Using MathMemos: A Model PD Module

MathMemos was developed to be a resource for both teachers and professional developers alike. Below is the basic frame for a meaningful PD experience using the site. Depending on your needs and time, you could certainly go deeper into any of these steps and take the training in other directions.

Goals:

- Teachers go through the stages of preparing for a class that uses a non-routine problem
    - Teachers work on a non-routine math problem and see a range of solution methods
    - Teachers talk about the benefits and challenges of using non-routine problems
    - Teachers predict student thinking
- Teachers work together analyze student work
- Teachers have a sense of what kinds of materials can be found at MathMemos.org and how those materials can help their instruction

1. Have teachers work on one of the problems from MathMemos.org. It is not necessary to say where it came from at this point.
2. Have teachers share their solution methods. This could be in small groups or as a whole group discussion. Try to make sure everyone gets to see at least 2-3 different approaches.
3. Ask teachers to share what they like about the problem and record their responses. Ask them to brainstorm the challenges of using the problem with their students.
4. Share samples of student work on the same problem. First have teachers write what they notice and what they wonder on each sample of student work. Then have teachers work in pairs or small groups to share their observations and discuss what they appreciate about each students' work.
5. Have a few volunteers share what they talked about.
6. Explain that all of the materials from the day come from MathMemos.org and give out the flyer
7. Give out a copy of the MathMemo write-up for the problem you chose and have teachers read through it, thinking about how MathMemos could support their instruction. Have a few volunteers share their ideas.
8. To keep the conversation going, ask teachers to try the problem with their students and share their experiences in the comment box under the MathMemo you worked on at the workshop.

