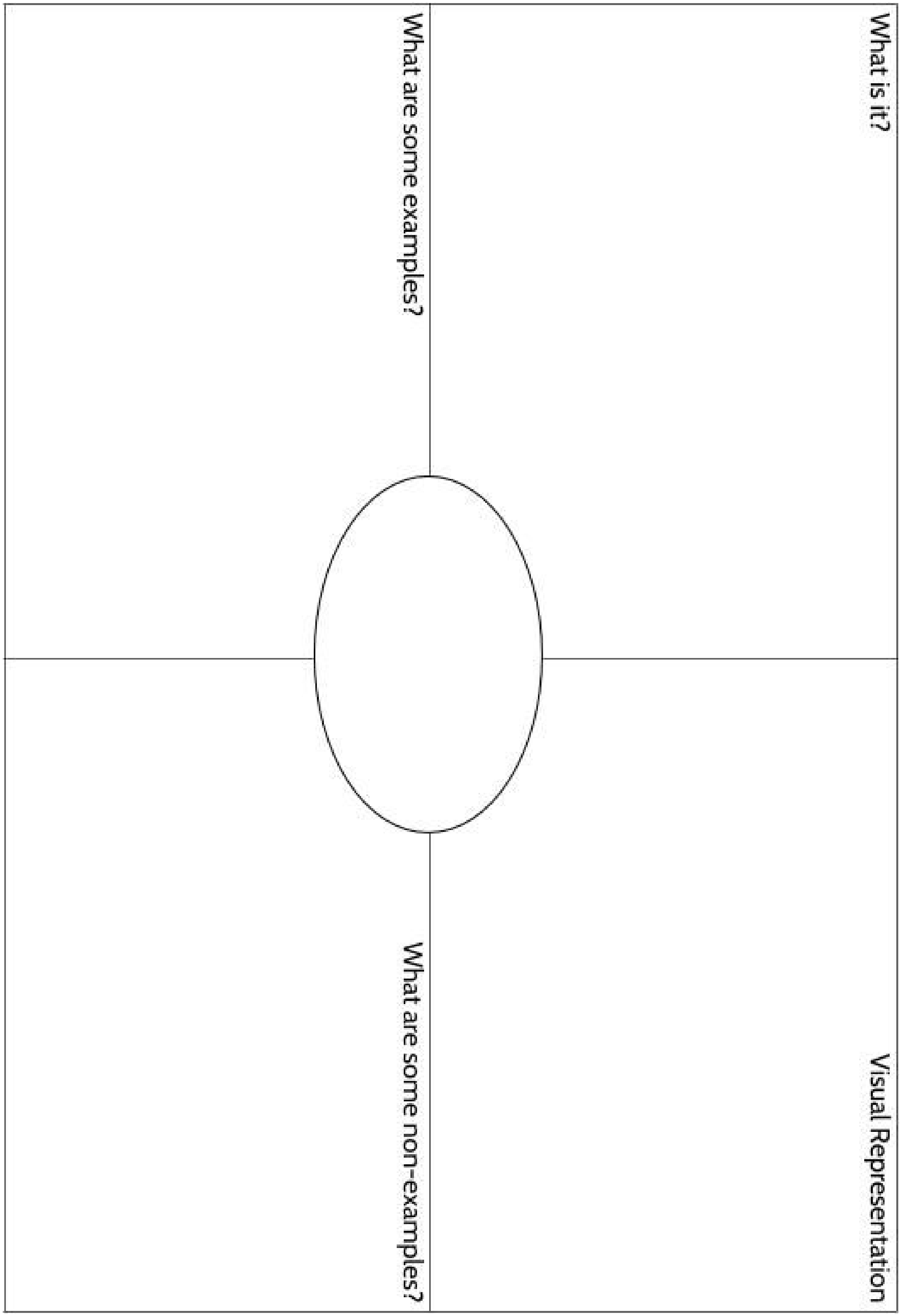
3.5 × 18 = 63

What is it?

What are some examples?

Visual Representation

What are some non-examples?


What is it?
What are some examples?
Visual Representation
What are some non-examples?

Why Exponents Matter

The math section of high school equivalency exams emphasize questions about exponents and roots. In this study packet, while learning about exponents and roots, you will break apart numbers and examine how they work. Then you will learn how to use exponents to solve problems. Finally, we will look at HSE practice questions involving exponents and roots.

But why should you care about exponents? Is it just because they are on the test? That might be the main reason you are studying them now, but exponents can also be useful when thinking about numbers in the real world. Exponents are tools to help us recognize patterns, find solutions, calculate with large and small numbers, and analyze numbers to understand science, finance and technology. Understanding how to use exponents will give you the power to make sense of scientific and financial situations.

In the next few pages, you will be introduced to problems that exponents help us explore. We'll come back to these problems later in the packet to understand them with exponents.

Counting Ancestors

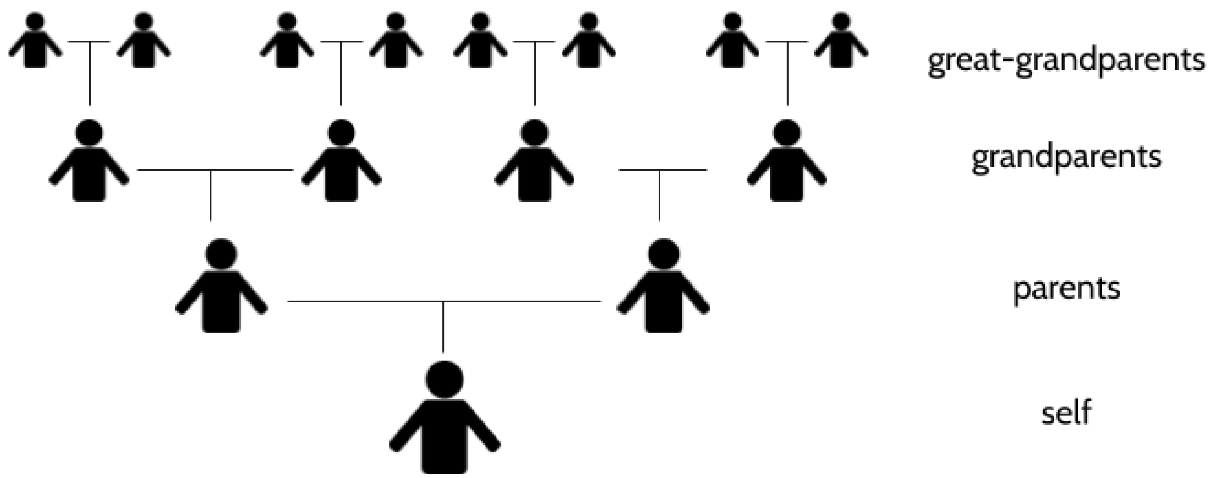
We each have two parents.[1] And each of our parents have two parents, which means that we have four grandparents. They may not be alive, and it's possible we never met them, but we have four grandparents who are our ancestors, biologically.

Your grandparents' parents are your great-grandparents. How many great-grandparents do we each have? How many great-great-grandparents? How would you figure it out?

Look at the following diagram and answer the questions below.

---

[1] There are many kinds of families. Not all families have two parents. Some families have one parent. Some families have two fathers or two mothers. And there are families with people who take on parenting roles in different ways. There are many ways to make a family. For this math problem, we are talking about biological parents.



1) What do you notice?

2) Complete the following table.

generation	number of people
self	1
parents	2
grandparents	4
great-grandparents	
great-great-grandparents	
great-great-great-grandparents	

You have probably noticed that the number of your ancestors in each generation doubles as you go back in time. You have 2 parents, 4 grandparents, 8 great-grandparents, 16 great-great-grandparents, etc.

What if you want to find out how many great-great-great-great-great-grandparents you have (also called *5th great-grandparents*)? You could multiply $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$. You have 128 5th great-grandparents!

Your *30th-great-grandparents* means your great-grandparents. They probably lived about 800 years ago.[2]

3) Take a wild guess at how many 30th great-grandparents you think you have.

   A. Less than five hundred

   B. Between five hundred and one thousand

   C. Between one thousand and a million

   D. More than one million

Did you try to calculate an answer? If you did, I hope you used a calculator! It's hard to multiply so many 2's, right? There has to be a faster way to calculate the answer.

Using exponents, you can quickly find the answer to a problem like this. By the time you finish this packet, you will be able to calculate the number of ancestors you have in any generation, even how many 30th great-grandparents you have!

[2] Genealogy is the study of ancestry. A genealogist looks at family records and finds connections between different family members to make a history of the family. As a rule of thumb, genealogists assume that there is an average of about 25 years between each generation. They would guess that there is about 25 years difference of age between parents and their children. This is only an average and isn't necessarily true for every family. For example, my mother is 29 years older than me and my father is 26 years older than me.

Really Large Numbers and Really Small Numbers

In science, we consider things in the universe that are really big, like the weight of the sun, the distance between the planets or the speed of light. We also study things that are really small, like the width of a blood cell, the size of a hydrogen atom or the speed that continents are moving on the surface of Earth. Here are some examples of those measurements in standard numbers:

Example	Measurement
Weight of the Sun	4,400,000,000,000,000,000,000,000,000,000 pounds
Distance between Earth and the Sun	93,000,000 miles
Speed of light	671,000,000 miles per hour
Width of a red blood cell	0.0003 inch
Diameter of an atom	0.000000004 inch
Speed of continental tectonic plates	0.000000006 miles per hour

What if you wanted to make some calculations with these measurements? For example, what is half the distance to the sun? How many miles would a tectonic plate move in 2 hours? How far does light travel in a year?

These are the kinds of calculations scientists often make, but they have a hard time working with numbers like these. The numbers are so long. It would be nice to be able to write the numbers in a shorter way, so that they are easier to organize on paper. It's also really hard to keep track of all those zeros. How can I make sure that I have the right number of zeros?

Scientists use exponents to keep their calculations accurate. In this packet, you will learn a method called scientific notation which uses exponents to handle large and small numbers.

Examples:

- The weight of the sun: 4.4×10^{30} pounds
- The speed of a tectonic plate: 6×10^{-9} miles per hour
- The speed of light: 6.71×10^{8} miles

Why Exponents Matter - Answer Key

1) There are different things you might notice. There is no wrong answer here.

2)

generation	number of people	
self	1	× 2
parents	2	× 2
grandparents	4	× 2
great-grandparents	8	× 2
great-great-grandparents	16	× 2
great-great-great-grandparents	32	× 2
great-great-great-great-grandparents	64	× 2
great-great-great-great-great-grandparents	128	

3) Amazingly, the answer is D) More than one million. Keep reading the packet to learn how this is possible.

Multiplication: Arrays, Factors and Multiples

Multiplication Practice

Understanding multiplication is essential to understanding exponents. Knowing your multiplication facts can also help you focus on other kinds of math while problem-solving. In the activities below, you will use multiplication facts and creative thinking to solve puzzles.

Look at the multiplication table below. You probably know many of the multiplication facts on this table. Circle the ones you already know.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

The numbers in the shaded boxes are called *factors*. You will learn more about factors later in the packet. The numbers you write in the white boxes are called *products* and are the results of multiplication.

$$\underbrace{3}_{\text{factor}} \times \underbrace{4}_{\text{factor}} = \underbrace{12}_{\text{product}}$$

1) Complete this 1-5 multiplication table.

×	1	2	3	4	5
1					
2					
3					
4					
5					

2) Look at these diagonal numbers (1, 4, 9, 16, 25). Do you notice anything special about these numbers?

3) Complete the mixed-up 1-5 multiplication table below. A few have been done for you.

×	5	3	1	4	2
2	10				
4					8
1		3			
5					
3				12	

The table below is called a Find the Factors[3] 1-5 puzzle. The goal is to write the numbers 1 through 5 in the correct shaded boxes, then use those factors to fill in the missing products. To solve the puzzle, look at each given product and think about what two numbers can be multiplied to get that number.

For example, to get 16, you can multiply 1 × 16 or 4 × 4. The number 16 isn't between 1 and 5, so it can't be one of the factors. The two numbers connected to 16 must both be 4.

4) Try it out. Use a pencil!

×	4				
					3
4	16		20		
		2			
			15		
				10	

Answer these questions after you have tried to solve the puzzle above.

5) What strategies did you use to figure out where to put the numbers 1 through 5 (the factors)?

If you're stuck: There are several ways to approach this puzzle. On the next page are the steps that I took to get started with this 1-5 puzzle.

[3] The Find the Factors puzzles in this section are inspired by or are from findthefactors.wordpress.com, by Iva Sallay. Non-commercial use/copying permitted.

×	4		5		
					3
4	16		20		
		2			
			15		
				10	

- With 4 on the left side, we now know that 4 × ? = 20. What can you multiply by 4 to get 20?
- Write the 5 in the factor row above.

×	4		5		
					3
4	16		20		
		2			
3	12		15		
				10	

- With the 5 on the factor row above, we now know that 5 × ? = 15. What can you multiply by 5 to get 15?
- Write the 3 on the left side.
- Now that we have 3 on the left and 4 above, we can multiply the two numbers to get 12.

×	4		5	2	
					3
4	16		20	8	
		2			
3	12		15	6	
5				10	

- Using the numbers 1 through 5, there is only one way to get a product of 10:

  2 × 5 = 10

- Do we put the 5 or the 2 on the line above? Since there is already a 5 on the factor row above, we can't put the 5 there. We can't repeat numbers on the shaded factor line. So the 2 must go above. That means the 5 will go on the left.
- Write the 2 above and the 5 on the left.
- With the 2 above, we can multiply by 4 to get 8 and by 3 to get 6.

Can you use the hints above to finish the rest of the puzzle now? The goal is fill every square with the right number.

6) When you're finished, try the puzzles below. This is called a Find the Factors 1-10 puzzle. Like the 1-5 puzzle, the goal is to write the correct number in each of the squares below. Use only one of each factor 1 through 10 in each shaded area. Use a pencil!

Example: The two numbers that multiply to make 36 must be 6 and 6, since there is no other way to get 36 by multiplying numbers between 1 and 10.

×	6									
						1				
							25			
									100	
6	36									
		49								
			4							
										81
								16		
					64					

7) What strategies helped you solve this puzzle? What was challenging?

8) Try this level ONE Find the Factors 1-12 puzzle. Write in the numbers 1 through 12 in the correct boxes vertically and horizontally.

×												
											1	
										144		
									4			
			121									
		81										
							9					
						100						
	64											
								25				
					36							
				16								

9) What strategies helped you solve this puzzle? What was challenging?

10) This is a level TWO Find the Factors 1-10 puzzle.

×										
						14				
						4				
	3	9	27		18		15	21	24	12
						12				
						2				
						18				
						16				
						10				
						20				

11) What strategies helped you solve this puzzle? What was challenging?

12) For a real challenge, try this level THREE 1-10 puzzle. Use a pencil! You will probably need to do some erasing or crossing out.

