Area Matters

A geometry lesson set for use in the pre-GED and GED classroom.

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Area Matters Lessons

Introduction to Area

Lesson #1

Estimated total teaching time—4 hours (6 activities)

Rationale:

Students may dutifully memorize formulas for calculating the area of various figures, but they often have little conceptual sense of area measurements, especially the significance of area notation and units. This lesson is designed to reveal that we measure area using squares. Concrete models for measuring area, including color tiles, centimeter grids, and even masking tape on the floor will be used to cement this concept. Students will also become familiar with conventional notation for reporting area measurement.

Objective:

The student will be able to distinguish instances where area, length, and volume measurements are appropriate. The student will be able determine the area of figures using centimeter grids, 1-inch color tiles, 1-inch graph paper, 1-foot floor tiles, and "unit" grids on index cards. The student will be able to report and interpret area measurements in conventional notation.

Materials:

Handout: *Area, Length and Volume*
Handout: *Measuring Area with Color Tiles*
Handout: *Measuring Area Using Grids*
Handout: *Creating Certain Areas, 1-Inch Grid Version*
Handout: *Creating Certain Areas, 1-Centimeter Grid Version*
Handout: *Taping Floor Area*
Handout: *Comparing Areas*
1-inch color tiles
Roll of 1-Inch or 1-Centimeter Graph Paper
5-6 rolls of masking tape and access to 1-foot square floor tiles (*if available*)
Large sheets of newsprint/butcher paper
1 classroom set of inch/centimeter rulers
Gridded index cards

Lesson Development:

1. Starting with the handout *Area, Length, and Volume,* ask your students to tell you what they imagine when you say you want to measure the <u>length</u> of something. You are likely to get a variety of responses, such as

   - How tall something is.
   - From here to there.
   - Length times width.
   - Perimeter.

When we say "length", it touches all sorts of student experiences. For students who were drilled into memorizing formulas when they were in school, they are more likely to respond by saying something such as "Length times width." When they do this, they associate "length" with the name of two sides of a rectangle. We are looking at the word "length" differently here—as a type of measurement. If students get stuck on this, encourage them to think about length as we use it in everyday life.

When I want to measure the length of an object, such as my arm, what am I doing? <u>How</u> do I do it?

2. A student who says something such as "Length is from here to there," or "How tall something is," is closer to the idea we are looking for. Ultimately, have students write the following beside "length":

The distance between two points.

Direct students to the image at the right of the line. It is a tape measure. If you can imagine measuring something with a tape measure (even if you need a *really* long one), then it is a measure of length. Question your students to asses their understanding.

Is the height of a building a measure of length? Why or why not?
Is the distance between you and me a measure of length?
How about the distance between you and Beijing, China?
Is the amount of apple juice in a cup a measure of length? Why or why not?
Can you give me another example of a measure of length?

3. This is an important moment in the area lesson set. Ask your students what they imagine when you say you want to measure the <u>area</u> of something. If students have ideas, they could include the following:

- Length times width.
- Base times height.
- Perimeter?
- Covering something.

Students who make one of the comments at the left are recalling formulas drilled into them. They are also only considering rectangular objects. Students who say perimeter probably know that there is some relationship between area and perimeter, but might not be sure what the relationship is. A student who says something such as "covering something" is on the right track. Have your students write the following beside "area" on the handout:

The size of a surface.

4. Examples of surfaces include the chalkboard, the tops of student desks/tables, and the walls. A surface can also include contours or bumps, such as a globe or a person's head.

5. It will help your students to remember the image of the paint can. A surface that can be measured using area is something you could imagine *painting*. Ask for more examples from your students.

6. It probably will not help for you to tell your students that area is 2-dimensional (and that volume is 3-dimensional). If you do, you will likely have to teach them what 2-dimensional means.

7. Your students will have the smallest number of experiences with volume. Look at the image—the pitcher pouring into a cup. If you can imagine pouring liquid (including enormous or tiny amounts) into something, then it can be measured using volume. Have students record the following:

   The space inside an object.

8. Examples of objects whose volume can be measured include a refrigerator, a closet, and a desk drawer. You have to use your imagination to realize a person's head has volume also, as it could also be filled with liquid.

9. It can be confusing for your students to realize that a single object can be measured using length, area, and volume. Choose an object in the classroom such as the trash bin as the focus for a series of questions.