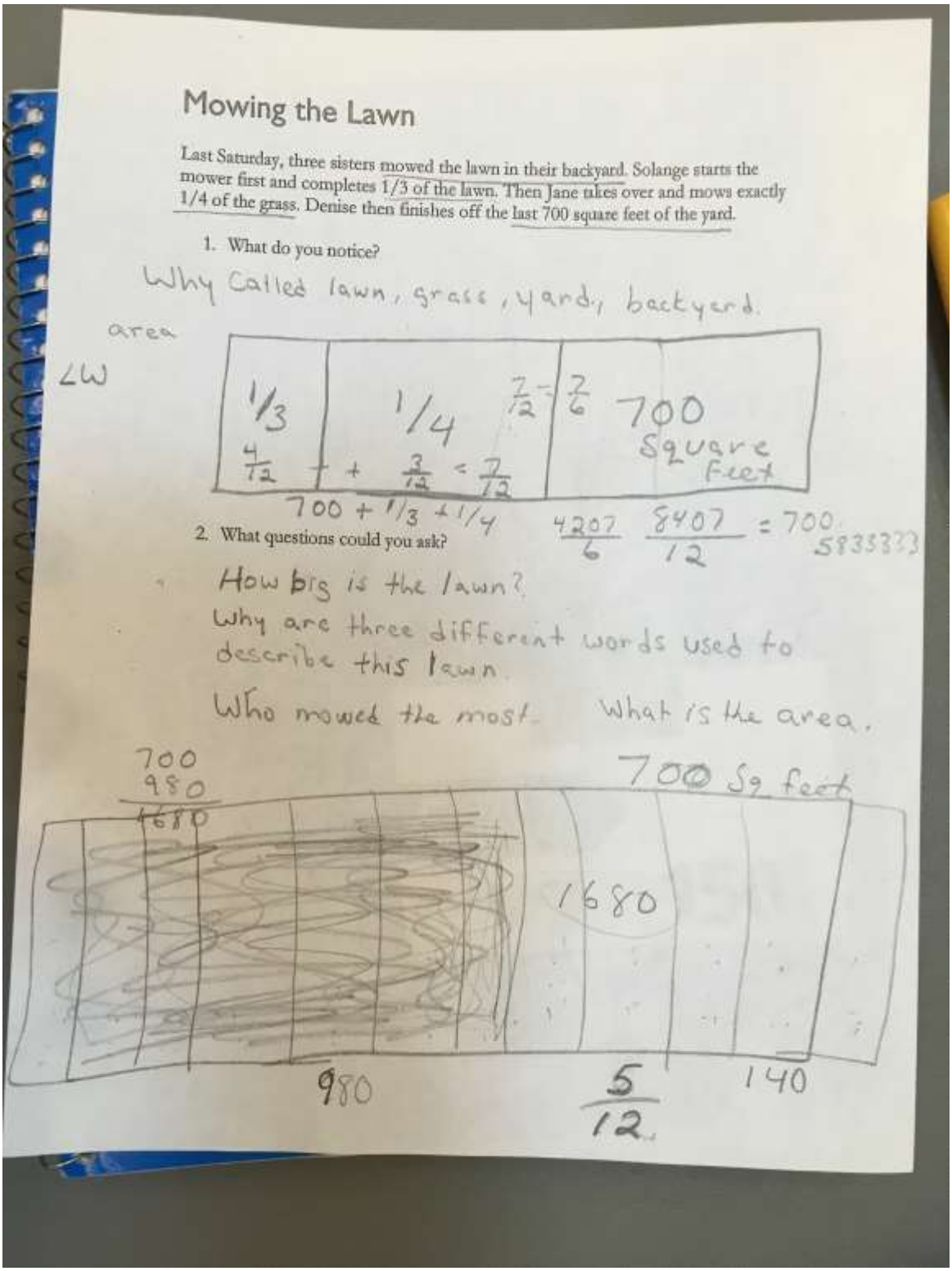
Mowing the Lawn

Last Saturday, three sisters mowed the lawn in their backyard. Solange starts the mower first and completes 1/3 of the lawn. Then Jane takes over and mows exactly 1/4 of the grass. Denise then finishes off the last 700 square feet of the yard.

1. What do you notice?

2. What questions could you ask?

Glenn



Mowing the Lawn

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1. What do you notice?

Why called lawn, grass, yard, backyard.

area

LW

1/3 | 1/4 | 7/12 = 7/6 | 700 Square Feet

4/12 + 3/12 = 7/12

700 + 1/3 + 1/4

4207/6 8407/12 = 700. 5833333

2. What questions could you ask?

How big is the lawn?

Why are three different words used to describe this lawn.

Who mowed the most. What is the area.