×										
	30	36								
									45	
					27					
			6							
										14
								28		
						32				
				8						
		60								

For many more Find the Factors puzzles, go to http://findthefactors.wordpress.com.

Working on these puzzles is a good way to practice the multiplication times tables, your problem-solving skills and your understanding of factors.

Arrays

An *array* is an arrangement of objects in columns and rows. You can often see arrays in the world, such as this pan of muffins.

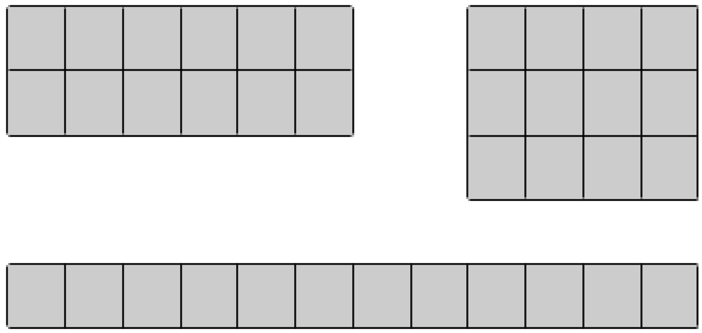
13) How many muffins are there? How do you know?

This tray of eggs is arranged in an array as well.

14) How many eggs are there? How do you know?

15) Look around you. What arrays do you see?

These shaded squares are arranged in three different arrays. Each represents the number 12.

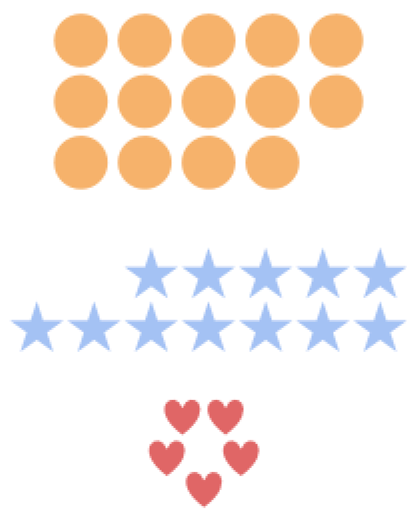


You might have noticed that each of the arrays above is in the shape of a rectangle. For that reason, they are sometimes called *rectangular arrays*.

If the rows and columns don't have an equal number of objects, it is not a rectangular array.

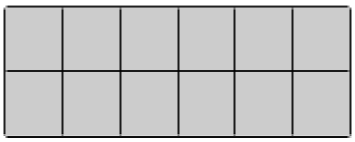
rectangular arrays

not rectangular arrays



Arrays can help us think about multiplication. They can also help us figure out the area of a shape. When we talk about the area of something we mean how many squares it would take to cover its surface. The length of a rectangle (or rectangular array) multiplied by its width gives the area (the number of squares).

Array A is 2 squares down and 6 squares across. 2×6 (or 6×2) is 12, so we can say the area is 12. You can count the number of squares to make sure.

A.


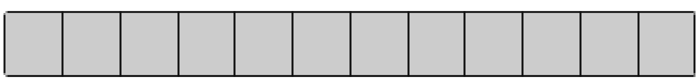
$2 \times 6 = 12$

$6 \times 2 = 12$

There are several different symbols for multiplication. An asterisk (*) or dot (·) is sometimes used to mean multiplication. Parentheses can also be used to indicate multiplication. Each of the following five equations all mean the same thing: *2 multiplied by 6*.

$2 \times 6 = 12$ $2 * 6 = 12$ $2 \cdot 6 = 12$ $(2)(6) = 12$ $2(6) = 12$

Array B could represent 1×12 (or 12×1).

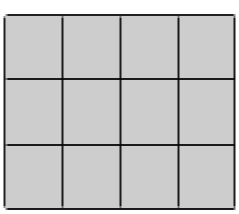
B.


Each equation below gives the number of the squares in the area of the long rectangle.

$1 \cdot 12 = 12$

$12 \cdot 1 = 12$

16) Look at array C. How could you calculate the area of the rectangle?

C.


____ × ____ = ____

____ × ____ = ____

Factors

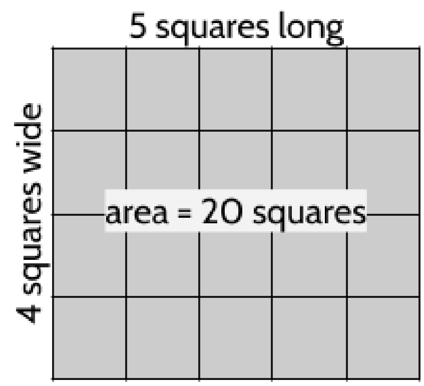
A factor is a number that can be multiplied to get another number. The numbers 1, 2, 3, 4, 6, and 12 are considered *factors* of 12. For example, 3 is a factor of 12 since 3 can be multiplied by 4 to get 12.

Factors are numbers that are multiplied to make a *product.* 4 and 5 are both factors of 20.

factor factor product

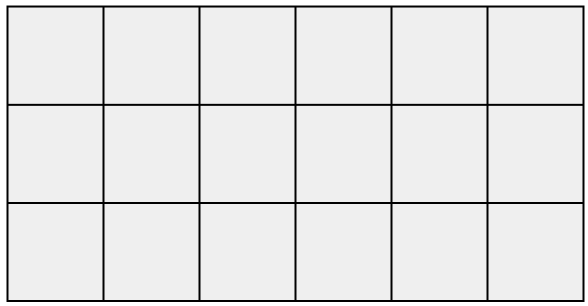
$$4 \cdot 5 = 20$$

The length of the sides of a rectangle are factors of the rectangle's area.



You can also see factors in the multiplication table. The numbers in the shaded boxes are factors. The numbers in the white boxes are products.

×	1	2	3	4	**5**	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	**20**	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36



17) If the area is of a rectangle is 18 squares and the length of one side is 3 squares long, what is the length of the other side?

18) If the area is of a rectangle is 28 squares and the length of one side is 4 squares long, what is the length of the other side?

Use the multiplication table to answer the questions below.

19) 5 × _____ = 30

20) 4 * 3 = _____

21) 24 = 6 · _____

22) _____ = 3 × 6

23) 16 = (4)(_____)

24) _____ · 5 = 20

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Division is another way to think about factors. A factor can be divided into a whole number to get an answer with no *remainder* (a number left over after division).

Example	Conclusion
12 ÷ 3 = 4	3 is a factor of 12
12 ÷ 6 = 2	6 is a factor of 12
20 ÷ 4 = 5	4 is a factor of 20
14 ÷ 4 = 3.5	4 is **not** a factor of 14
9 ÷ 6 = 1.5	6 is **not** a factor of 9

Something to remember: Factors are whole numbers such as 1, 3, 12, 0, and 451. A whole number is a number that can be written without using a fraction or a decimal. For example, ⅔ and 3.5 are not whole numbers. If you divide a number by one of its factors, you will get a whole number factor as an answer.

25) Is 7 a factor of 63? Please explain how you know.

26) Is 8 a factor of 20? Please explain how you know.

Multiples

Fill in the next three numbers in the following sequences.

	Sequence	What patterns do you see?
27)	2, 4, 6, ______, ______, ______,...	
28)	3, 6, 9, ______, ______, ______,...	
29)	5, 10, 15, ______, ______, ______,...	
30)	10, 20, 30, ______, ______, ______,...	

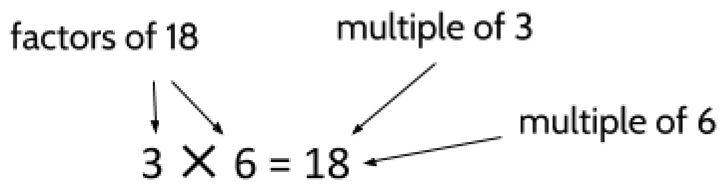
In the first series of numbers, you may have noticed that the numbers are going up by 2. Another way to look at this pattern is to see that the numbers are made by multiplying 2's.

$2 \cdot 1 = \mathbf{2}$ $\quad 2 \cdot 2 = \mathbf{4}$ $\quad 2 \cdot 3 = \mathbf{6}$ $\quad 2 \cdot 4 = \mathbf{8}$ $\quad 2 \cdot 5 = \mathbf{10}$ $\quad 2 \cdot 6 = \mathbf{12}$

The numbers 2, 4, 6, 8, 10 and 12 are *multiples* of 2. When you multiply 2 by another whole number, you will get a multiple of 2. Multiples of 2 are *even* numbers.

You can think of a multiple as the result of multiplication, similar to a product. The number 18 is a multiple of 3 because 3 six times is 18. This is because you can multiply 3 by a whole number to get 18. The number 18 is also a multiple of 6 because 6 three times is 18.

multiples of 3	**multiples of 6**
3, 6, 9, 12, 15, **18**, 21, 24, 27, 30, etc.	6, 12, **18**, 24, 30, 36, 42, 48, 54, 60, etc.



You might have heard multiples referred to as the "times table" for a particular number. For example, the multiples of 3 are sometimes called the 3 times table. Calling these numbers the multiples of 3 is more precise.

Math language: You can say, "The product of 3 and 6 is 18." Or you might say, "The product is 18." When you use the word *multiple*, you have to say what number is being multiplied. For example, you could say, "18 is a multiple of 6," or "All even numbers are multiples of 2."

31) Is 45 a multiple of 5? Please explain your answer.

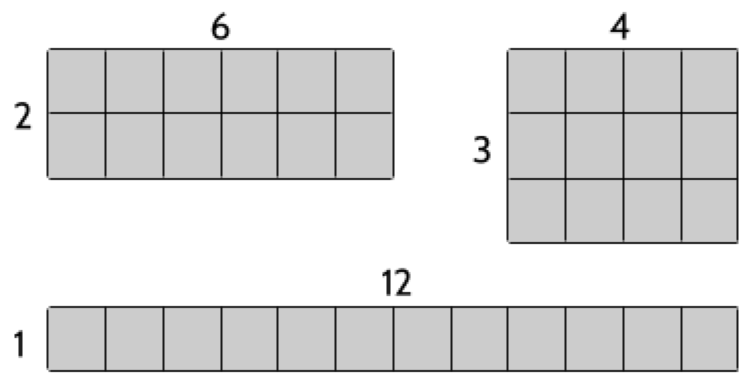
32) Is 62 a multiple of 12? Explain.

33) Is 5 a multiple of 25? Explain.

34) Is 12 a factor of 48? Explain.

Practice with Arrays, Factors and Multiples

The arrays below show all the factors of 12. There are six factors and three factor pairs.



35) The complete list of factors for 12 is 1, 2, ____, ____, 6, & 12.

Two factors that multiply together to make a number are called a *factor pair*. 4 and 5 are a factor pair of 20, since $4 \cdot 5 = 20$.

36) How many factor pairs does 12 have?

37) Which of the following are <u>not</u> factor pairs of 12? Circle more than one answer.

A. 1 and 12

B. 2 and 4

C. 3 and 4

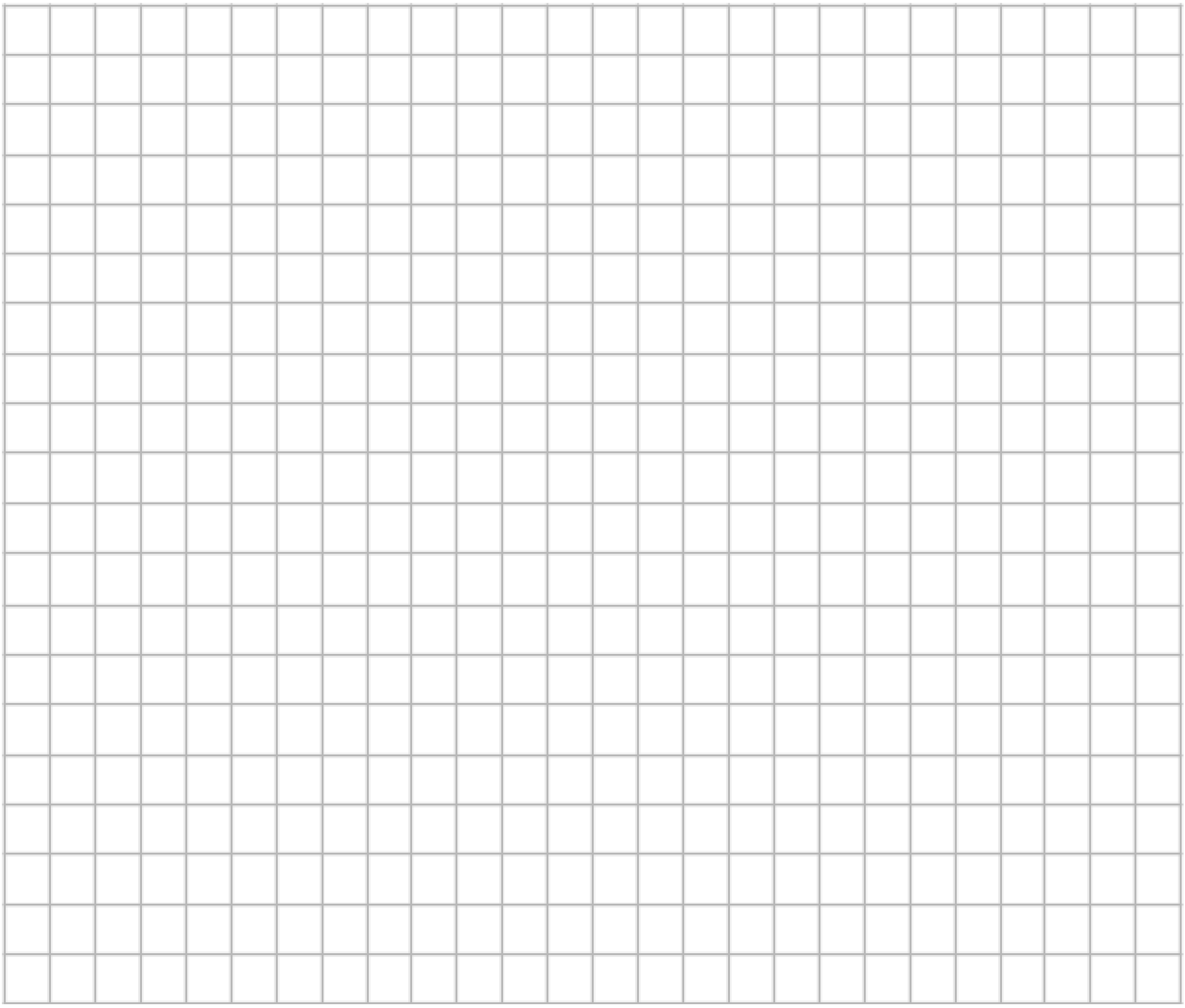
D. 3 and 6

E. 4 and 3

F. 4 and 6

38) 12 is a multiple of what 6 numbers? _____ _____ _____ _____ _____ _____

39) Shade in squares to make as many different rectangular arrays as you can to represent the number 24.