   What part of this trash bin could I measure that would be a measure of length?
   (It's height, among other possible answers.)

   What part of this trash bin could I measure that would be a measure of area?
   (The surface around the outside, among other possible answers.)

   What part of this trash bin could I measure that would be a measure of volume?
   (The space inside the can that I could fill with trash.)

10. When you ask your students to identify the appropriate measurements for the list in the handout *Area, Length, and Volume*, be aware that there may be more than one appropriate measurement. The key is that students must be able to defend their answer. In the case of the gap between the middle front teeth, that could certainly be a measure of length because the distance between the teeth could be measured with a ruler. However, you could also visit a dentist who could fill the gap with a false tooth substance—in this way, you could measure the size of the gap as volume.

11. Once we have established that area measurement concerns the size of a surface, we need to look more closely at how this is measured.

12. In order to complete the color tile activity, have rulers available for every student. Before giving them the handout, ask students to measure one tile. If they want to know which side of the ruler to use, ask

them to decide which makes the most sense. They are squares that measure 1 inch per side. Some students will need help using a ruler even to measure one inch. For example, students may think that you must begin measuring from the edge of the ruler, even though some rulers have a "0" mark inside of the edge. Put yourself near their desks as they do this. Students will be using rulers in most every one of these activities, giving them increasing power to measure with this tool, and also gradually giving them a "feel" for the size of 1 inch, 1 centimeter, 1 foot, and the related square measurements.

From here onward, have rulers available to every student during all *Area Matters* activities. It is not a bad idea to have a variety of rulers (as long as all have both cm and in scales). Being familiar with slightly different ruler formats will strengthen your students' measurement abilities.

13. Give students the handout *Measuring Area with Color Tiles* and allow students to cover the rectangle in #1 with tiles. Once they have done that, ask what they found. They should have used exactly 12 of the 1-inch square tiles. This is the moment to tell your students how mathematicians measure area.

Mathematicians measure area based on how many squares can cover a surface.

On the line below the figure, have students write the following:

Area of Rectangle ABDC = 12 square inches

This statement shows how we record area measurements.

What do you think the 12 stands for? Why do you think the word "square" is used? Why do you think it says square inches?

14. *In case any student asks why you labeled the rectangle ABDC instead of ABCD, turn it around and ask them why they think it is improper to refer to the rectangle as ABCD. In naming a figure such as the rectangle, you have to follow lettered points in order—you cannot jump across the figure. It does not matter where you start, or which direction you follow, but you cannot jump.*

15. You should have students write the area measurement in two more ways. See below.

Area of Rectangle ABDC = 12 square inches or 12 sq. in. or 12 in^2

Students should recognize the second one is simply abbreviating the words. The third version, however, can be very confusing. The 2 appears to be an exponent. Most of your students will not have much experience working with exponents, but even if they do, the purpose of the 2 is not to tell you to do any calculations. The 2 is a symbol for "square". This tells you the shape that is used to cover the surface. It might be useful for students to put the following diagram in their notes.

Squares are used to cover the surface.

12 in^2

Each square measures 1 inch per side.

12 of these squares can cover the surface without gaps or overlapping.

16. Students should be able to find the areas of the other three figures on their own. When students report their results, see if anyone remarks on the identical areas for rectangles #1 and #2. If not, ask about it:

Is this an interesting result? What does this demonstrate about area?

It shows that figures that have different shapes can have the same area. Beware that students may say that the figure in #1 is a rectangle, and that the figure in #2 is a square. Instead of contradicting this, see if the students themselves (using rulers) can decide whether or not that is the case.

17. The centimeter grid handout changes a few of the elements from the previous activity. For one, the squares that we are using to measure area are 1 centimeter per side, instead of 1 inch per side. Allow time for students to explore and decide this with rulers. Secondly, we are not fitting tiles inside the figure. In these cases the squares are already there—they simply need to be counted. Do not tell students that the triangular regions are halves in the third example. Also resist telling them how to combine the halves. Allow other students to give this advice. Students may accidentally continue to report area measurements in square inches. Watch out for this.

    This is also a good opportunity for you to introduce the concept of a rectangle. Ask students if any of the figures is a rectangle, and inquire about how they decided on their answer.

18. Try to gain access to 1-inch graph paper so you can do the inch version of *Creating Certain Areas*. If that is impossible, use the centimeter version and make photocopies of the centimeter grid that follows the instruction sheet. If you must use the centimeter version, make the groups smaller (for example, pairs) because the drawings will be much smaller. Make plenty of copies of the grids.

