Focused Strategies for Middle-Grades Mathematics Vocabulary Developme

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Principles and Standards for School Mathematics reminds us that communication is central to a broad range of goals in mathematics education (NCTM 2000). These goals include students' being able to (1) organize and consolidate mathematical thinking; (2) communicate coherently with teachers, peers, and others; (3) analyze and evaluate others' strategies; and (4) use language to express mathematics precisely. One part of communication is acquiring mathematical language and using it fluently. This article addresses learning vocabulary as one dimension of mathematics communication.

It is not surprising that even the narrow aspect of vocabulary usage presents an array of challenges to students (Kenney et al. 2005; Miller 1993; Rubenstein and Thompson 2002; Thompson and Rubenstein 2000). For example, some words are found only in mathematics, some words are shared with everyday English but have different meanings, and others are changed significantly by mathematical modifiers. **Table 1** lists several of these vocabulary challenges with examples that can be found at the middle-grades level.

Many authors have suggested general strategies to promote vocabulary development. These include developing concepts before introducing new terms, promoting cooperative learning, using journal writing, having students develop personal glossaries, making literature connections, implementing open-ended assessments that require explanations and justifications, and using cooperative reading (Borasi and Siegel 2000; Kenney et al. 2005; Rubenstein 1996; Rubenstein and Thompson 2002; Thompson and Rubenstein 2000; Usiskin 1996). Murray (2004) provides

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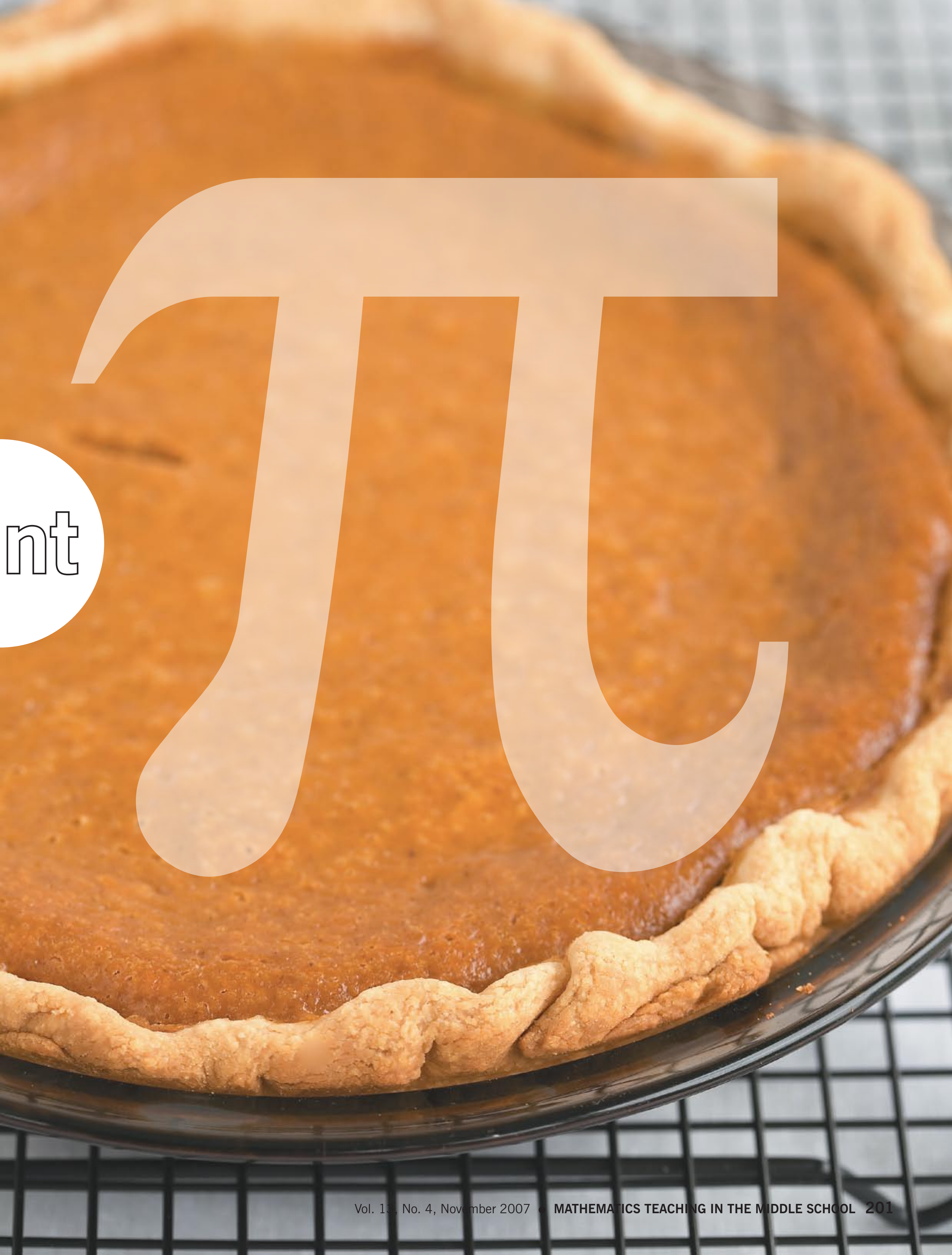
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Table 1
Categories of Mathematics Vocabulary Challenges, with Examples