40) Show all the ways that you can multiply two whole numbers to get 24. Use the arrays you drew on the grid above to find them all. The first factor pair is done.

<u>1</u> × <u>24</u> ___ × ___

___ × ___ ___ × ___

41) The complete list of factors for 24 is 1, ___, ___, ___, ___, ___, ___, & 24.

42) How many factors does 24 have? ________

43) In the grid below, create as many arrays as you can to represent the numbers 4, 5, 10, 11, 13, 14 and 15. Label each array with the factors and the number. One array for 4 was done for you.

Make more than one array for each number if you can.

44) Were you able to make more than one array for some of the numbers? Which ones?

The length of the sides of the rectangles are factors. The two lengths of a rectangle make a factor pair.

45) Using the arrays you drew on the previous page, complete this table. The first row is done for you.

Number	Factor Pairs	Factors
4	$2 \cdot 2$ and $1 \cdot 4$	1, 2, 4
5		
10		
11		
13		
14		
15		

46) Choose the true statement below.

A. All numbers have more than two factors.

B. All numbers that have more than two factors are even.

C. Some numbers have only two factors, 1 and the number itself.

D. Some whole numbers can't be drawn as a rectangular array.

E. Larger numbers always have more factors.

47) Which of the following is <u>not</u> a factor of 36?

A. 2
B. 6
C. 9
D. 12
E. 15
F. 36

Multiplication: Arrays, Factors and Multiples - Answer Key

1) You can check the multiplication if you're unsure of any answers.

2) They are made by multiplying a number by itself: 1 × 1, 2 × 2, 3 × 3, etc.

3)

×	5	3	1	4	2
2	10	6	2	8	4
4	20	12	4	16	8
1	5	3	1	4	2
5	25	15	5	20	10
3	15	9	3	12	6

4)

×	4	1	5	2	3
1	4	1	5	2	3
4	16	4	20	8	12
2	8	2	10	4	6
3	12	3	15	6	9
5	20	5	25	10	15

5) Are your strategies similar to the ones on the next page?

6)

×	6	7	2	3	8	1	5	4	10	9
1	6	7	2	3	8	1	5	4	10	9
5	30	35	10	15	40	5	25	20	50	45
10	60	70	20	30	80	10	50	40	100	90
6	36	42	12	18	48	6	30	24	60	54
3	18	21	6	9	24	3	15	12	30	27
7	42	49	14	21	56	7	35	28	70	63
2	12	14	4	6	16	2	10	8	20	18
9	54	63	18	27	72	9	45	36	90	81
4	24	28	8	12	32	4	20	16	40	36
8	48	56	16	24	64	8	40	32	80	72

7) One strategy is to look at each product in the table and think about what two numbers can be multiplied to get that number. For example, to get 49, you can multiply 1 × 49 or 7 × 7. The number 49 isn't between 1 and 10, so the numbers in the shaded rows connected to 49 must be 7.

8)

×	8	9	11	4	6	10	3	5	2	12	1	7
1	8	9	11	4	6	10	3	5	2	12	1	7
7	56	63	77	28	42	70	21	35	14	84	7	49
12	96	108	132	48	72	120	36	60	24	144	12	84
2	16	18	22	8	12	20	6	10	4	24	2	14
11	88	99	121	44	66	110	33	55	22	132	11	77
9	72	81	99	36	54	90	27	45	18	108	9	63
3	24	27	33	12	18	30	9	15	6	36	3	21
10	80	90	110	40	60	100	30	50	20	120	10	70
8	64	72	88	32	48	80	24	40	16	96	8	56
5	40	45	55	20	30	50	15	25	10	60	5	35
6	48	54	66	24	36	60	18	30	12	72	6	42
4	32	36	44	16	24	40	12	20	8	48	4	28

9) You might start with numbers that can be made with multiplication in just a few ways. For example, 25 is 5 × 5 . It is also 1 × 25 and 25 × 1, but these require numbers bigger than 10.

10)

×	1	3	9	10	6	2	5	7	8	4
7	7	21	63	70	42	**14**	35	49	56	28
2	2	6	18	20	12	**4**	10	14	16	8
3	**3**	**9**	**27**	30	**18**	6	**15**	**21**	**24**	**12**
6	6	18	54	60	36	**12**	30	42	48	24
1	1	3	9	10	6	**2**	5	7	8	4
9	9	27	81	90	54	**18**	45	63	72	36
8	8	24	72	80	48	**16**	40	56	64	32
4	4	12	36	40	24	8	20	28	32	16
5	5	15	45	50	30	**10**	25	35	40	20
10	10	30	90	100	60	**20**	50	70	80	40

11) One strategy is to look for numbers that have the same factor. For example, the numbers 3, 9, 27, 18, 15, 21, 24 and 12 are all the result of 3 multiplied by another number. So the shaded box on the left side must have a 3 for that row.

12)

×	5	6	2	1	3	8	10	4	9	7
6	**30**	**36**	12	6	18	48	60	24	54	42
5	25	30	10	5	15	40	50	20	**45**	35
9	45	54	18	9	**27**	72	90	36	81	63
3	15	18	**6**	3	9	24	30	12	27	21
2	10	12	4	2	6	16	20	8	18	**14**
7	35	42	14	7	21	56	70	**28**	63	49
4	20	24	8	4	12	**32**	40	16	36	28
8	40	48	16	**8**	24	64	80	32	72	56
1	5	6	2	1	3	8	10	4	9	7
10	50	**60**	20	10	30	80	100	40	90	70

13) You could count or multiply 3 rows × 4 columns to get 12 muffins.

14) You could count or multiply 5 rows × 4 columns to get 20 eggs.

15) What do you see with rows and columns?

16) 3 × 4 = 12 and 4 × 3 = 12

17) 6

18) 7

19) 6

20) 12

21) 4

22) 18

23) 4

24) 4

25) Yes. 7 is a factor of 63 because 7 × 9 equals 63. The number 9 is an whole number.

26) No. Even though 8 × 2.5 = 20, the number 8 is not a factor of 20 because 2.5 isn't a whole number.

27) 2, 4, 6, **8, 10, 12**, ... The numbers are going up by 2. They are all multiples of 2: 2 × 1, 2 × 2, 2 × 3, etc.

28) 3, 6, 9, **12**, **15**, **18**, ... The numbers are going up by 3. They are all multiples of 3: 3 × 1, 3 × 2, 3 × 3, etc.

29) 5, 10, 15, **20**, **25**, **30**, ... The numbers are going up by 5. They are all multiples of 5: 5 × 1, 5 × 2, 5 × 3, etc.

30) 10, 20, 30, **40**, **50**, **60**, ... The numbers are going up by 10. They are all multiples of 10: 10 × 1, 10 × 2, 10 × 3, etc.

31) Yes, because 5 × 9 = 45. Also, 45 divided by 5 equals 9, with no remainder.

32) No, because 62 divided by 12 equals 5.167. There is no whole number that can be multiplied by 12 to get 62.

33) No. 5 is a **factor** of 25. The number 25 is a multiple of 5.

34) Yes, because 12 × 4 = 48. The number 48 is a multiple of 12.

35) 1, 2, **3**, **4**, 6, 12

36) Three: 1 × 12, 2 × 6, and 3 × 4

37) B, D, F

38) 12 is a multiple of 1, 2, 3, 4, 6, and 12.

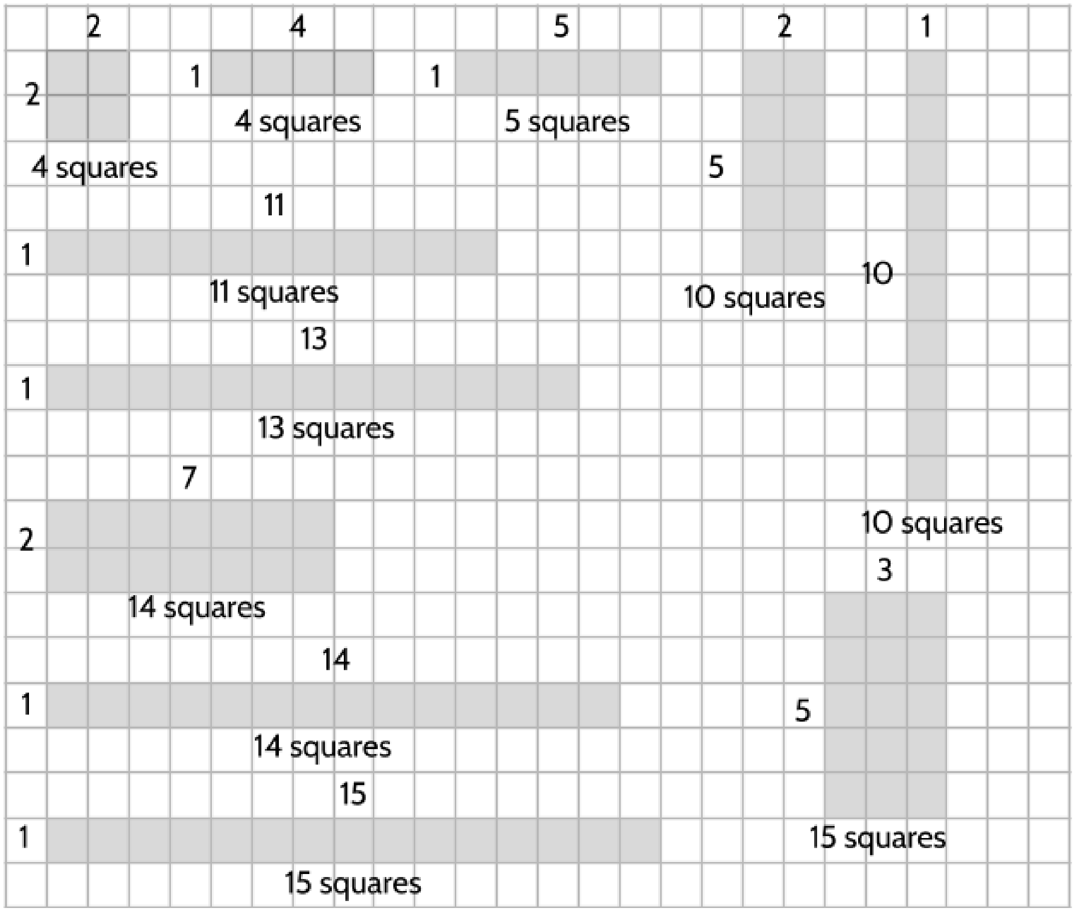
39) You should be able to draw 4 different rectangles: 1 by 24, 2 by 12, 3 by 8, and 4 by 6

40) 1 × 24, 2 × 12, 3 × 8, 4 × 6

41) 1, 2, 3, 4, 6, 8, 12, & 24

42) 8

43)



44) It is possible to make more than one rectangular array for the numbers 4, 10, 14 and 15. There is only one array that you can make for each of the numbers 5, 11 and 13.

45)

Number	Factor Pairs	Factors
4	1 · 4 and 2 · 2	1, 2, 4
5	1 · 5	1, 5
10	1 · 10 and 2 · 5	1, 2, 5, 10
11	1 · 11	1, 11
13	1 · 13	1, 13
14	1 · 14 and 2 · 7	1, 2, 7, 14
15	1 · 15 and 3 · 5	1, 3, 5, 15

46) C

47) E

Breaking Down Numbers

We will start this section with a review of factors.

1) Find all the factors for each of the numbers below.

   Ask yourself: What two numbers can I multiply to get _____? Are there any other ways?

   If it's helpful, you can use the multiplication table on the next page. Find each number on the multiplication table. Does it appear more than once?

Number	Factors
1	
2	
3	
4	1, 2, 4
5	1, 5
6	
7	
8	
9	
10	1, 2, 5, 10
11	1, 11
12	1, 2, 3, 4, 6, 12
13	1, 13
14	1, 2, 7, 14
15	1, 3, 5, 15
16	
17	
18	

Number	Factors
19	
20	
21	
22	
23	
24	1, 2, 3, 4, 6, 8, 12, 24
25	
26	
27	
28	
29	
30	
31	
32	
33	
34	
35	
36	

You may want to use this 15 × 15 multiplication table while working on the previous page.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	7	14	**21**	28	35	42	49	56	63	70	77	84	91	98	105
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Remember, the numbers in the shaded boxes are factors and the numbers in the unshaded boxes are products.

2) Which numbers from 1 to 36 have the most factors?

3) Which number has exactly 1 factor?

4) Which numbers have exactly 2 factors?

5) Which numbers have an odd number of factors?

In the next section, we will look at numbers that have exactly 2 factors and numbers that have more than 2 factors.

Prime Numbers

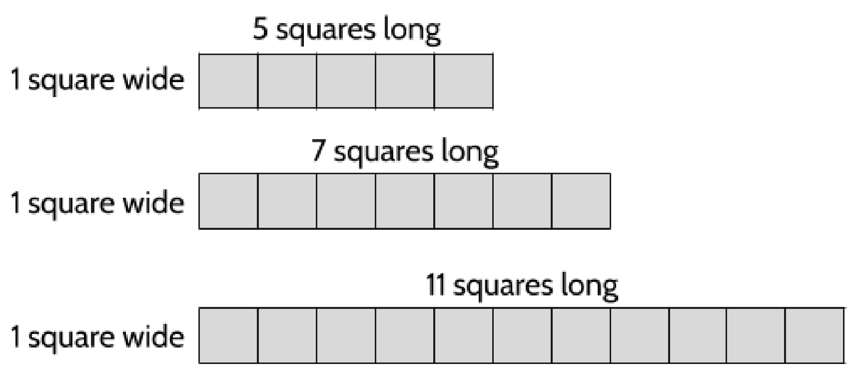
The numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29 are the first ten *prime* numbers. A prime is a number with only two factors, 1 and itself. This means it cannot be divided into smaller whole numbers. For example, 7 is *divisible* only by 7 and 1. These are the only two whole numbers that divide into 7 with no remainder.

7 ÷ 1 = 7	7 ÷ 4 = 1.75	**7 ÷ 7 = 1**
7 ÷ 2 = 3.5	7 ÷ 5 = 1.4	7 ÷ 8 = 0.875
7 ÷ 3 = 2.33...	7 ÷ 6 = 1.166...	7 ÷ 9 = 0.77...

Other than 1 and 7, there are no other whole numbers you can use to divide 7 to get a whole number answer.

You can use rectangles to think about prime numbers. Prime numbers can only be made into one kind of rectangle. Here are a few prime numbers as rectangles.

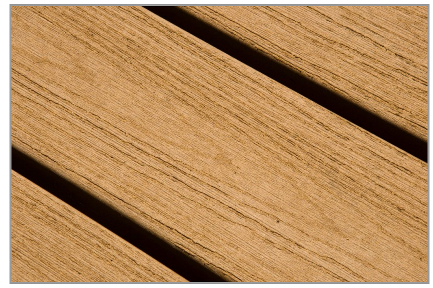
To make a rectangle with a prime number of squares, make the rectangle 1 square wide and the prime number of squares long. There is no other way to do it.