19. The first two activities asked students to find areas of given figures; in *Creating Certain Areas*, students must create figures that have a given area. This is a very rich group activity that can suffer from too much involvement by the teacher. When students are having difficulty, ask them to restate the problem, ask them what they have tried, ask them why their efforts have not worked, but resist telling them how to proceed.

    As students are working and are successfully drawing figures to solve these problems, interrogate them on their work with statements or questions such as the following:

    Tell me about what you have drawn here.
    Convince me that this figure is a solution to #4.

    It is important for you to make these kinds of statements as you wander between the groups, even when the groups have created accurate solutions to the problems. Students will develop better mathematical understanding if they are asked to articulate their reasoning to each other and to you.

20. These activities require students to create rectangles. Many students "know a rectangle when they see one" but have difficulty describing them in words. You have a couple of choices here. Firstly, you

could pass out the handouts and then engage students in what makes a figure a rectangle while they are working on the problems (my preference, because you are able to have discussions with more students). Alternatively, you could lead the class as a whole in a discussion of the properties of a rectangle. Remember, though, this is an activity whose focus is on area. Most students have enough of an idea of what a rectangle is that you do not need to get bogged down in definition work with your students. If you feel you must clear this up before proceeding, consider the following discussion sequence:

What makes a figure a rectangle?

Ask a student to sketch a rectangle on the board. Maybe also draw a 4-sided figure that is not a rectangle for contrast.

What makes *that* figure a rectangle?

Students will be more able to give some characteristics when the figure is in front of them. They may say that the opposite sides must be the same length. Sometimes they add that two sides must be "longer" and two sides "shorter". This second characteristic is not a requirement for rectangles, even though this is often true.

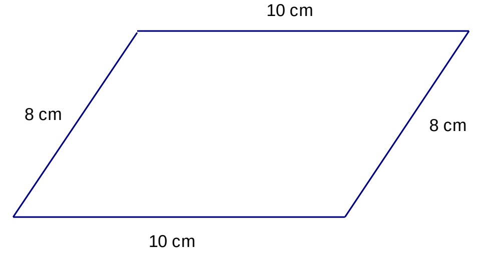
21. You can push your students further by asking the following:

    Is a square a rectangle?

    Most will say "no", but ask them to consider squares in light of the previous definition—that a rectangle is a 4-sided figure whose opposite sides have the same length. Draw a square and point to a pair of opposite sides:

    Are these opposite sides the same length? How about the other pair of opposite sides—are they the same length? Ultimately, the students should record that squares are a special kind of rectangle.

22. If you feel you must also address the angles of a rectangle, consider drawing the following figure on the board:

    With this figure, you can then focus a discussion on the requirement that rectangles always include 4 right angles.

23. It can be tempting to give students more help than they need to make the drawings in *Creating Certain Areas*. In particular, students may struggle with #5 which requires halves. Resist telling them to use halves. If they are struggling, ask them why they are finding it difficult. Encourage them to draw what they can of the solution, as a way of <u>showing</u> what is difficult. They will be able to show you that when two of the sides are 2 inches, they can only create a rectangle with 12 square inches or 14 square inches. Once you are looking at it in front of you, try the following questions:

Pointing to the larger rectangle… What is wrong with ending it here? (Too much area)

Pointing to the smaller rectangle… What is wrong with ending it here? (Too little area)

What can I do to create a rectangle whose area is just right?

Students are most likely to see the solution if they are working on an actual drawing. If they are only trying to visualize a solution, it is much more difficult. Students will often show great satisfaction at solving this problem without much of your help. Give them that chance.

24. Many classroom floors (or hallways) are covered with foot-square tiles, and if that is true in your classroom you should definitely do the masking tape activity. If you have access to nearby classrooms that also have these tiles, consider breaking the class up so that groups have plenty of room to work. Students should measure the tiles with a ruler to confirm their dimensions. Encourage creative figures in problem #3. You will need large newsprint/butcher paper that students can use to explain their estimates of the area of the classroom floor. Often, classrooms include partial tiles at the edges, and you should draw careful attention to how each group handle these.

    Once the groups have finished their work, bring the whole class to each group's area to review the work. Students should confirm the work done by each group. Pay careful attention to how they report units. Ask students to calculate the area of the figures created for #3 and confirm their solution with the group that created it.