Category of Challenge	Samples in Number Strand	Samples in Geometry and Measurement Strands	Samples in Algebra Strand	Samples in Data Analysis and Probability Strand
Some words are shared with everyday English, sometimes with distinct meanings, sometimes with more technical meanings in mathematics.	Fraction Product Power Prime	Similar Reflection Acute	Variable Function Origin Relation	Mode Average Median
Some words are shared with science or other disciplines.	Divide (continental) Degree	Prism Altitude Vertex	Power Degree Vertex	Experiment
Some words are found only in mathematics.	Denominator Percent	Isosceles Hypotenuse	Integer Polynomial	Outlier Histogram
Some words have multiple meanings in mathematics.	Round	Round Square Cube Tangent	Square Cube Range Tangent	Range
Some words are learned in pairs that often confuse students.	Factor and multiple At most and at least	Radius and diameter Complement and supplement Area and perimeter	Domain and range Horizontal and vertical Associative and commutative Solve and simplify	Dependent and independent Combination and permutation Horizontal and vertical
Some words sound like others (homonyms and near homonyms).	Sum, some Two, to, too Hundreds, hundredths	Pi, pie Plane, plain Complement, compliment Theorem, theory	Intercept, intersect	Leaf, leave
Modifiers change meanings of words in critical ways.	Fraction versus improper fraction Denominator versus common denominator Factorization versus the prime factorization	Bisector versus perpendicular bisector Polygon versus regular polygon Trapezoid versus isosceles trapezoid	Linear equation versus linear equation in slope-intercept form	Number versus random number Deviation versus standard deviation Score versus standard score

robust, highly contextualized strategies that are embedded in a problem-solving, inquiry-based classroom. This article moves beyond general strategies to more specific ones, focused on specific language difficulties.

Kamii and Warrington (1999) note that of Piaget's three kinds of knowledge (physical, social, and logico-mathematical), language is social knowledge, a convention that is created by people. Although acknowledging that language is a social construct, it is also important to recognize that words have etymologies and relationships to other words that provide some reason for their having been invented or selected to represent certain mathematical ideas or entities. Highlighting these connections in our teaching may help students see some of the logic or reasons behind specific terms. This article suggests how some of this logic can come forward when teachers acknowledge the challenges that students face and incorporate some focused language strategies into teaching.

WORDS SHARED WITH EVERYDAY ENGLISH OR OTHER DISCIPLINES

Many terms used in mathematics are also used in everyday language, which can present a source of confusion for students. For example, for many years I was puzzled why so many students in a general pre-assessment activity responded, "True," to the statement, "All rectangles are similar." Finally, it dawned on me that students were thinking of the nontechnical meaning of *similar*. In common English, *similar* describes things that have some commonality. In this everyday sense, all rectangles may be considered similar because they have four right angles and opposite sides that are parallel and congruent. On the other hand, in the world of mathematics, *similar* has a technical meaning that is more specific. In mathematics, figures are *similar* if they are identical in shape except that one is possibly a direct enlargement of the other. In effect, all corresponding angles are congruent and all corresponding linear measures are proportional. In this sense, a 4 in. × 6 in. rectangle and a 10 in. × 15 in. rectangle are similar, but a 4 in. × 6 in. and a 10 in. × 12 in. rectangle are not.

Another term with different everyday and mathematical meanings is the word *difference*. When asked the difference between 99 and 100, a young child responded, "Ninety-nine has two numbers, and 100 has three." This is a *difference* in the common English sense but not mathematically where *difference* is the result when two numbers are subtracted. In a related sense, *divide* in everyday parlance does not mean that a whole is divided into equal parts as the word intends in mathematics.