Check Your Understanding: What are the next three prime numbers after 29?

Composite Numbers

The numbers 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 are the first ten *composite* numbers. The word "composite" means something made from different parts. For example, composite wood is made in a factory from a combination of sawdust, natural fiber and plastic.



This looks like wood, but it's actually a *composite* of natural fibers and plastic.

A composite number is any number with more than 2 factors. For example, 14 is a composite number because it has four factors (1, 2, 7 and 14), which means that it can be divided by each of these numbers to get a whole number answer.

14 ÷ 1 = 14

14 ÷ 2 = 7

14 ÷ 3 = 4.66…

14 ÷ 4 = 3.5

14 ÷ 5 = 2.8

14 ÷ 6 = 2.33…

14 ÷ 7 = 2

14 ÷ 8 = 1.75

14 ÷ 9 = 1.55…

14 ÷ 10 = 1.4

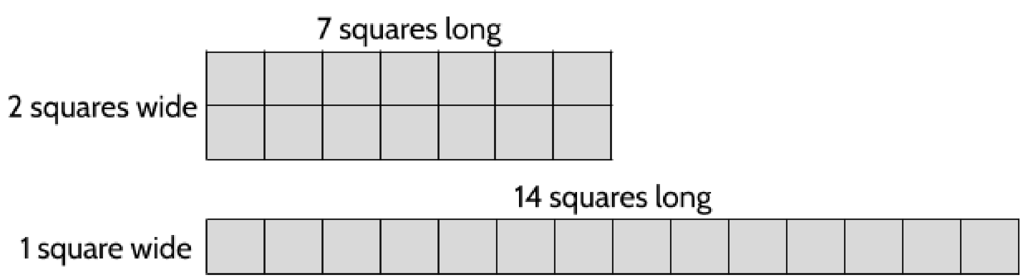
14 ÷ 11 = 1.2727…

14 ÷ 12 = 1.66…

14 ÷ 13 = 1.08

14 ÷ 14 = 1

You can use rectangles to think about composite numbers. Composite numbers can be made into at least two different rectangles. Two rectangles for 14 are below. The side lengths of the two rectangles represent the different factors of 14.



You can also see the factor pairs of 14 as the sides of these two rectangles. The two factor pairs are 2 & 7 and 1 & 14.

Composite numbers have three or more factors.

6) Complete this table.

Number	Prime or Composite?	How do you know?
55	Composite	It has four factors: 1, 5, 11 and 55.
21		
43		
49		
37		

7) Which of these numbers are composite? Choose more than one.

A. 2
B. 3
C. 6
D. 9
E. 17
F. 21

The number 1 is unique because it only has 1 factor. All other whole numbers have at least 2 factors. The number 1 is sometimes called a *unit*. It is not prime or composite.

Answer to Check Your Understanding: The next three prime numbers after 29 are 31, 37, and 41. There are many, many prime numbers. In fact, the prime numbers are infinite and go on forever. Some mathematicians spend their careers searching for new prime numbers. All the small ones have been found. A recently discovered prime number has almost 25 million digits. It would take about 8,000 pages to type this prime number!

Prime Factorization

Prime numbers can be thought of as the building blocks of all other numbers. The word "prime" comes from the Latin word *prima*, which means "first." All whole numbers can be written as the product of prime numbers.

For example, the number 6, which is a composite number, is made of the two prime numbers 2 and 3, since 2 × 3 = 6. The number 15 is made of the prime numbers 3 and 5, since they can be multiplied to get 15. All composite numbers can be made with prime numbers. It's like the prime numbers are hiding inside the composite numbers!

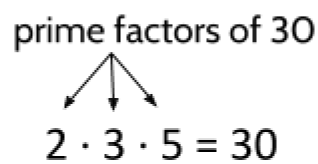
How can we find the prime numbers are "hidden inside" composite numbers?

Let's look at the number 30. You found out earlier that it has the factors 1, 2, 3, 5, 6, 10, 15, and 30. Some of these factors are prime and some are composite. The *prime factors* are 2, 3 and 5. The other numbers are composite.

The numbers 2, 3 and 5 are called the *prime factors* of 30 because:

- 2, 3, and 5 are each **factors** of 30: Each of these numbers can be divided into 30 with no remainder.
- 2, 3 and 5 are **prime** numbers: They each have only two factors and can't be divided into smaller whole numbers.

2, 3 and 5 is also called the *prime factorization* of 30. It's like a secret code for the original number. If you multiply all the prime factors of a number together, you will get the number.



It doesn't matter which order you multiply the prime factors. You should always get the original number in the end.

(2 × 3) × 5	(2 × 5) × 3	(3 × 5) × 2
6 × 5	10 × 3	15 × 2
30	30	30

Did you notice how the composite factors of 30 show up in the calculations above? Circle the 6, 10, 15, and 30.

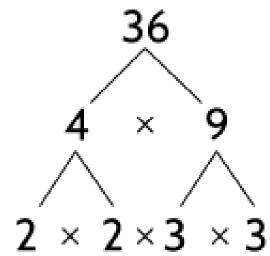
One way to find the prime factors of a number is to 1) find all the factors of the number and then 2) choose the factors that are prime numbers.

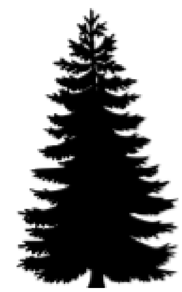
Another method for finding prime factors is called a *factor tree*. In this method, you use factor pairs to break up a number until all the numbers are prime. When you're done, the diagram looks similar to a tree.

★ This example shows how the prime factorization of 36 is done with a factor tree.

1. Write the number you're factoring at the top.
2. Write one of the factor pairs of that number. It doesn't matter which.
3. If one of the factors in the pair is a composite number, write a factor pair for that number below.
4. Continue until all the numbers at the end of the "branches" are prime.
5. The numbers on the bottom of the tree are the prime factors. For example:

$$2 \times 2 \times 3 \times 3 = 36$$

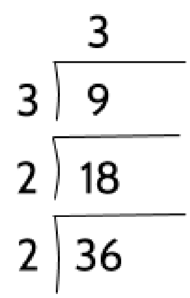


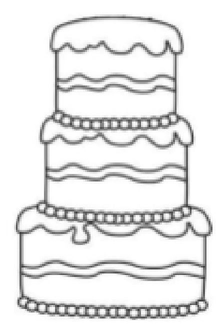


Another method to find the prime factorization of a number is with the *cake method*. In this method, you keep dividing a number by its prime factors until all the answers are prime. When you're finished, it looks a little bit like a cake.

★ This example shows how the prime factorization of 36 is done with the cake method.

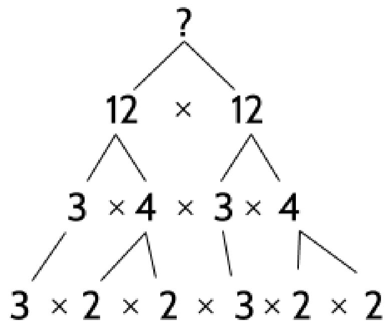
1. Write the number at the bottom under a division symbol.
2. Divide the number by one of its prime factors. Example: 36 ÷ 2 = 18.
3. If the answer isn't prime, divide it by one of its prime factors. Example: 18 ÷ 2 = 9.
4. Repeat until your final answer is a prime number. Example: 9 ÷ 3 = 3.
5. The numbers up the left side and on top are the prime factors. Example: 2 · 2 · 3 · 3 = 36



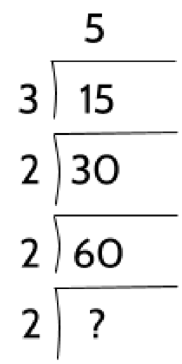


Either the factor tree method or the cake method will work for finding the prime factors of a number. You may also have your own method. Try them all and practice the one that works better for you.

8) What number is represented by the factor tree below?



9) What number is represented by the cake method below?



Using a factor tree, the cake method or a method of your own, complete the prime factorization for the numbers below. The prime factorization for prime numbers is just the number itself because the number 1 is not a prime number.

10) 60

12) 31

14) 64

11) 45

13) 56

15) 90

16) Which prime factorizations above include a prime number only multiplied by itself?

Common Factors

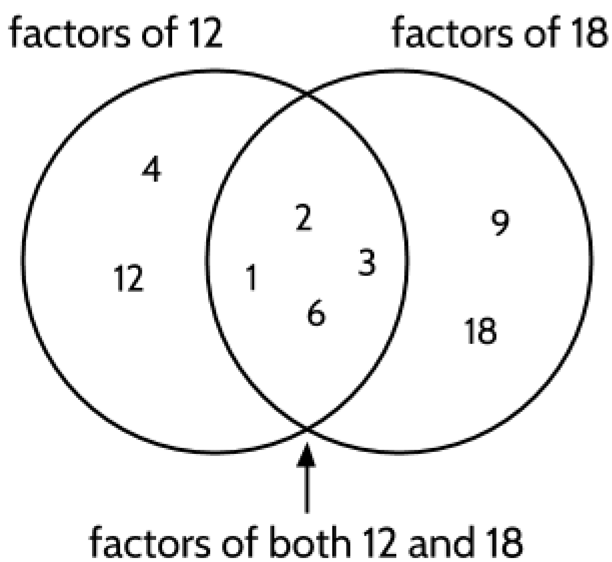
Understanding factors can be helpful in different kinds of mathematics. For example, finding *common factors* of two or three numbers is useful when solving problems in fractions and algebra. We'll start by looking at a couple numbers and their factors. From our work above, we know that…

12 has the factors: 1, 2, 3, 4, 6, 12 and 18 has the factors: 1, 2, 3, 6, 9, 18

What are the *common factors* of these numbers? In this situation, *common* means something that both numbers have. If two apartments share a wall *in common*, it means that the wall is connected to both apartments (and you might be able to hear your neighbors). If two countries have a border *in common*, it means that the two countries are connected at that border (like between the United States and Mexico). If two numbers have factors *in common*, it means they both have those numbers in their list of factors.

17) Which factors do 12 and 18 have in common?

The diagram with overlapping circles on the right is called a Venn diagram. It can show how two groups of information are connected. We are using a Venn diagram here to show how the numbers 12 and 18 are related. The circle on the left shows the factors of 12 and the circle on the right shows the factors of 18. In the overlapping area in the center are factors of both numbers (1, 2, 3 and 6).



Once we know which factors are shared by two numbers, we can find the *greatest* common factor. In this situation, *greatest* means "largest." This is similar to how the word *great* is used when we say that 15 is *greater* than 7. We mean that 15 is a larger number than 7.

12 and 18 have the shared factors 1, 2, 3, and 6. The largest number in that list is 6, which means that 6 is the *greatest common factor* of 12 and 18.

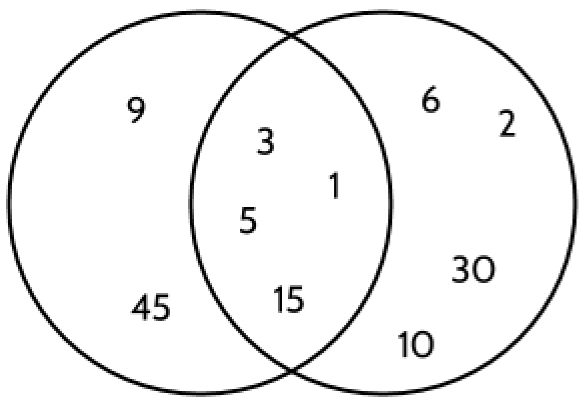
Answer the following questions. You can use the chart on page 39 if it's helpful..

18) What are the common factors of 8 and 20?

19) What is the greatest common factor of 9 and 27?

20) What is the largest number that divides evenly into both 12 and 21? How do you know?

Two numbers and their greatest common factor are shown in this Venn diagram.



21) The factors in the left circle (9 and 45) belong to which number?

22) The factors in the right circle (2, 6, 10, 30) belong to which number?

23) What is the greatest common factor of the two numbers?

Breaking Down Numbers - Answer Key

1)

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
11	1, 11
12	1, 2, 3, 4, 6, 12
13	1, 13
14	1, 2, 7, 14
15	1, 3, 5, 15
16	1, 2, 4, 8, 16
17	1, 17
18	1, 2, 3, 6, 9, 18

Number	Factors
19	1, 19
20	1, 2, 4, 5, 10, 20
21	1, 3, 7, 21
22	1, 2, 11, 22
23	1, 23
24	1, 2, 3, 4, 6, 8, 12, 24
25	1, 5, 25
26	1, 2, 13, 26
27	1, 3, 9, 27
28	1, 2, 4, 7, 14, 28
29	1, 29
30	1, 2, 3, 5, 6, 10, 15, 30
31	1, 31
32	1, 2, 4, 8, 16, 32
33	1, 3, 11, 33
34	1, 2, 17, 34
35	1, 5, 7, 35
36	1, 2, 3, 4, 6, 9, 12, 18, 36

2) 36 has nine factors, which is the most. 24 and 30 are in second place, each with eight factors.

3) 1

4) The bold numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31 all have exactly two factors: 1 and the number itself. For example, the two factors for 13 are 1 and 13.

5) 1, 4, 9, 16, 25, and 36. Do you notice anything special about these numbers? These are the only numbers on the list that can be represented by arrays that are perfect squares.

6)

Number	Prime or Composite?	How do you know?
55	Composite	It has four factors: 1, 5, 11 and 55.
21	Composite	It has four factors: 1, 3, 7 and 21.
43	Prime	It has only two factors: 1 and 43.
49	Composite	It has three factors: 1, 7 and 49.
37	Prime	It has only two factors: 1 and 37.

7) C, D, F

8) 144

9) 120

10) $2 \cdot 2 \cdot 3 \cdot 5$

11) $3 \cdot 3 \cdot 5$

12) 31 (The number 31 is prime so it can't be broken down any more.)

13) $2 \cdot 2 \cdot 2 \cdot 7$

14) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

15) $2 \cdot 3 \cdot 3 \cdot 5$

16) 64, which is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

17) 1, 2, 3 and 6

18) 1, 2, and 4

19) 9

20) 3. The factors of 12 are 1, 2, 3, 4, 6 and 12. The factors of 21 are 1, 3, 7, and 21. These are the numbers that divide evenly into each number. 3 divides evenly into both numbers, with no remainder. There aren't any larger numbers that divide evenly into both numbers. (This question is another way of asking for the greatest common factor.)

21) 45

22) 30

23) 15

Using The Power of Exponents

Multiplication is Repeated Addition

Multiplication is a way to show addition that repeats over and over. Imagine it's Halloween and you want to give 3 pieces of candy each to 7 kids. How many total pieces of candy will you give away?