25. Your students may be confused about "3.5 square feet". Do not simply tell them that 3.5 means three-and-one-half. As an alternative, tell them that 3.5 is the same as 3.50. Ask them to consider 3.50 in money terms, and ask until you hear from them that 3.50 is "three and one-half dollars". Now you are in a position to ask them what .5 or .50 indicates.

    For the challenge problem, the money connection can also be very helpful.

26. Index cards with grid designs allow more concrete opportunities to explore area (and later, perimeter). Draw different figures on the cards, 1 per student. Hand them out to the class, and ask them to organize themselves along one wall in ascending order, according to the area of their figures. If two or more students have figures with the same area, have them stand in front of each other.

    Once they are organized, you might ask them to report the area of their figure. Are they square inches? Square centimeters? The students should be able to confirm with a ruler that they are neither square inches nor square centimeters. In some cases, the size of the squares is not given or known. In these cases, we report area as the number of "square units" and length as the number of "units."

27. For the activity *Comparing Areas*, begin by placing all of the following items on a table at the front of the classroom:

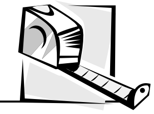
    Rulers

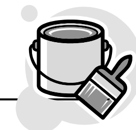
1-inch graph paper
Centimeter paper (photocopy full-page centimeter grids from Lesson #2 handouts)
Color tiles
Masking tape
(If 1-foot floor tiles are available, that is also beneficial.)

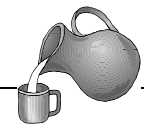
Invite the students to use any of the materials they wish in order to display and explain their solution. Don't tell the students the sizes of the various grids. They have rulers and at this point they should be able to confirm measurements for themselves.

28. When your students are largely finished, ask them to hold up their depictions of 120 cm^2. Have them look at each other's work, especially the different ways that the same area was represented. Do the same with the other two measurements. Once you have reviewed all three, ask your students to compare the three and decide which is the largest and which is the smallest. Ask them to explain their reasoning.