Another example is the word *fraction.* In everyday parlance, someone might say, "I make a fraction of what Bill Gates makes." In common language, *fraction* refers to a very small portion of some whole. In mathematics, in contrast, 9/10 could be a fraction, as could 20/20 or 15/4. Mathematically, a fraction need not be a value less than 1, let alone a value close to 0.

Further difficulties arise when we find words that are shared with science or other disciplines. For example, *prism* is used quite differently in the two subjects. In mathematics, a prism is a three-dimensional shape with two congruent parallel polygon faces (called *bases*) and parallelograms joining their corresponding points that make the lateral surfaces. In science, a prism is a piece of transparent material, often cut glass, that bends light; a pendant or a piece of a chandelier might be a prism. The shape of the prism in science may be a mathematical prism or it may be a mathematical pyramid or some other less special shape.

How do we help students recognize these nuanced differences of usage? One first step is being aware of the potential for confusion and being alert to words that are used differently in different realms. Next, we need to find ways to make explicit to students that words may have narrower or more specific meanings in mathematics than they do in everyday life or other disciplines. They need to see the mathematical meaning and compare it, both in terms of commonalties as well as differences, to the everyday English word. Through such conversations, students can learn the rationale behind why someone may have chosen a particular word. For example, the word *median* is both the center section on a divided highway *and* the center of a set of data when sequenced from the least to the greatest values. Both words share the meaning of "middle." But the mathematical meaning is more precise, in that we are first sequencing our set of data, then finding the middle value or the average of the two middle-most values.

Challenge yourself

Consider the word *acute* as used in everyday English (acute pain, acute vision) and as used to describe angles. How are the two meanings alike? How are they distinct? How can you help students recognize the everyday meaning and how that meaning may help them make sense of the mathematical meaning?

Words found only in mathematics

Some words are found solely within mathematics. When students encounter words like *denominator, isosceles,* or *polynomial*, they sometimes wonder, "Who thought up these hard words?" Words that are used solely within mathematics make some students believe that mathematics is a foreign language. How do we help students make sense of these terms and feel comfortable using them? One powerful strategy is to introduce students to the etymologies, or historical roots of words. A good source for mathematical word origin explanations is Schwartzman's *The Words of Mathematics* (1994). He provides a dictionary of nearly every term used at every level of mathematics, its etymology, related everyday words, a mathematical definition, and often graphics or other tools to support the understanding of meanings. He notes that "technical terms aren't just arbitrary syllables designed to make [students'] lives more difficult" (1994, p. 1). He reveals the clues within word origins that can help students *and* teachers see why someone chose or created that word to describe a mathematical idea. Other sources contain specific ideas for middle school and high school teachers (Rubenstein 2000; Rubenstein and Schwartz 2000, 2002, 2003).

Word origins provide a valuable link to the hidden meanings in words, and they help us understand why related words have the same roots. For example, *denominator* comes from the root for "name." From the same root, we have *nominate,* which can mean "to name someone for an office." In addition, when someone uses a *misnomer,* they are using a "wrong name" or "wrong word." When we work with fractions, the denominator *names* the unit. For example, to understand the meaning of 3/5, we must imagine some whole that is broken into five equal parts. Each of these parts is called *one-fifth*. Then we need to replicate that one-fifth three times to produce three-fifths. The 5 names the unit, which is fifths, and the 3 tells how many (the number, called the *numerator*) of those fifths. The words *denominator* and *numerator* are far less strange when we connect them to their different roles. [Siebert and Gaskin (2006) provide an article that focuses carefully on these two basic fraction concepts of partitioning a whole (corresponding to a denominator) and iterating that unit fraction (corresponding to a numerator).]

Many terms used in mathematics are also used in everyday language

Another strictly mathematical word is *isosceles.* It derives from the roots *isos* meaning "equal" and *skelos* meaning "leg." Both an isosceles triangle and an isosceles trapezoid have two legs, or sides, that are equal. Students may be familiar with weather-related words having the same root, like *isotherm*, or *isobar*, referring to lines on a weather map that join points with the same temperature or barometric pressure.

Polynomial, another specialized mathematics term, is used in algebra to describe expressions like $x^2 + 3x + 5$ or $xy - 2$ where two or more terms are added (or subtracted). *Poly* means "many" and *nomial* derives from a root meaning "part" or "name" (as we earlier saw in denominator). A polynomial is an expression with "many parts" or "many terms." Related mathematical words are *binomial* ("having two terms") and *trinomial* ("having three terms.")