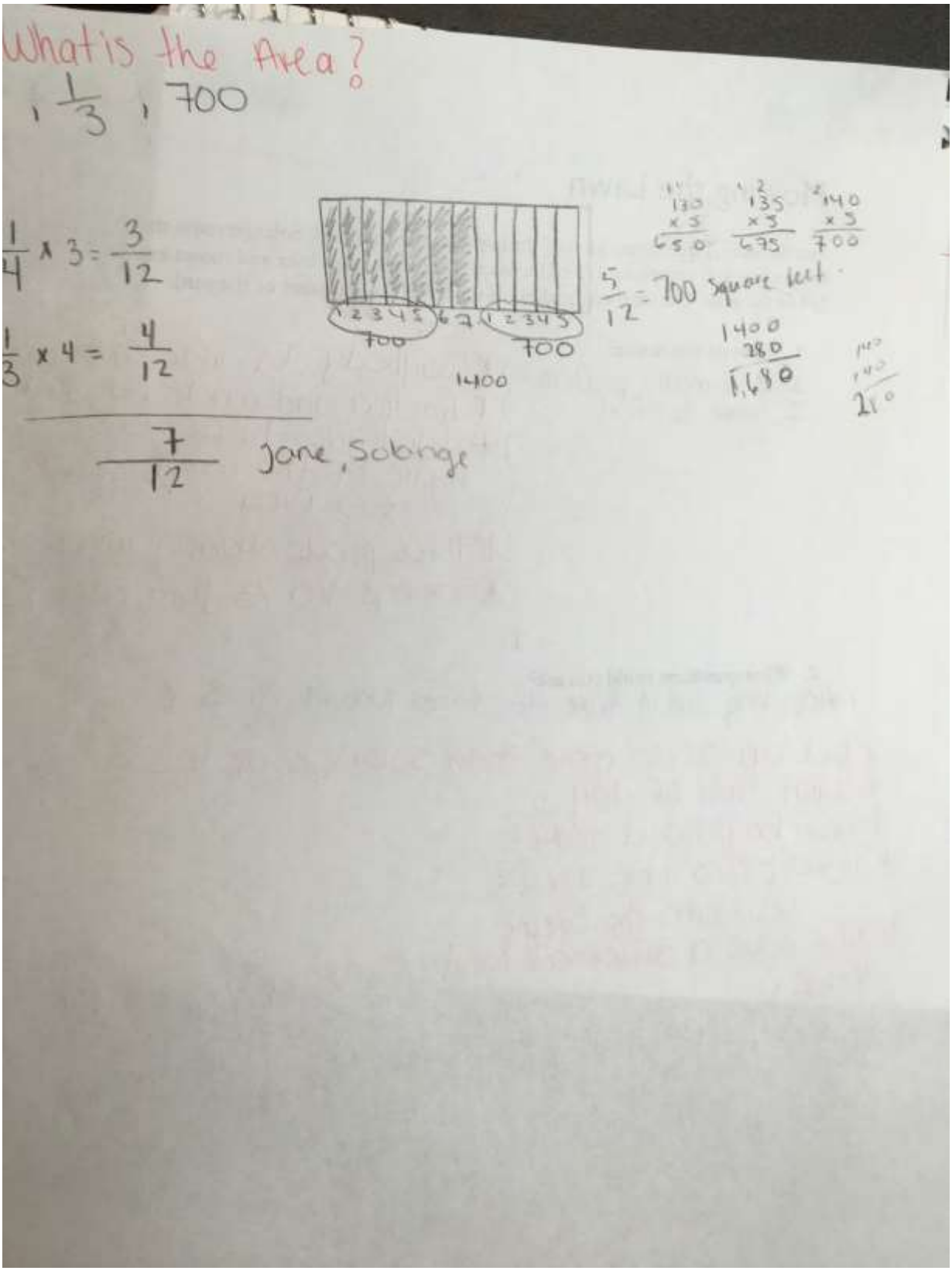
700
980
1680

700 Sq feet

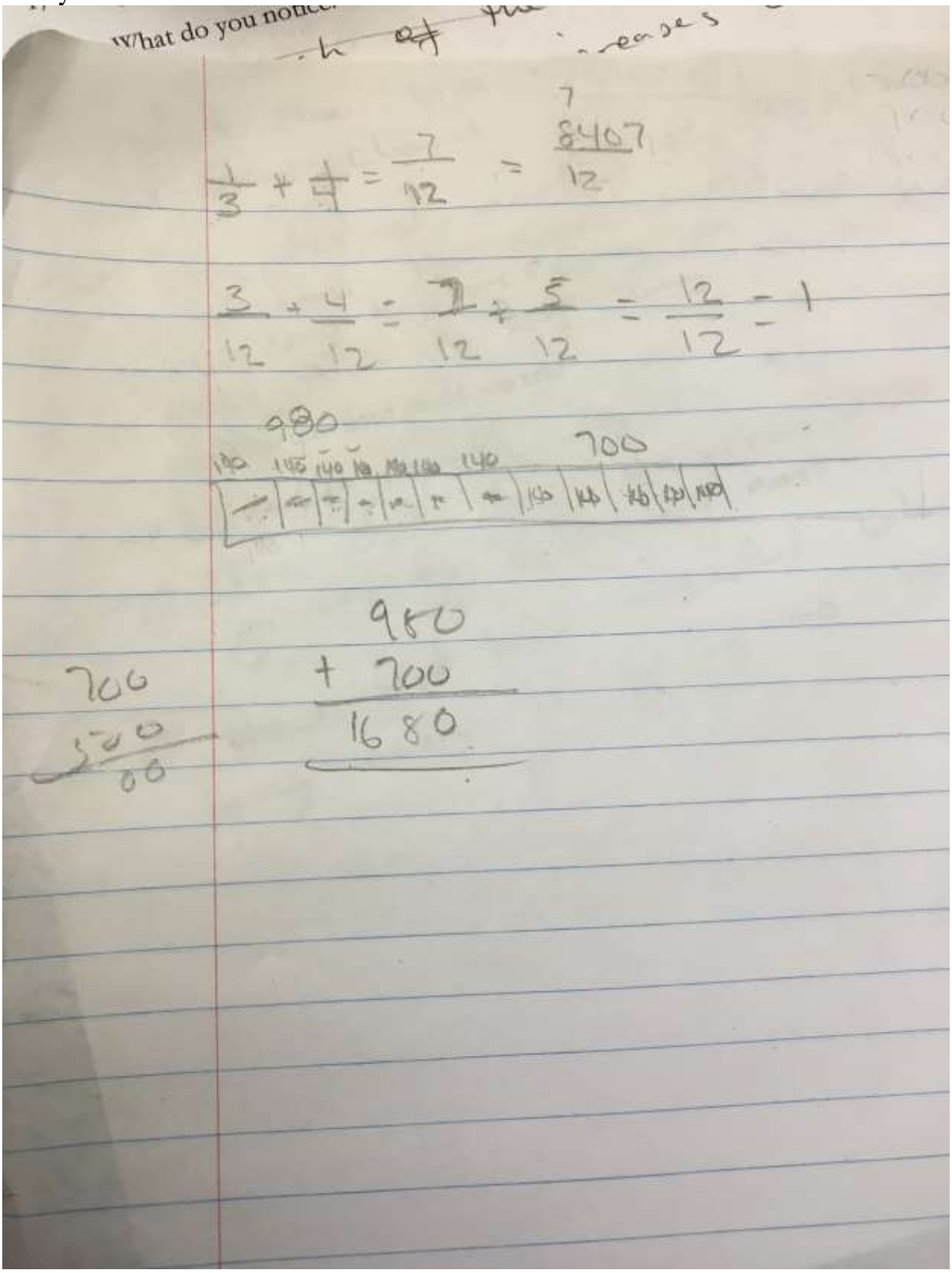
1680

980 5/12 140

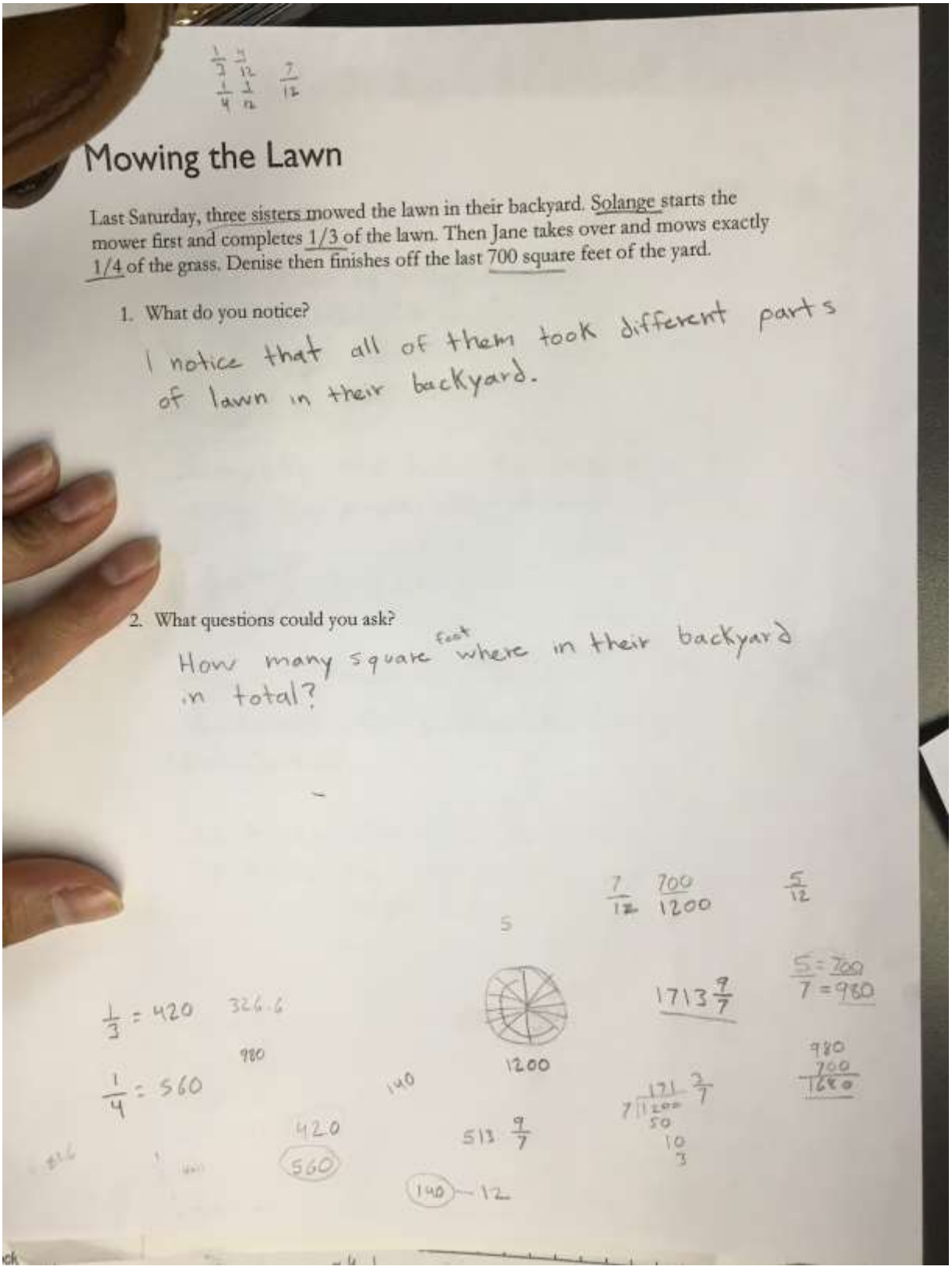
Priscilla


What is the Area?
, 1/3 , 700
1/4 x 3 = 3/12
1/3 x 4 = 4/12
7/12
Jane, Solange
1 2 3 4 5 6 7 1 2 3 4 5
700
700
1400
5/12 = 700 square feet
130 x 5 = 650
135 x 5 = 675
140 x 5 = 700
1400
280
1,680
140
140
210

Tracy


What do you notice
1/3 + 1/4 = 7/12 = 8407/12
3/12 + 4/12 = 7/12 + 5/12 = 12/12 = 1
980
700
980
+ 700
1680
700
500
00

