My Special Number

Many people have a number they find interesting. Choose a whole number between 10 and 100 that you especially like.

In your journal

- record your number
- explain why you chose that number
- list three or four mathematical things about your number
- list three or four connections you can make between your number and your world

As you work through the investigations in *Prime Time*, you will learn lots of things about numbers. Think about how these new ideas apply to your special number, and add any new information about your number to your journal. You may want to designate one or two "special number" pages in your journal, where you can record this information. At the end of the unit, your teacher will ask you to find an interesting way to report to the class about your special number.

The Factor Game

Today Jamie is 12 years old. Jamie has three younger cousins: Cam, Emilio, and Ester. They are 2, 3, and 8 years old respectively. The following mathematical sentences show that Jamie is

6 times as old as Cam,   4 times as old as Emilio,   and $1\frac{1}{2}$ times as old as Ester

$$12 = 6 \times 2 \qquad\qquad 12 = 4 \times 3 \qquad\qquad 12 = 1\frac{1}{2} \times 8$$

Notice that each of the whole numbers 2, 3, 4, and 6 can be multiplied by another whole number to get 12. We call 2, 3, 4, and 6 *whole number factors* or *whole number divisors* of 12. Although 8 is a whole number, it is not a whole number factor of 12, since we cannot multiply it by another whole number to get 12. To save time, we will simply use the word **factor** to refer to whole number factors.

1.1 Playing the Factor Game

The Factor Game is a two-person game in which players find factors of numbers on a game board. To play the game you will need Labsheet 1.1 and colored pens, pencils, or markers.

Problem 1.1

Play the Factor Game several times with a partner. Take turns making the first move. Look for moves that give the best scores. In your journal, record any strategies you find that help you to win.

▪ Problem 1.1 Follow-Up

Talk with your partner about the games you played. Be prepared to tell the class about a good idea you discoved for playing the game well.

The Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Factor Game Rules

1. Player A chooses a number on the game board and circles it.
2. Using a different color, Player B circles all the proper factors of Player A's number. The **proper factors** of a number are all the factors of that number, except the number itself. For example, the proper factors of 12 are 1, 2, 3, 4, and 6. Although 12 is a factor of itself, it is not a proper factor.
3. Player B circles a new number, and Player A circles all the factors of the number that are not already circled.
4. The players take turns choosing numbers and circling factors.
5. If a player circles a number that has no factors left that have not been circled, that player loses a turn and does not get the points for the number circled.
6. The game ends when there are no numbers remaining with uncircled factors.
7. Each player adds the numbers that are circled with his or her color. The player with the greater total is the winner.

A sample game is shown on the following pages.

1.2 Playing to Win the Factor Game

Did you find that some numbers are better than others to pick for the first move in the Factor Game? For example, if you pick 22, you get 22 points and your opponent gets only $1 + 2 + 11 = 14$ points. However, if you pick 18, you get 18 points, and your opponent gets $1 + 2 + 3 + 6 + 9 = 21$ points!

Make a table of all the possible first moves (numbers from 1 to 30) you could make. For each move, list the proper factors, and record the scores you and your opponent would receive. Your table might start like this:

First move	Proper factors	My score	Opponent's score
1	none	lose a turn	0
2	1	2	1
3	1	3	1
4	1, 2	4	3

Problem 1.2

Use your list to figure out the best and worst first moves.

A. What is the best first move? Why?

B. What is the worst first move? Why?

C. Look for other patterns in your list. Describe an interesting pattern that you find.

Problem 1.2 Follow-Up

1. List all the first moves that allow your opponent to score only one point. These kinds of numbers have a special name. They are called **prime numbers.**

2. Are all prime numbers good first moves? (A number is a good first move if the player picking the number scores more points than his or her opponent.) Explain your answer.