You might use addition, adding 3 + 3 + 3 + 3 + 3 + 3 + 3 to get 21. A shorter way would be to multiply 7 × 3 to get 21. Adding 7 copies of 3 is the same as multiplying 7 × 3.

number of kids

number of candies to each kid

$$3 + 3 + 3 + 3 + 3 + 3 + 3 = 7 \times 3$$

Another example: If a school has 5 classrooms of students and each classroom has 20 students, how many students are there in the school?

number of classrooms

number of students in each classroom

$$20 + 20 + 20 + 20 + 20 = 5 \times 20$$

1) Complete this table.

written as multiplication	written as repeated addition	written as a single number
3 × 2	2 + 2 + 2	6
4 × 7	7 + 7 + 7 + 7	
	6 + 6 + 6 + 6 + 6	
4 × 9		

What if you wanted a shorter way to write repeated *multiplication*? What is another way to write 2 × 2 × 2 × 2 × 2 × 2?

Exponents Show Repeated Multiplication

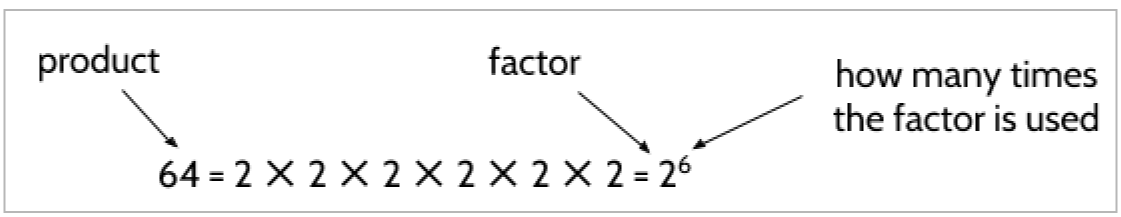
As we read in the last section, multiplication is a way to show addition that repeats. Instead of writing 5 + 5 + 5 + 5 + 5 + 5, we can write 6×5 as a shorthand way of writing 6 copies of 5.

$$5 + 5 + 5 + 5 + 5 + 5 = 6 \times 5$$

Now, I want a fast way to write multiplication that repeats. For that, we can finally use exponents! First, let's remind ourselves of factors, which are used in multiplication.

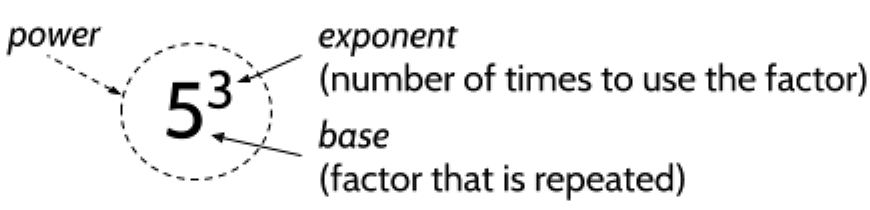
2) What is the prime factorization of 64? Show your method.

You should have found out that the prime factors of 64 are 2, 2, 2, 2, 2 and 2. This means that $2 \times 2 \times 2 \times 2 \times 2 \times 2$ is one way to write 64. (Check this with a calculator.) You can also write this repeated multiplication as 2^6, which is read as "two raised to the sixth power." When you see 2^6, it means 2 is used as a factor 6 times.



Writing 64 as a power

The number 5^3 is another example of a number written as a *power*. 5^3 means $5 \times 5 \times 5$ or 125. When a number is written as a power, it has a *base* and an *exponent*. The exponent (the little number on top) tells us how many times the base (the number on the bottom) should be used as a factor in multiplication. This is also called *exponential form*.



In this number, the 5 should be used in multiplication 3 times: $5 \times 5 \times 5 = 125$.

There are many ways to read 5^3. The formal way to say it is:

five raised to the third power

It might also be shortened to *five to the third power* or *five to the third*. Another way to say the same thing is *five cubed*.

3) Complete the following table.

Power	Repeated Multiplication	Number	Written in Words
2^3	$2 \times 2 \times 2$	8	two raised to the ________ power
	$3 \times 3 \times 3 \times 3$		________ to the ________ power
5^2			________ raised to the ________
	$2 \times 2 \times 2 \times 2$		________ to the ________
			Four raised to the third power
1^3	$1 \times 1 \times 1$		

4) Paul says 5^2 is equal to 10. How would you help Paul understand his mistake?

5) True or False: $3^4 = 3 \times 3 \times 3 \times 3$

Explanation:

6) Which is greater, 4^3 or 3^4?

Explanation:

7) True or False: $5^4 = 625$

   Explanation:

8) Which exponential expression is equivalent to $8 \times 8 \times 8 \times 8$?

   A. 4^4

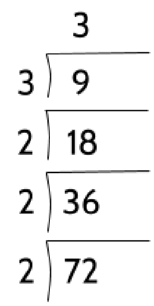
   B. 4^8

   C. 8^4

   D. 8^8

So far, we have only written numbers using one base and one exponent. For example, 81 is 3^4 $(3 \cdot 3 \cdot 3 \cdot 3)$ and 32 is 2^5 $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$.

We can also write some numbers as the product of different powers. For example, what if we wanted to write 72 with exponents? The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ (shown on the side). 72 has a mix of 2's and 3's as factors. Since 72 has three 2's and two 3's as factors, we can write it in exponential form like this:



$$2^3 \times 3^2$$

This is much shorter than $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$, but it means the same thing. We could even get rid of the multiplication symbol and write it like this:

$$2^3 3^2$$

When quantities like 2^3 and 3^2 are written next to each other, it means that the two quantities should be multiplied.

9) What number can be written as $2^3 5^2$?

10) How would you write the number 100 in exponential form?

When numbers are written in their exponential form, we usually write the prime factors in order from smallest to largest. For example, 100 would be written $2^2 5^2$. If you wrote $5^2 2^2$, other people would know what you mean, but it might look a little strange.

11) Complete the table.

Number	Prime Factorization	Exponential Form
36	$2 \cdot 2 \cdot 3 \cdot 3$	
500		$2^2 5^3$
	$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$	
		$3^1 5^2$
90		

12) What is 1^5? Explain your answer.

13) Figure out the prime factorization of 144. Which of the following show 144 in exponential form?

A. $2^2 4^2$

B. $2^2 3^4$

C. $2^3 3^2$

D. $2^4 3^2$

14) The exponential form of a number is $3^2 5^1 7^1$. What is the number?

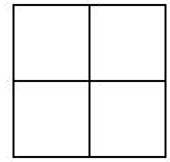
A. 19

B. 210

C. 315

D. 945

Area and Exponents

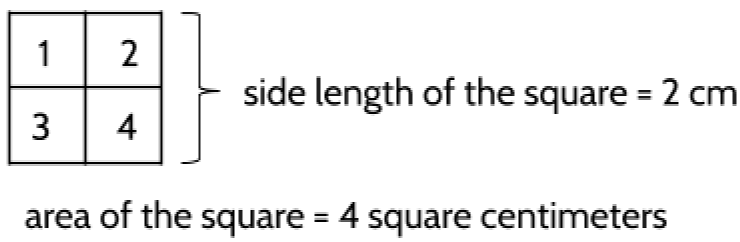


Area is a way to represent a number raised to the 2nd power. Measurements of area are written in square units. Look at the square on the right. Notice that the length of each side is the same.

15) What is the side length of the square? ____________ centimeters

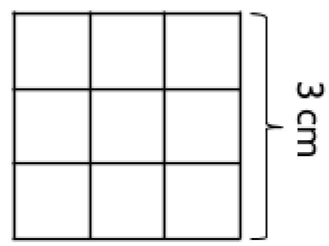
16) What is the area of the square? ____________ square centimeters

The side length of the square is 2 centimeters and there are a total of 4 square centimeters that make up the square. One way to find the area of the shape is to count the square centimeters that is made of.



17) What would be the area of a square with a side length of 3 centimeters?

____________ square centimeters



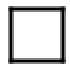
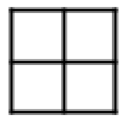
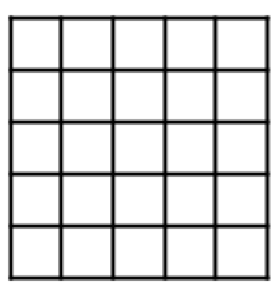
Another way to find the area of the square is to multiply the side length by the side length. For a square with a side length of 3cm, the area is 3×3 or 9 square centimeters.

18) If the side length of a square is 10 centimeters, what would the area of the square be?

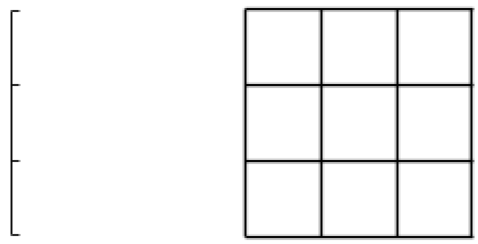
___________ square centimeters

Look at a multiplication table (page 40). Here is the diagonal line of numbers that starts in the top left and goes towards the bottom right: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, etc. These are called *square numbers* because they can each be drawn as square arrays.

19) Complete the table below by drawing squares and filling in blanks.

Area (cm^2)	Drawing	Side length (cm)
1		1
4		
9		
16		
		

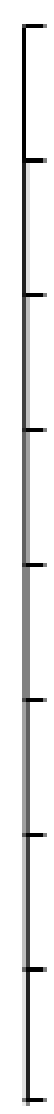
If we are thinking about area, to *square* a number means to start with a measurement of length and then draw a square where each side has that same length.



side length = 3 cm   side length *squared* = 9 square centimeters

If you have a side length of 3, then the square will have an area of 9. The number 3 *squared* equals 9.

For each of the given side lengths below, draw the rest of the square and calculate the area.

20) 

side length =

area =

21) 

side length =

area =

(You will find more opportunities to practice area in the **Population Density** packet and in the **Lines, Angles, and Shapes: Measuring Our World** packet.)

The area of a square can be calculated using the formula: $A = s^2$.

This formula can be read as "**A**rea equals the **s**ide length squared" or "**A**rea equals the **s**ide length to the second power." To find the area, substitute the side length for *s* and then calculate the value of the exponent.

Here's an example:

$$A = s^2$$
$$A = 5^2$$
$$A = 5 \cdot 5$$
$$A = 25$$

22) Complete the table below.

Side Length (*s*)	Area (*A*)	Calculation	Exponential Form
5 cm	25 cm^2	$5 \cdot 5$	5^2
2 cm			
3 cm			3^2
4 cm			
8 cm			
10 cm			
1 cm			
	36 cm^2		
		$7 \cdot 7$	
			9^2
0.5 cm			

23) Complete this table for $A = s^2$.

Side length (*s*)	1	2	3	4	5	6
Area (*A*)						

Square Roots

When we hear the word "roots", we usually think of the underground parts of plants. In mathematics, a *root* is the solution to an equation. Usually, the word *root* refers to the solution of an equation similar to this one:

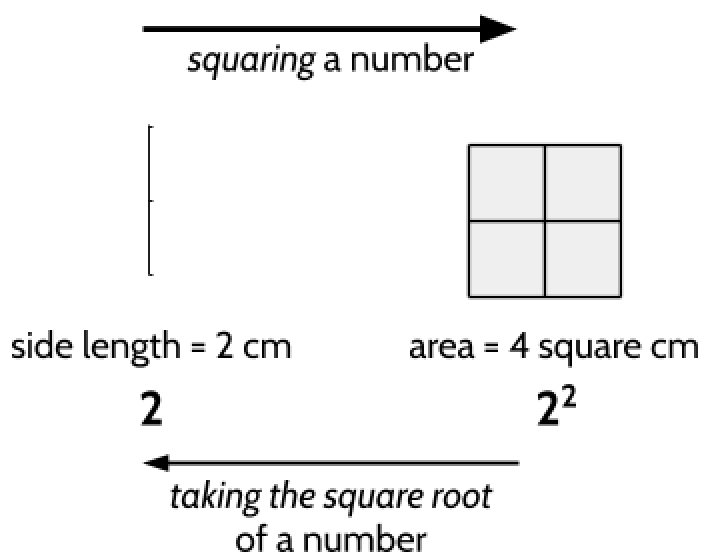
$$a^2 = 25$$

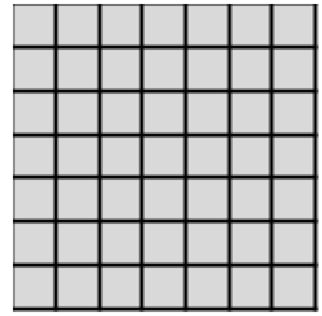
The root is a value of a that makes the expression $a^2 = 25$ true. To find the value of a in this equation, you can ask yourself, "What number times itself is equal to 25?" Since $5 \times 5 = 25$, a must be 5 (or -5, since -5 multiplied by -5 also equals 25.)

24) If $b^2 = 36$, what is b?

The values of a and b in the equations above are examples of *square roots*. As we saw above, you can think of *squaring* as starting with a number as the length of one side and then creating a square. The area is the square of the original length. Th*e square root of* a number is the opposite. It means starting with a number that is the area of a square and then finding the length of one side.

For example, if a square has an area of 4, the side length is 2. The square root of 4 is 2.





25) What is the square root of the number represented by the grid to the right?

26) If the length of one side of a square is 11 (not pictured), what is the area of the square?

27) If the area of a square is 64, what is the length of one side?

The *radical* symbol ($\sqrt{\quad}$) is used to indicate a square root. For example:

$\sqrt{25} = 5$ (This equation can be read as "The square root of 25 is 5.")

$\sqrt{36} = 6$

Another way to say square root is the *2nd root*. To find the square root or the 2nd root of a number, you can ask yourself, "What number can I multiply by itself to get the number I want?"

28) What is the square root of 81?

29) What is $\sqrt{49}$?

30) What is $\sqrt{10 \cdot 10}$?

31) What is the 2nd root of 9?

32) What is $\sqrt{3^2}$?

33) What is $\sqrt{x \cdot x}$?

34) What is $\sqrt{3 \cdot 5 \cdot 3 \cdot 5}$?

35) What is $\left(\sqrt{16}\right)^3$?

Volume and Exponents

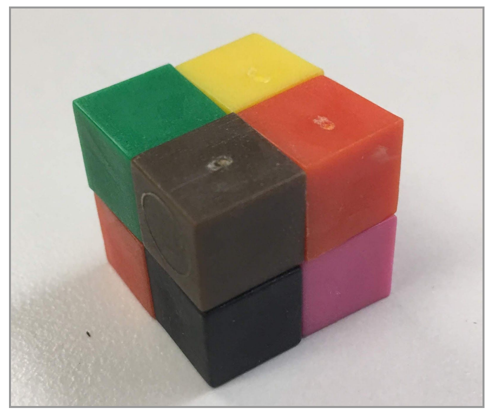
Volume can be used to represent a number raised to the 3rd power. Remember the different ways to say 2^3:

2 raised to the third power

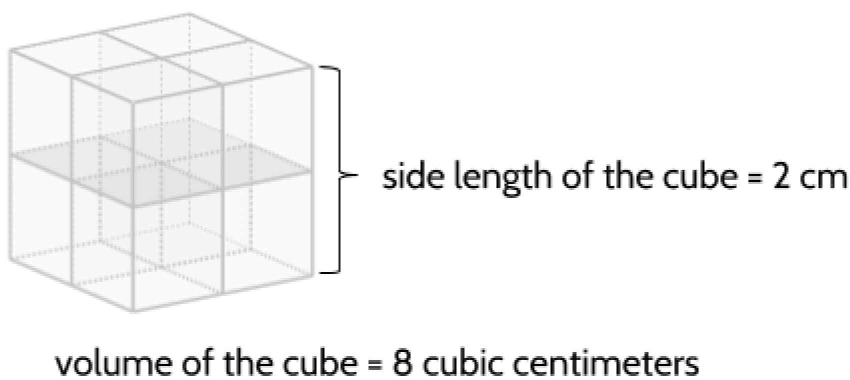
2 to the third power

2 to the third

You can also say 2 *cubed*! Where does that come from? Similar to *squared*, this language can be understood by thinking about shapes.