Edith



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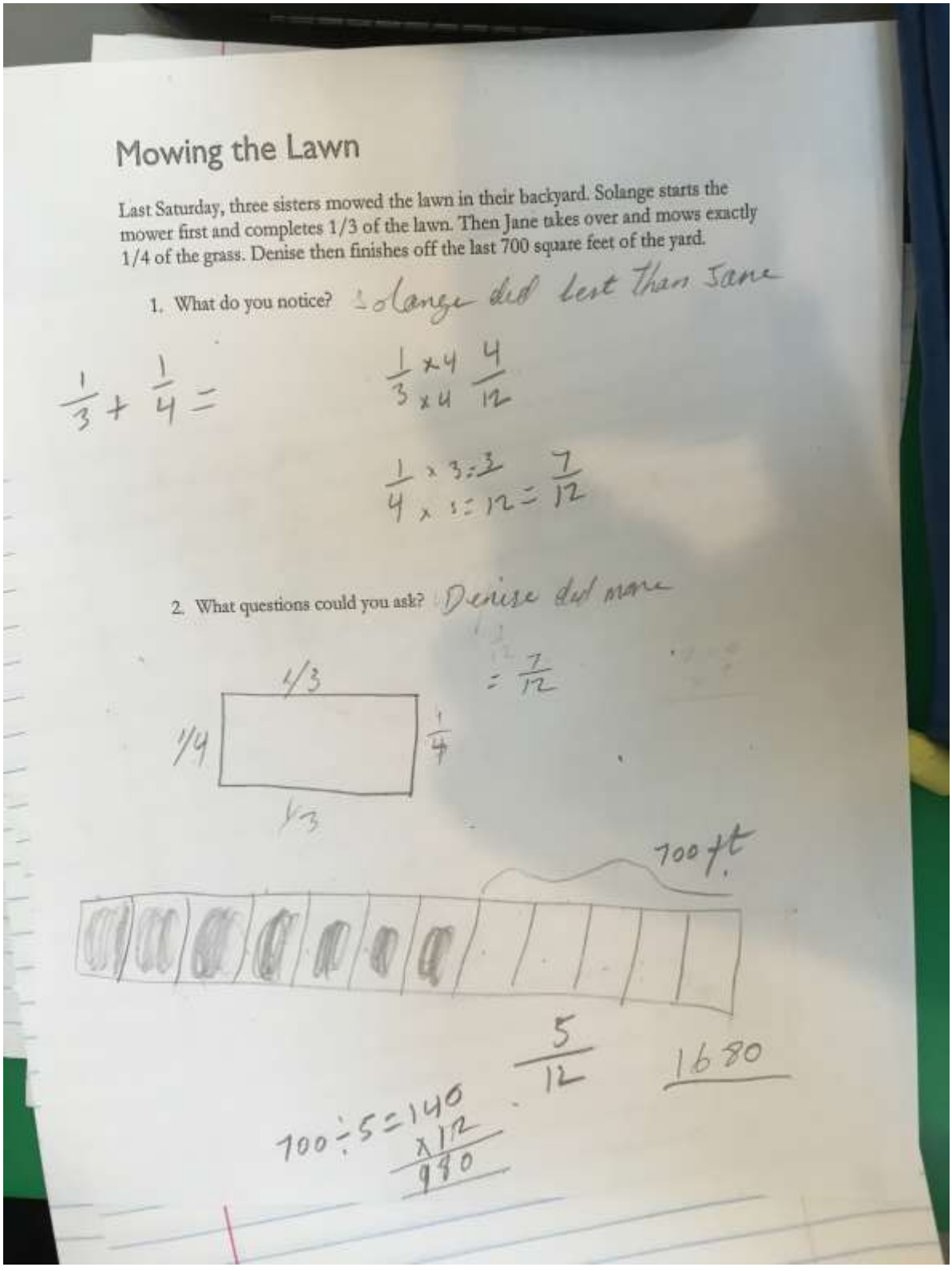
1. What do you notice?

I notice that all of them took different parts of lawn in their backyard.

2. What questions could you ask?

How many square feet where in their backyard in total?

Tameka



Mowing the Lawn

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1. What do you notice? Solange did less than Jane