Area, Length, and Volume

1. **Length** ____ 

2. **Area** ____ 

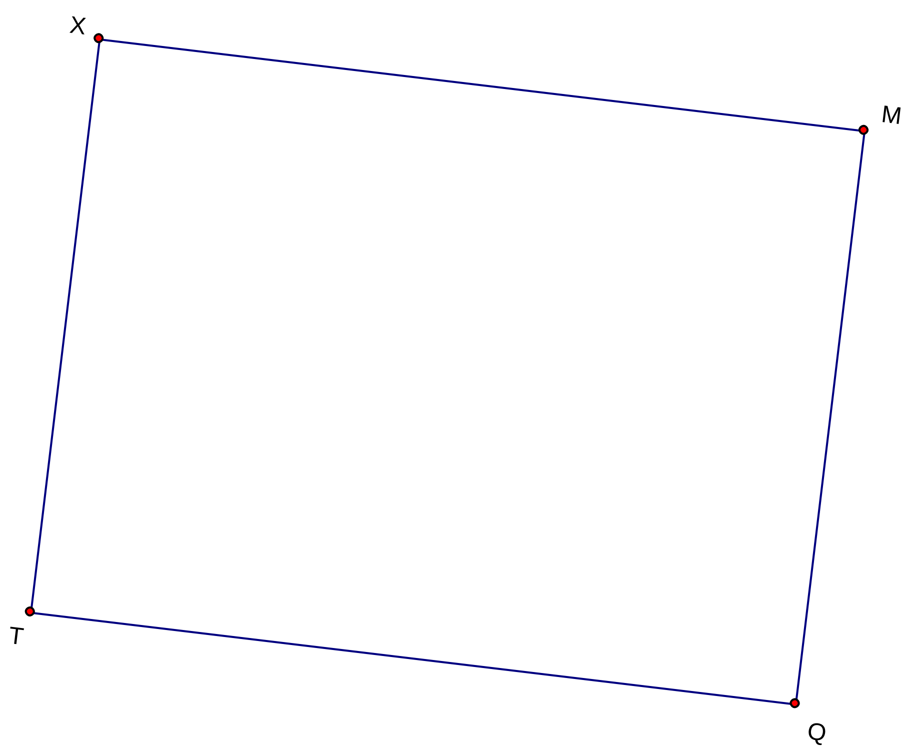
3. **Volume** ____ 

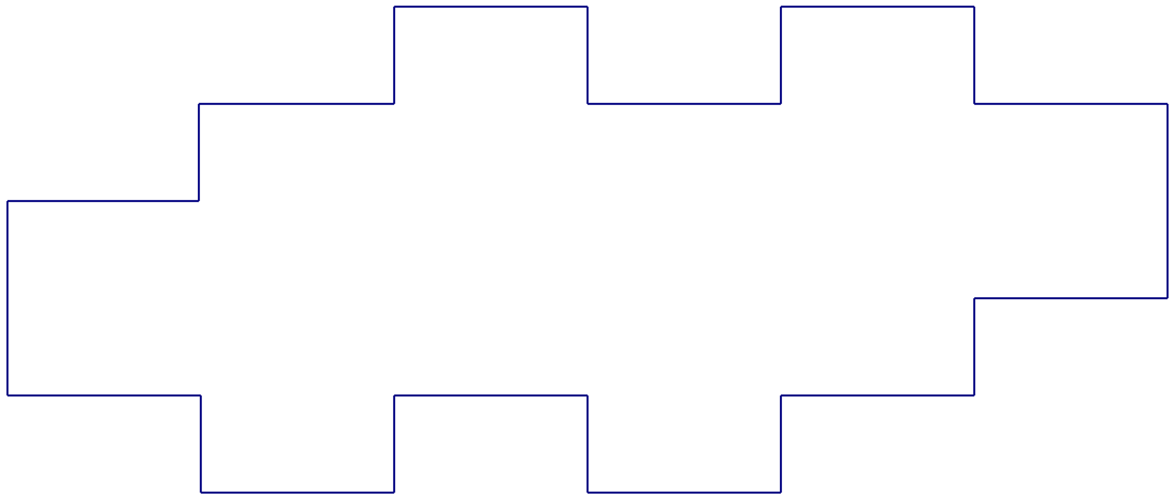
4. Which would you use to measure the following—length, area, or volume?

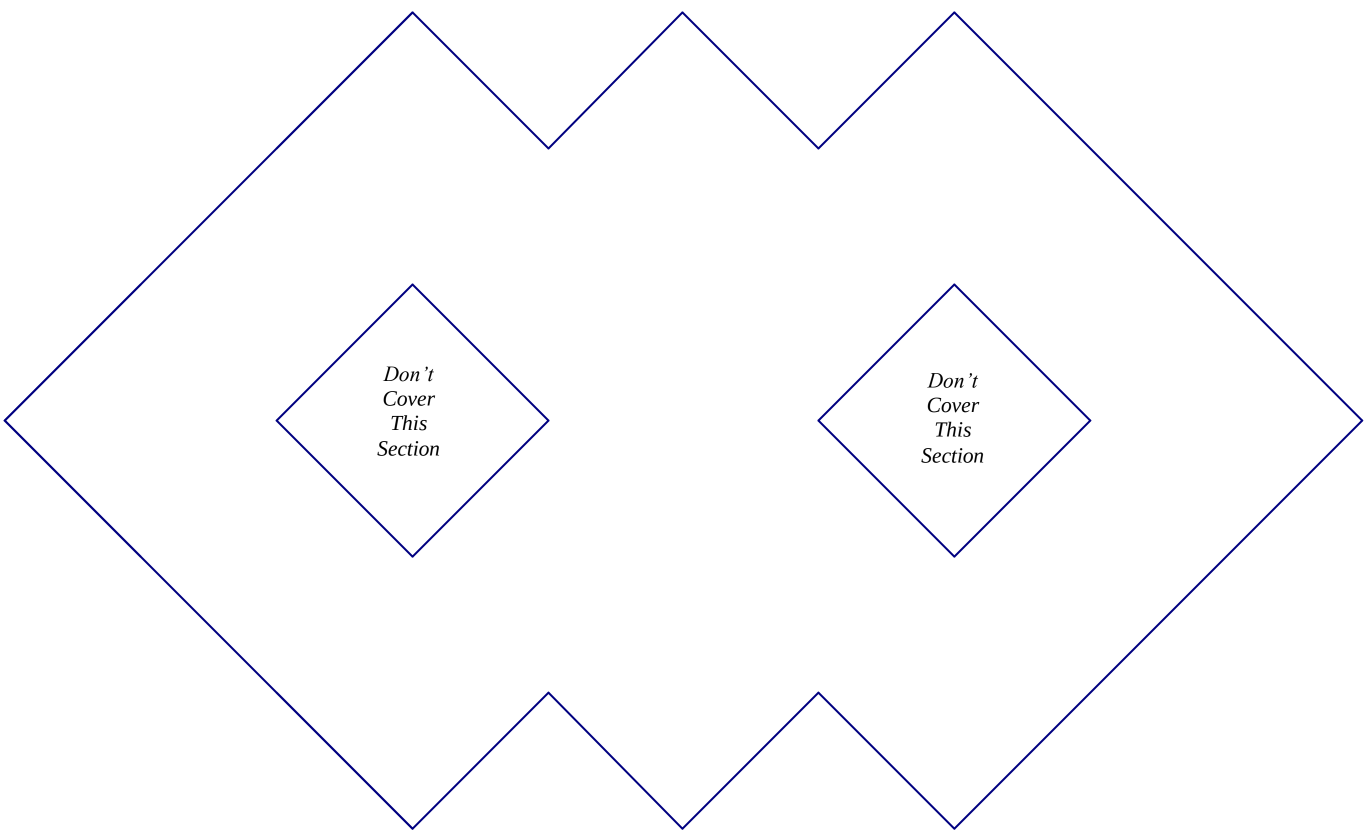
- The distance from the floor to the ceiling.
- The size of the chalkboard.
- Size of a pool—how much water it will hold.
- Your teacher's height.
- The amount of skin on your body.
- The floor space in an apartment.
- The gap between your middle front teeth.
- Space for juice in a container.
- The tissue paper needed to wrap a present.
- Truckloads needed to carry away a pile of dirt.
- Number of shingles needed to cover a roof.