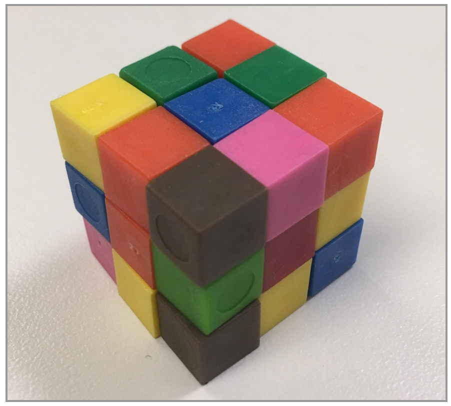
In the multicolored cube on the right, the length of each side is 2 centimeters. This results in a total of 8 cubic centimeters that make up the cube. Each of the small, colored cubes is 1 cubic centimeter in volume. One way to find the volume of the multicolored cube is to count the number of the cubic centimeters. Do you see 8 different cubes inside the big cube?



(You will find more opportunities to practice volume in **The Density of Matter** and **Lines, Angles, and Shapes: Measuring Our World**.)

36) The cube below is 3 centimeters tall. What is the volume of the cube?

_____________ cubic centimeters

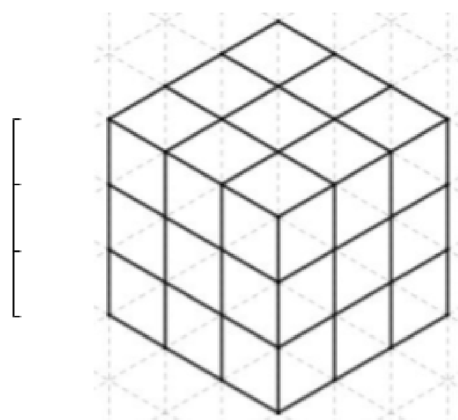


Another way to find the volume of a cube is to multiply the side length by the side length by the side length (in a cube, all three side lengths are the same). For a cube with a side length of 3 cm, the volume is 3 $\times$ 3 $\times$ 3 or 27 cubic centimeters. We found the answer by *cubing* 3.

37) If the side length of a cube is 5 centimeters, what is the volume?

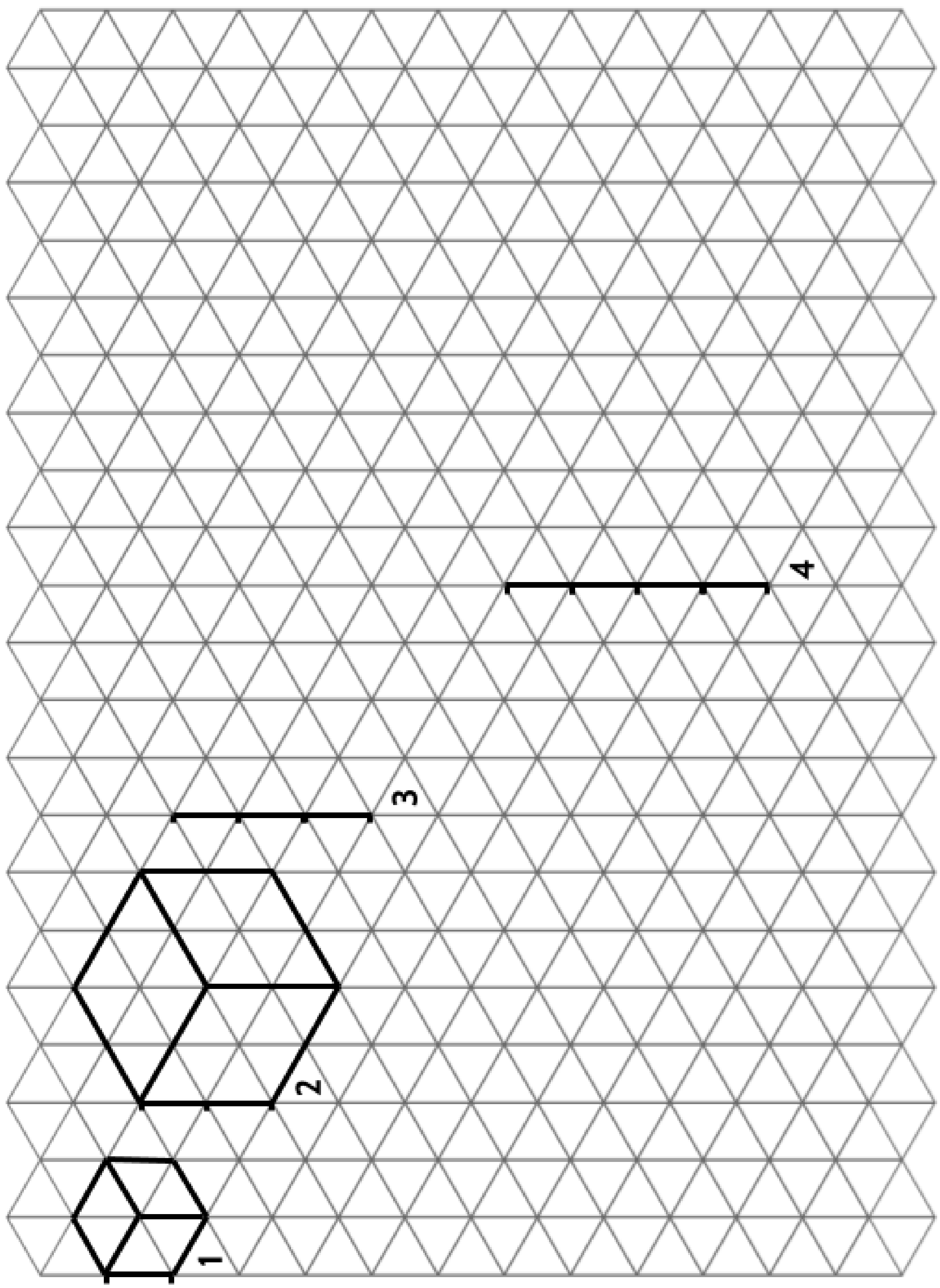
____________ cubic centimeters

When thinking about volume, to *cube* a number means to start with a measurement of side length and then make a cube where each side has that same length.



side length = 3 cm side length *cubed* = 27 cubic centimeters

38) The first two cubes are drawn for you. Use the side lengths to draw the next two cubes. How many cubic centimeters are there in each one? (Turn the paper sideways.)



The volume of a cube can be calculated using this formula: $V = s^3$.

This formula can be read as "Volume equals the side length cubed" or "Volume equals the side length raised to the third power." To find the volume, substitute the side length for *s* and then calculate the value of the exponent.

Here's an example:

$$V = s^3$$
$$V = 2^3$$
$$V = 2 \cdot 2 \cdot 2$$
$$V = 8$$

39) Complete the table below.

Side Length (*s*)	Volume (*V*)	Calculation	Exponential Form
2	8	$2 \cdot 2 \cdot 2$	2^3
3			3^3
4			
8			
10			
1			
		$6 \cdot 6 \cdot 6$	
	343		
			9^3
0.5			

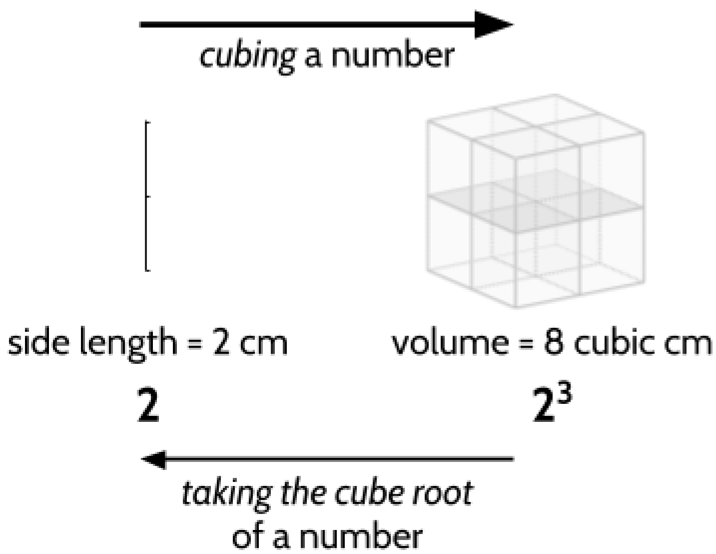
40) Complete this table for $V = s^3$.

Side length (*s*)	1	2	3	4	5	6
Volume (*V*)						

Cube Roots

Just as square roots are based on squares, *cube roots* are related to cubes. A number cubed is the volume of a cube with the number as a side length. The volume is the cube of the original length. The cube root of a number is the opposite. If the number is the volume of a cube, the cube root is the length of one side of that cube.

For example, if a cube has a volume of 8, the side length is 2. The cube root of 8 is 2.



A cube root is also a value of b that makes an equation like $b^3 = 8$ true. To find the value of b in this equation, you can ask yourself, "What number times itself and then times itself again is equal to 8?" Since $2 \times 2 \times 2 = 8$, in this equation b must be 2. This means that the cube root of 8 is 2.

41) If the length of one side of a cube is 4, what is the volume of the cube? (In other words, what is the value of 4^3?)

42) If the volume of a cube is 125, what is the length of one side? (In other words, what is the cube root of 125?)

The *radical* symbol is also used to indicate a cube root. The only difference with the radical used for a square root symbol is that the cube root symbol has a small ^{3} in the radical:

$$\sqrt[3]{}$$

For example:

$\sqrt[3]{27} = 3$ (This equation can be read as "The cube root of 27 is 3.")

$\sqrt[3]{64} = 4$

Another way to say cube root is the *3rd root*. That's why the radical includes a 3. To find the cube root or the 3rd root of a number, you can ask yourself, "What number can I use as a factor in multiplication three times to get the number I want?"

43) What is the cube root of 216?

44) What is $\sqrt[3]{343}$?

45) What is $\sqrt[3]{10 \cdot 10 \cdot 10}$?

46) What is the 3rd root of 125?

47) What is $\sqrt[3]{8^3}$?

48) What is $\sqrt[3]{x \cdot x \cdot x}$?

49) What is $\sqrt[3]{2 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 3}$?

50) What is $\left(\sqrt[3]{27}\right)^2$?

51) If $\sqrt[3]{}$ means the 3rd root, what do you think $\sqrt[4]{}$ means?

What is $\sqrt[4]{16}$?

Finding Roots Using Factors

Finding Square Roots

There are several ways to find the root of a number. One way is “guess and check.” For example, if you want to know the square root of 256, you can try multiplying numbers together until you find the number that multiplies with itself to give 256. Take a few minutes to try that now. You might want to use a calculator.

We can also use factors to figure out the square root of any perfect square: 1, 4, 9, 16, 25, 36, etc. Look back at the earlier section on prime factorization if you want a refresher.

Perfect Square	Prime Factors	Square Root
4	2 · 2	2
9	3 · 3	3
16	2 · 2 · 2 · 2	4
25	5 · 5	5
36	2 · 3 · 2 · 3	6
49	7 · 7	7
64	2 · 2 · 2 · 2 · 2 · 2	8
81	3 · 3 · 3 · 3	9
100	2 · 5 · 2 · 5	10
256	2 · 2 · 2 · 2 · 2 · 2 · 2 · 2	16

Look at the prime factors of each perfect square and compare them to the square roots. The square root is always equal to the product of half of the prime factors of the number. For example, the prime factors of 100 are 2 · 5 · 2 · 5. The square root of 100 is 10, which is 2 · 5 (half of the factors of 100).

What if you want to find the square root of 225? You can start by finding its prime factorization. As shown on the right, the prime factors of 225 are 3, 3, 5, and 5.

To find the square root of 225, split the factors into two equal groups of factors (see below). If the two groups of factors on the left and the right are the same, the square root is one of them.

$$3 \cdot 5 \cdot 3 \cdot 5$$

You might notice that 3 × 5 is 15, so the square root is 15. Since you have the factors 3 and 5 twice in the group of factors for 225, the product is the same as 15 × 15.

$$(3 \cdot 5) \cdot (3 \cdot 5) = 15 \cdot 15$$

$$\sqrt{3 \cdot 5 \cdot 3 \cdot 5} = 3 \cdot 5$$

$$\sqrt{225} = 15$$

52) The prime factorization of 324 is $2 \cdot 3 \cdot 3 \cdot 2 \cdot 3 \cdot 3$. What is the square root of 324?

Be careful! Make sure that you have two sets of the same factors. For example, $5 \cdot 5$ is <u>not</u> the square root of $5 \cdot 5 \cdot 3 \cdot 3$. It may be easier to write 225 as $3 \cdot 5 \cdot 3 \cdot 5$ so that you can see two groups of $3 \cdot 5$.

Also, you can only use the factor method of finding a square root with numbers that are perfect squares. It won't work for numbers that don't have two even sets of factors. For example, the prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$. There is no way to split these factors into two groups of the same factors. The square root of 48 is about 6.9 (not a whole number).

There are a few different ways you will see square roots represented in test questions.

with a radical	as the solution to an equation	$\frac{1}{2}$ as the exponent
$\sqrt{4}$	$c^2 = 4$	$4^{\frac{1}{2}}$
$\sqrt{100}$	$d^2 = 100$	$100^{\frac{1}{2}}$

Each of these ways of writing the square root mean the same thing. For example, each of the following means, "The square root of 4 is 2."

$\sqrt{4} = 2$ If $c^2 = 4$, then $c = 2$ $4^{\frac{1}{2}} = 2$

The last example is read as "Four to the one-half power," which is the same as "The square root of four." Seeing ½ as an exponent probably looks pretty strange. What do square roots have to do with half of something? Half of what? To get the square root of a number, you can use half of its prime factors. This is what we did on the last page.

53) Complete the following table.

Number	Prime Factors	Square Root (½ of factors)	Square Root (numeral)	Square Root (as a power)
16	$2 \cdot 2 \cdot 2 \cdot 2$	$2 \cdot 2$	4	$16^{\frac{1}{2}}$
100	$2 \cdot 5 \cdot 2 \cdot 5$			
121		11	11	$121^{\frac{1}{2}}$
225			15	

Decide whether the following statements are True or False. If the statement is false, write a correction.