$\frac{1}{3} + \frac{1}{4} =$

$\frac{1}{3} \times 4 \quad \frac{4}{12}$

$\frac{1}{4} \times 3 = 3$

$4 \times 3 = 12 = \frac{7}{12}$

2. What questions could you ask? Denise did more

$= \frac{7}{12}$

4/3

1/4

1/4

1/3

700 ft

$\frac{5}{12}$

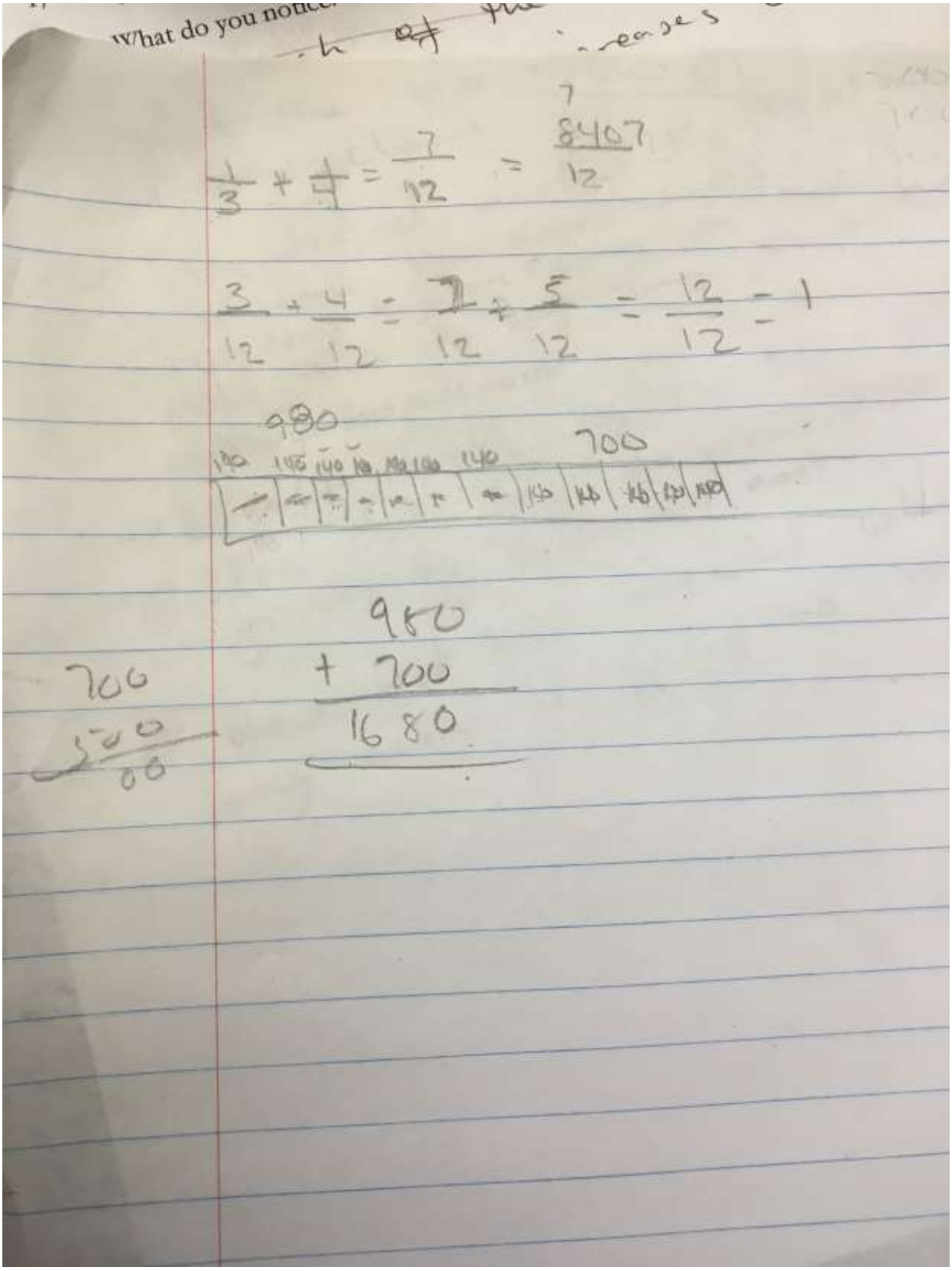
1680

$700 \div 5 = 140$

$\times 12$

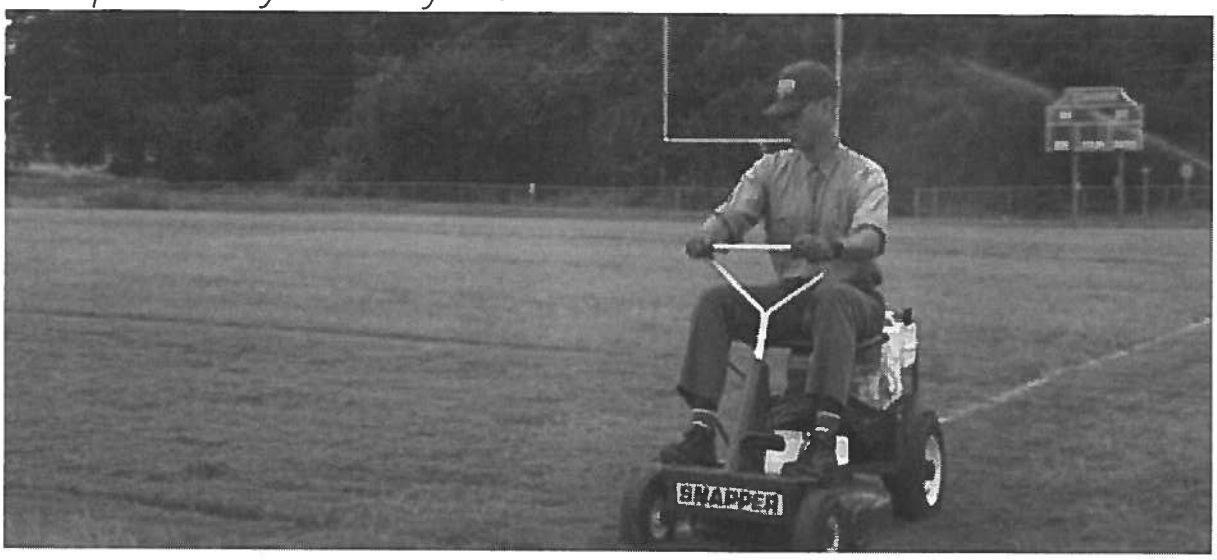
980

Araceli


What do you notice
increases
1/3 + 1/4 = 7/12 = 8407/12
3/12 + 4/12 = 7/12 + 5/12 = 12/12 = 1
980
700
980
+ 700
1680
700
500
00

CollectEdNY

resources for New York State educators preparing adult students for high school equivalency and beyond



Mowing the Lawn: Let Students Ask the Questions

Last Saturday, three sisters mowed the lawn in their backyard. Solange starts the mower first and completes 1/3 of the lawn. Then Jane takes over and mows exactly 1/4 of the grass. Denise then finishes off the last 700 square feet of the yard.