3. List all the first moves that allow your opponent to score more than one point. These kinds of numbers also have a special name. They are called **composite numbers.**

4. Are composite numbers good first moves? Explain your answer.

5. Which first move would make you lose a turn? Why?

Mathematical Reflections

In Investigation 1, you played and analyzed the Factor Game. These questions will help you summarize what you have learned:

1. Which numbers are good first moves? What makes these numbers good moves?

2. Which numbers are bad first moves? What makes these numbers bad moves?

3. What did your analysis of the factor game tell you about prime numbers?

Think about your answers to these questions, discuss your ideas with other students and your teacher, and then write a summary of your findings in your journal.

Have you remembered to write about your special number?
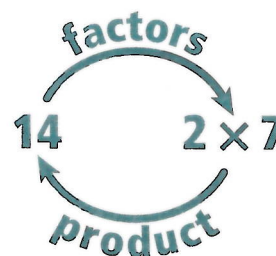
Names: _____

Date: _____

The Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

INVESTIGATION 2

The Product Game

In the Factor Game, you start with a number and find its factors. In the Product Game, you start with factors and find their product. The diagram shows the relationship between factors and their product.

factors
14   2 × 7
product

2.1 Playing the Product Game

The Product Game board consists of a list of factors and a grid of products. Two players compete to get four squares in a row—up and down, across, or diagonally. To play the Product Game, you will need Labsheet 2.1, two paper clips, and colored markers or game chips. The rules for the Product Game are given on the next page.

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:
1 2 3 4 5 6 7 8 9

Problem 2.1

Play the Product Game several times with a partner. Look for interesting patterns and winning strategies. Make notes of your observations.

Product Game Rules

1. Player A puts a paper clip on a number in the factor list. Player A does not mark a square on the product grid because only one factor has been marked; it takes two factors to make a product.
2. Player B puts the other paper clip on any number in the factor list (including the same number marked by Player A) and then shades or covers the product of the two factors on the product grid.
3. Player A moves *either one* of the paper clips to another number and then shades or covers the new product.
4. Each player, in turn, moves a paper clip and marks a product. If a product is already marked, the player does not get a mark for that turn. The winner is the first player to mark four squares in a row—up and down, across, or diagonally.
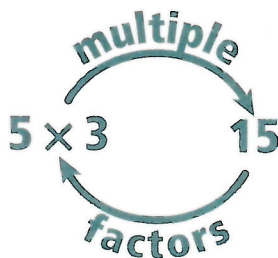
■ Problem 2.1 Follow-Up

1. Suppose one of the paper clips is on 5. What products can you make by moving the other paper clip?

The products you listed in question 1 are multiples of 5. A **multiple** of a number is the product of that number and another whole number.

If a number is a multiple of 5, then 5 is a factor of that number. These four sentences are all ways of expressing $5 \times 3 = 15$:

5 is a factor of 15.
3 is a factor of 15.
15 is a multiple of 5.
15 is a multiple of 3.

$$multiple$$
$$5 \times 3 \qquad 15$$
$$factors$$

2. List five multiples of 5 that are not on the game board.
3. Suppose one of the paper clips is on 3. What products can you make by moving the other paper clip?
4. List five multiples of 3 that are not on the game board.

2.2 Making Your Own Product Game

Suppose you want to create a product game that takes less time to play or, perhaps, more time to play than the game with the 6 × 6 product grid. You would have to decide what numbers to include in the factor list and what products to include in the product grid.

Problem 2.2

Work with your partner to design a new game board for the Product Game.
- Choose factors to include in your factor list.
- Determine the products you need to include on the game board.
- Find a game board that will accommodate all the products.
- Decide how many squares a player must get in a row—up and down, across, or diagonally—to win.

Make the game board. Play your game against your partner; then make any changes you both agree would make your game better.

Switch game boards with another pair, and play their game. Give them some written suggestions about how they can improve their game. Read the suggestions for improving your game, then make any changes you and your partner think are necessary.

■ Problem 2.2 Follow-Up

Write a paragraph about why you think your game board is interesting to use for playing the Product Game. In the paragraph, describe any problems you ran into while making the board, and explain how you solved them.

Names: _____

Date: _____

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:

1 2 3 4 5 6 7 8 9