Measuring Area with Color Tiles

Cover the following figures using 1-inch color tiles.



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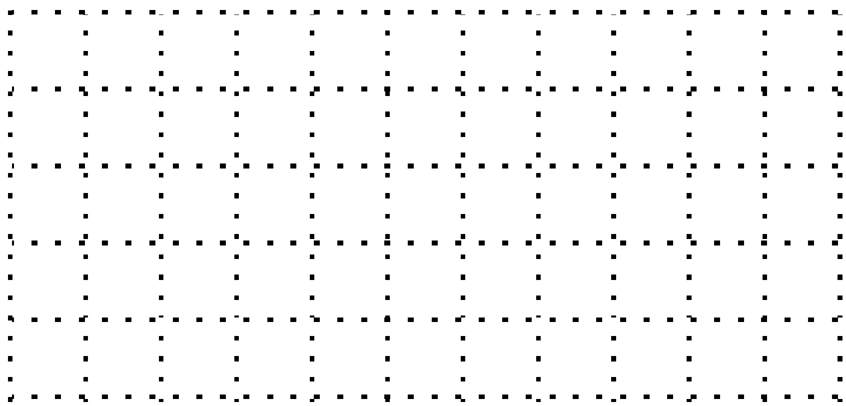
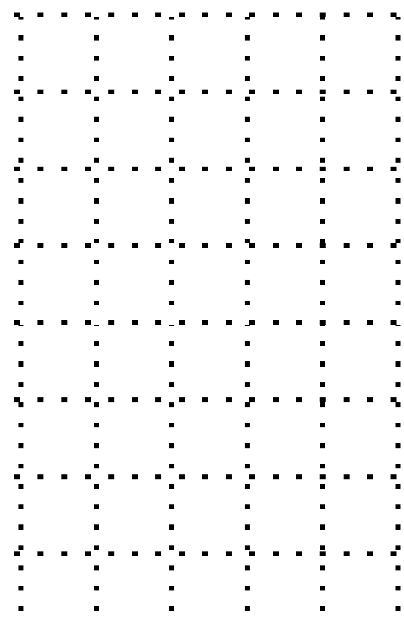
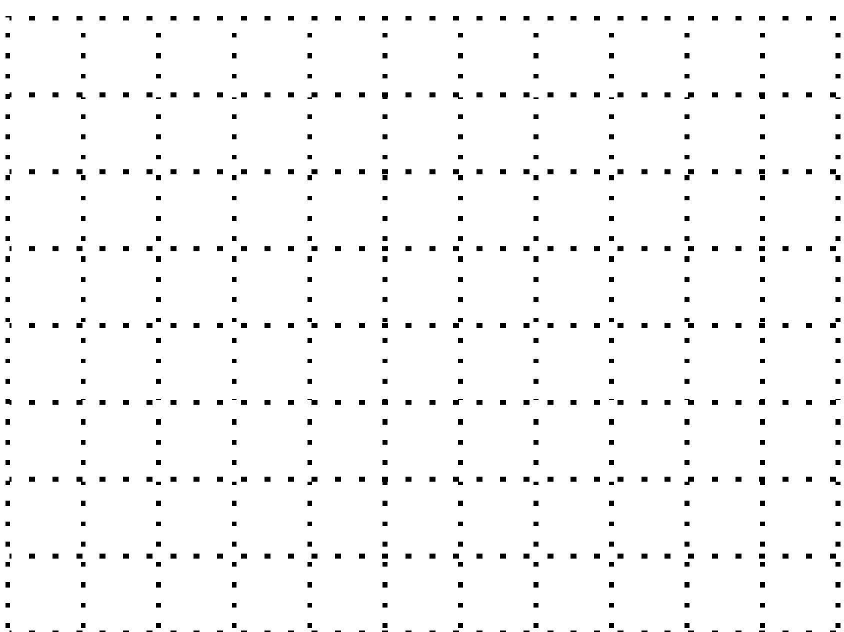
Measuring Area Using Grids