54) $\sqrt{900} = 900^{\frac{1}{2}}$

55) The square root of $2 \cdot 2 \cdot 3 \cdot 3$ is $2 \cdot 2$.

56) $400^{\frac{1}{2}} = 20$

57) $\sqrt{144} = 2 \cdot 3 \cdot 3$

58) $\sqrt{3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7} = 3 \cdot 5 \cdot 7$

Finding Cube Roots

You can also use "guess and check" to find the cube root or 3rd root of a number. For example, if you want to know the cube root of 343, you can try multiplying numbers together until you find the number that multiplies three times to give 343. Try that now. You might want to use a calculator.

We can also use factors to figure out the cube roots of some numbers. This will only work with numbers that are equal to a whole number raised to the 3rd power. These are called *perfect cubes*. You can take the cube root of any number, but only perfect cubes will have cube roots that are whole numbers without decimals or fractions. Here are some examples:

A "cube" number	Prime Factors	Cube Root
8	$2 \cdot 2 \cdot 2$	2
27	$3 \cdot 3 \cdot 3$	3
64	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	4
125	$5 \cdot 5 \cdot 5$	5

59) Remember that a square root has half the original factors of the original perfect square. The cube root has __________ of the "cube" number.

A. one-half of the factors

B. one-third of the factors

C. twice as many factors

D. three times as many factors

Look at the prime factors of each cube and compare them to the cube roots. The cube root is equal to the one-third of the prime factors of the cube number.

What if you want to find the cube root of 216? After we find that its prime factors are $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$, we could separate the factors into three equal groups:

$$(2 \cdot 3)(2 \cdot 3)(2 \cdot 3)$$

The cube root is 6.

60) The prime factorization of 27,000 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5$. What is the cube root of 27,000?

When using factors to find the cube root of a number, make sure that you have three sets of the same factors. For example, $5 \cdot 5$ is <u>not</u> the cube root of $5 \cdot 5 \cdot 5 \cdot 2 \cdot 2 \cdot 2$. It may be easier to write 1,000 as $2 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 5$ so that you can see three groups of $2 \cdot 5$.

$$(2 \cdot 5)(2 \cdot 5)(2 \cdot 5) = 10 \cdot 10 \cdot 10$$

Also, you can only use the factor method of finding a square root with cubed numbers (a whole number raised to the 3rd power). It won't work for numbers that don't have three sets of the same factors. For example, the prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$. There is no way to split these factors into three groups of the same factors.

You can write cube roots in many ways. Each of the following mean the same thing:
"The cube root of 216 is 6."

$\sqrt[3]{216} = 6$ If $c^3 = 216$, then $c = 6$ $216^{\frac{1}{3}}$

Notice that $8^{\frac{1}{3}}$ means the same as $\sqrt[3]{8}$. Both of these notations mean "the cube root of 8" or "the 3rd root of 8." You can also say this as "8 to the one-third power."

Decide whether the following statements are True or False. If the statement is false, write a correction.

61) $\sqrt[3]{2197} = 2197^{\frac{1}{3}}$

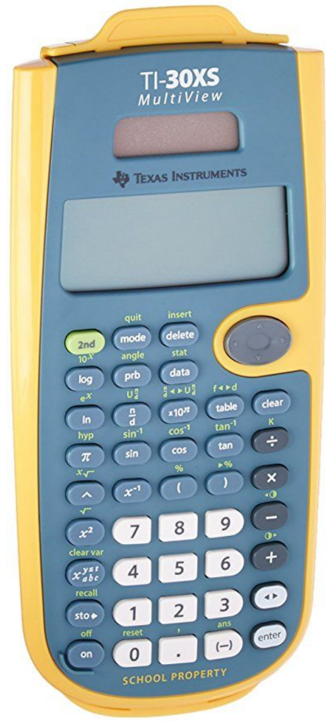
62) The cube root of $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$ is 10.

63) $216^{\frac{1}{3}} = 72$

64) $\sqrt[3]{1728} = 2 \cdot 2 \cdot 3$

65) $\sqrt[3]{3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7} = 3 \cdot 5 \cdot 7$

Calculator Practice



We recommend that you use a calculator as you work on the remainder of this packet. If you take the HSE exam, you will probably use the TI-30XS calculator. If you have access to this calculator, this is a good opportunity to start learning how to use it.

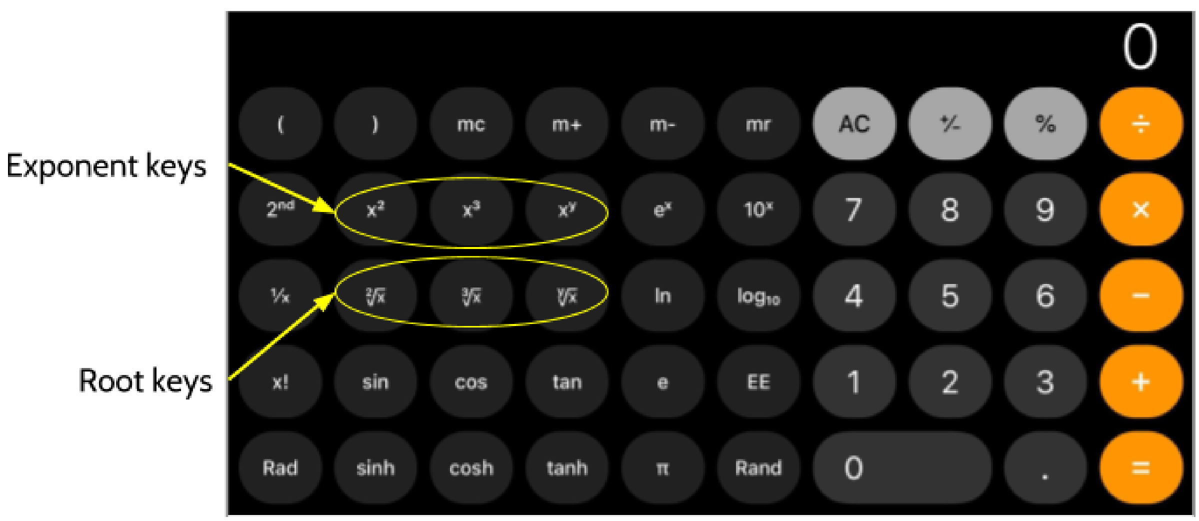
It's okay if you don't have the same calculator, but it is helpful if your calculator has functions for exponents and roots. You might notice that calculators have different buttons for exponents and roots.

Exponent keys: x^y or ^

Root keys: $\sqrt{\ }$ or $\sqrt[x]{\ }$ or $\sqrt[y]{x}$

For practice on the TI30XS, go to http://bit.ly/TI30XShelp.

If you have a smartphone, your calculator probably has exponents and roots also. Open the calculator app and then turn your phone sideways. The scientific calculator should display.



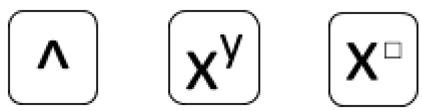
Finding Squares and Cubes with a Calculator

In this exercise, you will practice finding square and cube numbers with a calculator.

The variable *b* stands for *base* and can represent any number we want to use for the base. On the TI-30XS calculator used on HSE exams, we use this symbol **^** **(**called a *caret*) for entering exponents. The calculator uses this symbol because b^3 is another way to write b^3.

square numbers	**cube numbers**
"to the second power"	"to the third power"
b^2	b^3
b^2	b^3
$b \times b$	$b \times b \times b$

You can remember to use ^ by imagining it pushing the exponent up above the base. Other calculators may use a different button for roots. Here are some common calculator buttons for entering exponents.



calculator exponent buttons

All of these buttons mean "to the _____ power." To use the exponent function, enter the number you are using for the base (*b*), then press the exponent button, depending on your calculator. Then, enter the exponent. Finally, press **=** or **enter**, depending on your calculator. Here is an example of 4^3 on two different calculators:

Press **4**, press **^**, press **3**, press **enter**. Display should read **64**. (TI-30XS)

or

Turn phone, press **4**, press **x^y^**, press **3**, press **=**. Display should read **64**. (smartphone)

Note: You can also use the  button to raise numbers to the 2nd power.

Calculate the following numbers with exponents.

66) What is 15^2? __________

67) What is 25^2? __________

68) What is 15^3? __________

69) What is 25^3? __________

70) You can use these buttons to calculate any power. What is 4^5?

Finding Square Roots and Cube Roots with a Calculator

In this exercise, you will use a calculator to quickly find square and cube roots. On the TI-30XS calculator, $\sqrt{}$ is used for square roots and $\sqrt[x]{}$ is used for cube roots (and other roots). You can actually use $\sqrt[x]{}$ for all roots. Enter **2** for square roots, **3** for cube roots, **4** for 4th roots and so on.

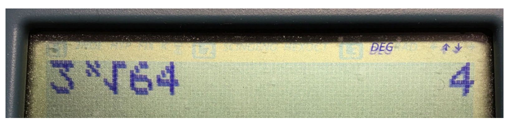
square root	**cube root**
"the second root"	"the third root"
$\sqrt{x}$	$\sqrt[3]{x}$
$x^{\frac{1}{2}}$	$x^{\frac{1}{3}}$

Here is an example of $\sqrt[3]{64}$ (the cube root of 64) on two different calculators:

Press **3**, press 

, press 

, press **64,** press **enter**. (TI-30XS)

Display: 

or

Turn phone, press **64**, press $\sqrt[y]{x}$, press **3**, press **=**. Display should read **64**. (smartphone)

Calculate the following numbers with exponents.

71) What is $\sqrt{1024}$? ___________

73) What is $\sqrt[3]{512}$? ___________

72) What is $441^{\frac{1}{2}}$? ___________

74) What is $729^{\frac{1}{3}}$? ___________

75) You can use these buttons to calculate any root. What is the 5th root of 7776?

76) The cube root of some numbers is not a whole number. What $\sqrt[3]{48}$?

When the Exponent is 1 or Less

At this point, you have a lot of practice with powers like 2^2. The exponent tells you how many times to use 2 as a factor in multiplication, so $2^2 = 2 \times 2$. But what about powers like these?

$$2^1 \qquad 2^0 \qquad 2^{-1}$$

What happens when you multiply a number by 1?

$5 \times 1 =$ $\quad$ $30 \times 1 =$ $\quad$ $1{,}000 \times 1 =$ $\quad$ $1.5 \times 1 =$ $\quad$ $.25 \times 1 =$

Any number multiplied by 1 is itself. Multiplying by 1 doesn't change the number.

To the Power of 1

If you look at the powers of 2 below, you might see a pattern that will help you understand powers that use exponents 1 or less. The number 1 can be included as a factor below because multiplying by 1 doesn't change the value.

Exponential Form	Factors	Number
2^5	$1 \times 2 \times 2 \times 2 \times 2 \times 2$	32
2^4	$1 \times 2 \times 2 \times 2 \times 2$	16
2^3	$1 \times 2 \times 2 \times 2$	8
2^2	$1 \times 2 \times 2$	4
2^1	1×2	2
2^0	1	1
2^{-1}		$\frac{1}{2}$
2^{-2}		$\frac{1}{4}$
2^{-3}		$\frac{1}{8}$

77) What do you notice?

The number 16 can be written as 2^4 because the number 2 is used as a factor four times ($2 \times 2 \times 2 \times 2$).

78) How many times is 2 used as a factor in 8?

79) How many times is 2 used as a factor in 4?

80) How many times is 2 used as a factor in 2?

Look back at the table on the previous page. If you follow the pattern in the factor column, 1×2 should be in the row for 2^1. The number 2 has 2 as a factor once. It is just itself. This is why 2^1 is 2. A number raised to the 1st power equals the number. This is true of any number that is raised to the first power.

$$3^1 = 3 \qquad\qquad 15^1 = 15 \qquad\qquad x^1 = x$$

To the Power of 0

Your first instinct might be to say that a number raised to the zero power is 0.

Actually 2^0 doesn't equal 0 and 3^0 doesn't equal 0 either. The symbol $\neq$ means "not equal to." $2^0 \neq 0$ and $3^0 \neq 0$. So, what do they equal?

Look back at the table on the previous page.

81) Think about the number 1. How many times is 2 used as a factor in the number 1?

These may seem like a strange question. 1 doesn't have a 2 as a factor at all. That's what 2^0 means. It's a number with 0 factors of 2. This is one way to understand why 2^0 equals 1. The number 1 doesn't have any 2s as factors. Other than 1, which is always a factor, it doesn't have other numbers as factors. So, 2^0 is 1.

82) How many times is 3 used as a factor in the number 1?

83) What is the value of *x* in this equation? $3^x = 1$

A. 0

B. 1

C. 2

D. 3

If you chose $3^0 = 1$, you are correct. 3^0 means that the number has no factors of 3. The only factor it has left is 1, so the value of the number is 1. In fact, any base to the zero power is 1.

84) What is the value of 5^0?

85) What is the value of 35^0?

86) What is the value of x^0?

Negative Exponents

Let's look at the powers of 2 again for a way to understand 2^{-1} and numbers raised to negative exponents.

multiply by 2 →

Exponential Form	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3
Number	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

← divide by 2

As we move to the right, the numbers are doubling. As the exponent increases by 1, the numbers are multiplied by 2. For example, $2^2 \times 2$ is 2^3 and 4×2 is 8. The number for each power of 2 is doubling as the exponent increases by 1.

Now, look at what happens to the numbers as we move to the left in the table. The numbers are divided in half with each step to the left. We can see that $2^3 \div 2$ is 2^2 just as $8 \div 2$ is 4. As we move to the left in the table, we are dividing the number by 2. As the exponent goes down by 1, the number is divided by 2.

Can you see another reason why 2^0 equals 1? $2^2 \div 2$ is 2. And $2^1 \div 2$ is 1. So, 2^0 is 1.

There are other patterns in the powers with negative exponents:

Powers of 2		Powers of 3	
Positive Exponents	Negative Exponents	Positive Exponents	Negative Exponents
$2^1 = 2$	$2^{-1} = \frac{1}{2}$	$3^1 = 3$	$3^{-1} = \frac{1}{3}$
$2^2 = 4$	$2^{-2} = \frac{1}{4}$	$3^2 = 9$	$3^{-2} = \frac{1}{9}$
$2^3 = 8$	$2^{-3} = \frac{1}{8}$	$3^3 = 27$	$3^{-3} = \frac{1}{27}$
$2^4 = 16$	$2^{-4} = \frac{1}{16}$	$3^4 = 81$	$3^{-4} = \frac{1}{81}$

87) What do you notice?

One thing you might notice is that these negative exponents don't result in negative numbers. The negative exponents make the numbers smaller, but ½, ¼, and ⅛ are positive numbers. So, the negative exponents aren't making the numbers negative. They are doing something else.