In general, sharing word origins serves many purposes. It helps students perceive a sense of history, that words and meanings have evolved over time. It helps students build their general language knowledge and word-attack skills where they may be able to use some roots that are already known to ferret out new terms. But our major goal is that word origins may support students' making sense of mathematical terms. A suggestion for these mathematics-only words is to focus on the meaning of the roots and to relate them to familiar English words sharing the same roots.

Challenge yourself

Consider the word *percent*. What are the meanings of the roots of percent? What other familiar words use one or the other of the two roots? How does knowing the meanings of the roots help us make sense of the word's meaning?

Words with multiple mathematics meanings

Sometimes we find the same word, for example, *round, range,* or *square,* with distinct meanings in different strands of mathematics. Usually there is some reason why the same term is used in the different contexts. As with earlier cases, we want to help students see both the concept that links the two uses and the distinctions among them.

Consider, for example, the term *range* found both in statistics and algebra. In general, *range* refers to the breadth or extent of something, as in how far someone can throw a ball. In music, *range* describes how high or how low someone can sing. In statistics, the *range* of a set of data is usually the difference between the largest and the smallest values, although sometimes range refers to the interval defined by those endpoints. In algebra, the *range* of a function is the set of values that the function produces, the outputs of the function. What links these different ideas? The etymological root means "ring." The word *ranch,* describing a large western farm, comes from the same root. So one might imagine a ring of rope or some other material that loops or encompasses all of the range of a function or a data set.

Other examples are the words *square* and *cube* that arise in both geometry and algebra. Students are usually familiar with the shapes in geometry; with some help, they will see the relationship to words in algebra. Algebraically, the square of 7, or 7^2, is the area of a *square* ("shape") with edge length 7. Similarly, the cube of 7, or 7^3, is the volume of a *cube* ("shape") with an edge length of 7. Recognizing these relationships is more than vocabulary development; these connections are important mathematical ideas in their own right.

For those terms that have multiple mathematical meanings, we need to recognize and acknowledge that the same word is used in two different contexts so that students will learn to use context to realize which meaning is intended. We can identify and provide meanings in both contexts but need to find the links for ourselves so that we can then help students see them. The links may arise in the origin of the word or in the mathematics itself.

Challenge yourself

Consider the word *base* used both in geometry and in algebra. What is the meaning of base in each strand? What links the two meanings?

Word pairs that students often confuse

Factor and *multiple, radius* and *diameter,* and *combination* and *permutation* are examples of mathematical pairs that are usually learned together and that often confuse students. One strategy to reduce confusion would be to teach one term and use it so that students are comfortable, then, several lessons later, introduce the other term; in other words, do not teach the terms together. For example, radius might be emphasized in early work with circles before diameter is introduced. Students could complete activities involving the radius of a circle, such as using a compass to draw circles with different radii and using the radii to inscribe hexagons. They could construct the perpendicular bisector of a segment and think about why they can use the segment as the common radius of two congruent circles constructed from the endpoints as the centers of the two circles. Emphasize the root of *radius*, which is "ray," and make connections to familiar words: a *ray* of light, the circle *radiates* from its center by way of its *radius,* and *radio* waves *radiate*. In a later lesson, students might study secants, or segments joining two points on a circle. Describe the length of the longest secant as the *diameter*.

Another teaching strategy is to acknowledge that the terms are easy to confuse and invite all students to help one another keep them straight. For example, in work with number theory, students commonly confuse *factor* and *multiple*. One teaching strategy is to emphasize that multiples are numbers found on a multiplication table; for example, multiples of 4 are 4, 8, 12, and so on and are derived from multiplying 4 by 1, 4 by 2, 4 by 3, and so on. We can also help students see the analogy to related English words, for example, that *factors* produce products, just as *factories* produce products. Despite these efforts, students often misspeak and confuse these terms. Both factor and multiple come from multiplication, and they are commonly confused. We have to help one another use the right word in the right place. Whenever the words are used, ask yourself and others to double-check the usage.

Challenge yourself

Consider the word pair *horizontal* and *vertical*. What word origins might help students distinguish the two terms? How might students support one another in distinguishing the terms?