Find the area of each figure. Report the areas with appropriate units.

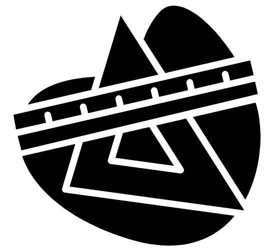






Creating Certain Areas

1-Inch Grid *Graph Paper Version*



In groups of 2-3 students, complete the following problems using a large sheet of graph paper whose squares measure one inch per side. Label each drawing according to the problem number. Be sure that all group members agree on a drawing before you continue to the next one.

1. Create a rectangle whose area is 14 in^2.

2. Create two rectangles with different shapes that both have an area of 15 square inches.

3. Create a figure that is not a rectangle, and that has an area of 6 square inches.

4. Create a rectangle with two sides that each measure 4 inches, and whose area is 36 square inches.

5. Create a rectangle with two sides that each measure 2 inches, and whose area is exactly 13 in^2.

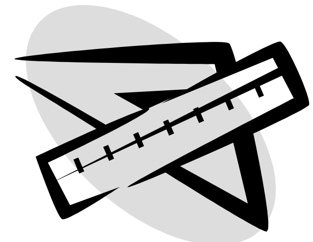
6. Create a rectangle with two sides that are each longer than 5 inches, and whose area is 8 square inches.

7. Create a rectangle with two sides that each measure 4 inches, and whose area is 22 square inches.

Creating Certain Areas

***1-Centimeter Grid** Graph Paper Version*

In groups of 2-3 students, complete the following problems using a large sheet of graph paper whose squares measure one centimeter per side. Label each drawing according to the problem number. Be sure that all group members agree on a drawing before you continue to the next one.



1. Create a rectangle whose area is 28 cm^2.
2. Create two rectangles with different shapes that both have an area of 22 square centimeters.
3. Create a figure that is not a rectangle, and that has an area of 14 square centimeters.
4. Create a rectangle with two sides that each measure 4 centimeters, and whose area is 36 square centimeters.
5. Create a rectangle with two sides that each measure 2 centimeters, and whose area is exactly 13 cm^2.
6. Create a rectangle with two sides that are each longer than 9 centimeters, and whose area is 16 square centimeters.
7. Create a rectangle with two sides that each measure 4 centimeters, and whose area is 22 square centimeters

Taping Floor Area

This activity requires a room with square floor tiles that measure one foot per side.

1. In a group formed by your teacher, use masking tape to create a rectangle on the floor whose area is 6 square feet.

2. Use masking tape to create a figure on the floor that is <u>not</u> a rectangle and whose area is 3.5 square feet.

3. Create a figure on the floor using masking tape and calculate its area. Other groups will be asked to find this area.

4. Estimate the area of the entire classroom floor. One member should record the process your group used in determining this.

<u>Challenge</u>: Use masking tape to create two rectangles that have different shapes and that each have an area of 1.25 square feet.

Comparing Areas

120 cm^2 45 in^2 2 ft^2

Using any of the tools we have encountered in these area lessons, display each of the three area measurements shown above. Be prepared to explain how your creations represent the given measurements.