If you compare two powers, one with a positive exponent and one with the negative version of the same exponent, you can see that the two numbers are very similar. For example, 3^2 is 9 and 3^{-2} is $\frac{1}{9}$. In mathematics, this relationship is called a *reciprocal*. $\frac{1}{9}$ is the reciprocal of 9. $\frac{1}{16}$ is the reciprocal of 16. The number 27 is also considered the reciprocal of $\frac{1}{27}$. When you see 2^{-x}, ask yourself, "What is 1 divided by 2^x?"

88) What is 4^{-3}?

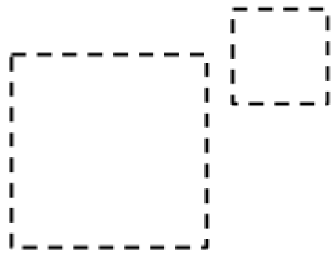
89) If x^{-2} is $\frac{1}{25}$, what is x^2?

90) $\square^{-4} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3}$

91) $6^{-2} = \frac{1}{\square}$

Practice with Powers

92) Directions: Use any two different numbers 1 through 10, using each number only once, to fill in the boxes below to make the largest value possible. Record your attempts below.



We tried 5 for the base and 4 for the exponent and got an answer of 625. Can you use different numbers between 1 and 10 to make a larger value?

Attempt	Base and Exponent	Value
1st try	5^4	625

Now use two different numbers from 1 to 10 to make the smallest value possible.

Attempt	Base and Exponent	Value

93) Directions: Use the digits 1 to 9 to fill in the boxes to make this equation true. You can use a digit more than once. Record your attempts below. There is more than one way to make this equation true. How many solutions can you find?

$$\square^{\square} = 64$$

We tried 6 for the base and 3 for the exponent and got an answer of 216, which is not equal to 64. What digits will you try?

Attempt	Base and Exponent	Value	True or False?
1st try	6^3	216	False

Thanks to openmiddle.com for these activities.

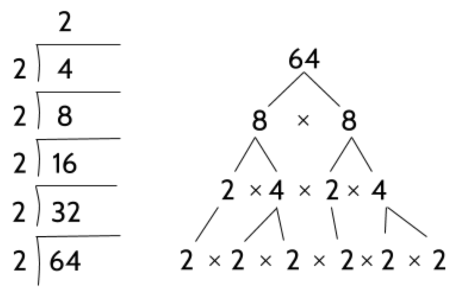
94) How many different ways are there to make this equation true with the numbers 1 through 9? How do you know?

Using The Power of Exponents - Answer Key

1)

written as multiplication	written as repeated addition	written as a single number
3 × 2	2 + 2 + 2	6
4 × 7	7 + 7 + 7 + 7	28
5 × 6	6 + 6 + 6 + 6 + 6	30
4 × 9	9 + 9 + 9 + 9	36

2) Your method may look like one of these:



3)

Power	Repeated Multiplication	Number	Written in Words
2^3	2 × 2 × 2	8	two raised to the third power
3^4	3 × 3 × 3 × 3	81	three to the fourth power
5^2	5 × 5	25	five raised to the second
2^4	2 × 2 × 2 × 2	16	two to the fourth
4^3	4 × 4 × 4	64	Four raised to the third power
1^3	1 × 1 × 1	1	one to the third power

4) Your explanation could use these facts:
$5^2 = 5 \times 5 = 25$
$5 \times 2 = 10$

5) True.
3^4 means 3 used as a factor 4 times: $3 \times 3 \times 3 \times 3$.

6) $4^3 = 4 \times 4 \times 4 = 64$
   $3^4 = 3 \times 3 \times 3 \times 3 = 81$
   3^4 is greater than 4^3.

7) True.
   $5^4 = 5 \times 5 \times 5 \times 5 = 625$.
   625 is also 25^2, Do you know why?

8) 8^4

9) 200

10) $2^2 5^2$

11)

Number	Prime Factorization	Exponential Form
36	$2 \cdot 2 \cdot 3 \cdot 3$	$2^2 3^2$
500	$2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$	$2^2 5^3$
72	$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$	$2^3 3^2$
75	$3 \cdot 5 \cdot 5$	$3^1 5^2$
90	$2 \cdot 3 \cdot 3 \cdot 5$	$2^1 3^2 5^1$

12) $1 \times 1 = 1$, no matter how many times you multiply it. 1 to any power equals 1.

13) $2^4 3^2$

14) 315

15) 2 centimeters

16) 4 square centimeters

17) 9 square centimeters

18) 100 square centimeters

19)

Area (cm^2)	Side length (cm)
4	2
9	3
16	4
25	5

20) 8 centimeters
64 square centimeters

21) 7 centimeters
49 square centimeters

22)

Side length (*s*)	Area (*A*)	Calculation	Exponential Form
5 cm	25 cm^2	5×5	5^2
2 cm	4 cm^2	2×2	2^2
3 cm	9 cm^2	3×3	3^2
4 cm	16 cm^2	4×4	4^2
8 cm	64 cm^2	8×8	8^2
10 cm	100 cm^2	10×10	10^2
1 cm	1 cm^2	1×1	1^2
6 cm	36 cm^2	6×6	6^2
7 cm	49 cm^2	7×7	7^2
9 cm	81 cm^2	9×9	9^2
0.5 cm	.25 cm^2	$.5 \times .5$	$.5^2$

23)

Side length (s)	1	2	3	4	5	6
Area (A)	1	4	9	16	25	36

24) 6

25) 7

26) 121

27) 8

28) 9

29) 7

30) 10

31) 3

32) 3

33) x

34) $3 \cdot 5$ or 15

35) 64

36) 27 cubic centimeters

37) 125 cubic centimeters

38) Your drawings should look similar to the first two cubes. There should be 1, 8, 27, and 64 cubic centimeters shown in the drawings.

39)

Side length (*s*)	Volume (*V*)	Calculation	Exponential Form
2	8	$2 \times 2 \times 2$	2^3
3	27	$3 \times 3 \times 3$	3^3
4	64	$4 \times 4 \times 4$	4^3
8	512	$8 \times 8 \times 8$	8^3
10	1,000	$10 \times 10 \times 10$	10^3
1	1	$1 \times 1 \times 1$	1^3
6	216	$6 \times 6 \times 6$	6^3
7	343	$7 \times 7 \times 7$	7^3
9	729	$9 \times 9 \times 9$	9^3
0.5	.125	$.5 \times .5 \times .5$	$.5^3$

40)

Side length (s)	1	2	3	4	5	6
Volume (V)	1	8	27	64	125	216

41) 64

42) 5

43) 6

44) 7

45) 10

46) 5

47) 8

48) x

49) 2 · 3 or 6

50) 9. The cube root of 27 is 3 and 3 squared is 9.

51) The 4th root or "What number can be used has a factor 4 times to get 16?" For example, the 4th root of 16 is 2. This means that 2 ✕ 2 ✕ 2 ✕ 2 = 16.

52) 2 · 3 · 3 or 18

53)

Number	Prime Factors	Square Root (Factors)	Square Root (Numeral)	Square Root (as a Power)
16	2 · 2 · 2 · 2	2 · 2	4	$16^{\frac{1}{2}}$
100	2 · 5 · 2 · 5	2 · 5	10	$100^{\frac{1}{2}}$
121	11 · 11	11	11	$121^{\frac{1}{2}}$
225	3 · 5 · 3 · 5	3 · 5	15	$15^{\frac{1}{2}}$

54) True

55) False. The square root of 2 · 2 · 3 · 3 is 2 · 3. In other words, the square root of 36 is 6, not 4.

56) True. The square root of 400 is 20.

57) False. The square root of 144 is 2 · 2 · 3, which is 12.

58) True.

59) B

60) 2 · 3 · 5 or 30

61) True

62) True

63) False. 1/3 of 216 is 72, but the cube root of 216 is 6.

64) True

65) True

66) 225

67) 625
68) 3375

69) 15,625

70) 1024

71) 32

72) 21

73) 8

74) 9

75) 6

76) Approximately 3.6

77) There are different things you might notice. Here are a few:
    - This chart includes negative exponents (-1, -2, -3)
    - These negative exponents produce fractions
    - 2^0 is 1, 2^{-1} is $\frac{1}{2}$, 2^{-2} is $\frac{1}{4}$
    - The fractions are being divided in half as we go down the chart

78) 3

79) 2

80) 1

81) 0. The number 1 doesn't have 2 as a factor.

82) 0. The number 1 doesn't have 3 as a factor.

83) A

84) 1

85) 1

86) 1

87) There are different things you might notice. Here are a few:

- The chart shows positive exponents and negative exponents.
- Negative exponents don't result in negative number values.
- The negative sign in the exponent means you should put a 1 over the positive version of the exponent

88) $\frac{1}{64}$

89) 25

90) 3

91) 36

92) Try at least 4 different combinations of numbers. Remember that you can only use each number once. For example, 7^7 is not allowed.

93) See below.

94) There are three ways to make this equation true with the numbers 1 through 9:

$8^2 = 64$ $4^3 = 64$ $2^6 = 64$

For explanation of these three solutions, think about the prime factorization of 64:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ is the same as 2^6, which is one of the solutions. 2 and 6 are both numbers between 1 and 9.

You could also group the factors of 64 in a couple different ways:

$(2 \cdot 2) \cdot (2 \cdot 2) \cdot (2 \cdot 2)$ is the same as $4 \cdot 4 \cdot 4$ or 4^3, which is another solution. 4 and 3 are both numbers between 1 and 9.

$(2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$ is the same as $8 \cdot 8$ or 8^2, which is another solution. 8 and 2 are both numbers between 1 and 9.

Vocabulary Review

You can use this section to look up words used in this math packet.

area (noun): The size of a flat surface, measured in square units

array (noun): An arrangement of objects in columns and rows

> *rectangular array*: An *array* in the shape of a rectangle

base (noun): In a quantity represented as a power, the *base* is the factor being multiplied. For example, in the power 2^3, the *base* is 2.

composite number: A number that has more than two factors

cube (noun): A box-shaped solid object that has six identical square faces

cube number (noun): A number which is the product of three numbers which are the same

> *perfect cube*: Numbers like 1, 8, 27, 64, and 125 which can be formed into a *cube* of this number of blocks. All *perfect cubes* are a whole number to the third power. For example, $27 = 3^3$ and $125 = 5^3$.

digit (noun): The numbers 0-9 and the numerals that represent them: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. There are 5 digits in this number: 14,692. *Digit* can also mean "finger."

divisible (adjective): A number is *divisible* by another number if it can be divided into the second number with no remainder. 21 is divisible by 3. 25 is <u>not</u> divisible by 3.

equation (noun): An expression that shows two mathematical expressions are equal (using = sign). $2^3 = 8$ is an *equation*. $5x + 3$ is an expression, <u>not</u> an *equation*.

equivalent (adjective): Having the same value. 4^3 and 64 are *equivalent*

estimate (verb): To make a rough guess at a number, usually without making written calculations

evaluate (verb): To calculate the value of something. If asked to *evaluate* 4^3, your answer should be 64.

even (adjective): Numbers that are divisible by 2.

expanded form (noun): A way of showing all the factors that are multiplied in a power. The *expanded form* of 3^4 is $3 \times 3 \times 3 \times 3$.

expression (noun): Numbers and symbols that show the value of something. 100, $5x + 3$, and 2^3 are all *expressions*. $5x + 3 = 23$ is an equation, not an *expression*. *Expressions* don't use equal signs.

exponent (noun): In a quantity represented as a power, the *exponent* shows how many times the base is multiplied. The exponent is shown as a smaller number up and to the right of the base. For example, in the power 2^3, the *exponent* is 3.

> *exponential form*: A way of writing values as a power using exponents. 3^4 is an *exponential form* of 81.
>
> *exponential growth*: A growth pattern in which the unknown quantity or variable is an exponent and the base (factor) is larger than 1. The power 2^x produces *exponential growth* based on doubling or multiplying by 2. The power 3^x produces *exponential growth* based on tripling or multiplying by 3.
>
> *exponential decay*: A change in numbers where the variable is an exponent and the base (factor) being multiplied is less than 1. As the exponent gets larger, the values get smaller. The power $(.5)^x$ represents *exponential decay*.

factor (noun): Whole numbers that are multiplied together to get another number. A number that can be divided <u>into</u> another number evenly, with no remainder.

> *prime factor*: A factor of another number that itself has only 2 factors. 2, 3, and 5 are the prime factors of 30. The number 15 is a factor of 30, but it is <u>not</u> a *prime factor* of 30.
>
> *prime factorization*: A way of representing numbers with all of their prime factors. $2 \times 3 \times 5$ is the *prime factorization* of 30. $2 \times 3 \times 3$ is the *prime factorization* of 18.

factor (verb): To split a number into its factors (see above definition of factors).

formula (noun): A rule or fact written with mathematical symbols. A formula usually has an equals sign $=$ and two or more variables.

multiple (noun): A number that can be divided by another number evenly, with no remainder. 25 is a *multiple* of 5.

numeral (noun)**:** A symbol or name for a number. 12 and twelve are both *numerals*.

place value (noun)**:** The value of each position in a number. In the number 4,967,285, the *place value* of the 7 is 1,000.

power (noun): A way of showing repeated multiplication. The base of a *power* shows what value is being multiplied. The exponent shows how many times it is multiplied.

powers of ten: 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, etc.

powers of two: 2^{-2}, 2^{-1}, 2^{0}, 2^{1}, 2^{2}, etc.

prime number (noun): A number that has exactly two factors (1 and itself)

product (noun): The result of multiplication. 4 times 5 gives a *product* of 20.

quotient (noun): The result of division. 20 divided by 5 gives a *quotient* of 4.

radical (noun): A symbol that means "root." *Radicals* are used for square roots, cube roots, and other roots.

square root symbol: $\sqrt{}$

cube root symbol: $\sqrt[3]{}$

reciprocal (noun): Equal to 1 divided by a number or value. The *reciprocal* of 10 is $\frac{1}{10}$. The *reciprocal* of x^3 is $\frac{1}{x^3}$.

remainder (noun): A number left over after division. 20 divided by 8 equals 2 with a *remainder* of 4.

root (noun): The solution to an equation, usually similar to $a^2 = 25$ or $a^3 = 8$

square root: A square root of a number is a value that, when multiplied by itself, gives the number. The *square root* of 25 is 5.

cube root: A cube root of a number is a value that, when multiplied by itself and then multiplied by itself again, gives the number. The *cube root* of 8 is 2.

square (noun): a 4-sided, flat shape which has four straight and equal sides, and four right (90°) angles

square number (noun): A number which is the product of two numbers which are the same

> *perfect square*: Numbers like 1, 4, 9, 16, 25, and 36, which can be formed into a square array of rows and columns. All *perfect squares* are a whole number to the second power. For example, $9 = 3^2$ and $25 = 5^2$.

variable (noun): A symbol that represents any number or a specific number. In the expression x^3, *x is a variable* that could mean any number. In the equation $2v + 3 = 15$, the *variable* v must represent 6.

volume (noun): A measurement of the 3-dimensional space something takes up, measured in cubes