Words that sound alike

Students are sometimes confused by homonyms or near homonyms. An apple pie may be circular, but that is not why the ratio of a circle's circumference to diameter is called *pi*. Pi is a Greek letter, written π, and the first letter of the Greek word for *perimeter* or *periphery*, which corresponds to the circumference, the numerator of the ratio on which pi is based. We want students to distinguish *pi* and *pie* even though they sound alike. One strategy is to say, spell, and use the two words in context. Students might even be invited to produce cartoons in which homonyms are the source of a joke.

Word origins, again, may be helpful. For example, why does the *complement* of an angle, the amount needed to make 90 degrees, sound like *compliment*, telling someone they look good? Both words are derived from a root meaning "to fill." When we find the complement of 30 degrees (which is 60 degrees), we are "filling up" the first angle so that it is a complete 90 degree angle. When we say something positive about individuals, a *compliment*, we are filling them up or making them feel fulfilled.

In general, for terms that sound alike, as in other cases, we need to say them clearly, acknowledge that other words sound the same, identify those other words, say and spell each, define them, and use each in context.

Challenge yourself

Consider *intersect* versus *intercept*. Find the origins of each word, and think about the mathematical meaning. How might the word origins and the mathematics help students understand how the words are alike and how they are different?

Words with modifiers

Mathematics is terse. Every word matters. This is a major difference between everyday English usage and mathematics. Built-in redundancies occur in languages. Subject-verb agreement, tense, case, and other structures help us reconfirm that we are hearing the intended message. In contrast, in mathematics each symbol or word

Table 2
Summary of Focused Language Strategies

Category of Challenge	Focused Strategy
Some words are shared with everyday English, science, or other disciplines and may have distinct or more technical meanings within mathematics.	Be aware of potential confusion. Distinguish the technical from the everyday meanings. Help students understand what is shared by the terms and the reason why the common language term was adopted for mathematics.
Some words are found only in mathematics.	Help students see the roots and origins. Point out common English words with the same root. Help them see how the roots build the mathematical meaning.
Some words have multiple mathematics meanings.	Remind students of the multiple usages and to use context clues to know which meaning is intended. Help them see why the word makes sense in each context.
Some words are learned in pairs that often confuse students.	If possible, separate the learning of the two terms so that one is well understood before the second is introduced. Continue to use word origins and relations of each of the two terms to related everyday English words. Acknowledge the challenge, and have students double-check one another when they use the words.
Some words sound like others (homonyms and near homonyms).	Say the words clearly. Spell them. Distinguish them. Use them each in their particular context.
Sometimes modifiers change meanings of words in critical ways.	Have students explore the unmodified term, the modifier, then the full phrase (e.g., *bisector, perpendicular, perpendicular bisector*). Help them see the broad category as well as the specific case within it.

matters. In particular, a modified noun means something different from the unmodified word. For example, a *perpendicular bisector* of a segment is vastly different from a *bisector*. We want students to pay close attention to modifiers and to the differences they make. In the case of bisector versus perpendicular bisector, ask students to show many different bisectors for a segment and many different perpendiculars to a segment; any that fit both qualities will be a *perpendicular bisector*.

We could also ask students to create many different hexagons, then have them consider what it would mean for a hexagon to be *equiangular* (but not equilateral), *equilateral* (but not equiangular), and both *equilateral* and *equiangular*. The last case is so special that we call it a *regular hexagon*. (This is another instance in which the curriculum sequence is important. For example, the terms *equilateral* and *equiangular* are often first introduced in reference to triangles. But in a triangle, the two different properties always come together, so we have no way of distinguishing them.)

In general, we want to engage students in exploring the general case (an unmodified term, e.g., *trapezoid*) and the specific case (a modified term, e.g., *isosceles trapezoid*). We want to have students explain what is special about the specific case and how it is a part of the broader category.

Challenge yourself

Consider the pair *factorization* and *prime factorization*. What are some ways to engage students in exploring the distinction between these two phrases?

CONCLUSIONS

Communicating is a broad goal in mathematics learning that provides means to develop and share understanding. Even the narrow strand of vocabulary within communication presents challenges. We want to explore mathematical concepts with students in ways that support their understanding of the nuances of meaning, to distinctions from uses of the same words in common everyday English or other disciplines, and to the logic of that choice of term. We want them to work respectfully and thoughtfully to support one another in their language-learning endeavors. As well as the general language and communication-supportive strategies of journals, open-ended assessments, cooperative learning, and others, the focused strategies suggested in this article and summarized in **table 2** may serve as more refined tools. Language is a major medium of teaching and learning mathematics; we serve students well when we support them in learning mathematical language with meaning and fluency.

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