1. What do you notice?

2. What questions do you have?

Source: From a problem suggested by Fawn Nguyen
Subjects: Geometry, Number and Quantity
Tags: area, equality, fractions, proportion, ratio, visual strategies
Problem: docx · pdf **Student Work**: docx · pdf

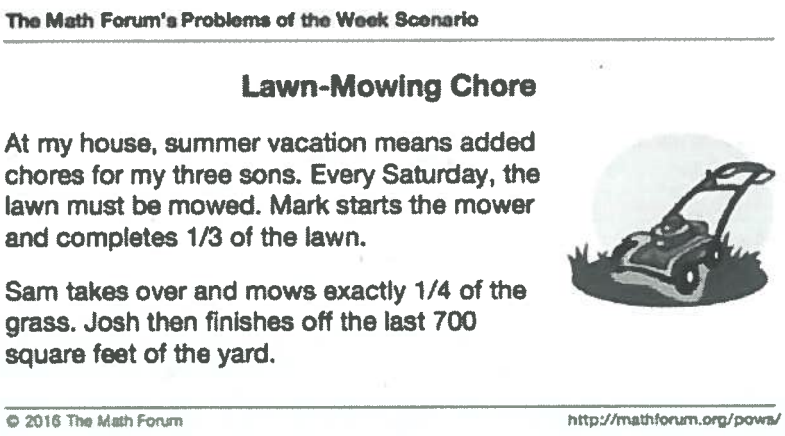
Edit · August 23rd, 2016 · Tyler Holzer · 3 Comments »

A couple months ago, one of my favorite math Tweeters and bloggers, Fawn Nguyen, posted this, which I promptly liked and retweeted:

Fawn Nguyen
@fawnpnguyen

Following

REMOVE the ? from the problem initially as Ss will always come up with ? you had in mind! Do "I notice/wonder" first



The Math Forum's Problems of the Week Scenario

Lawn-Mowing Chore

At my house, summer vacation means added chores for my three sons. Every Saturday, the lawn must be mowed. Mark starts the mower and completes 1/3 of the lawn.

Sam takes over and mows exactly 1/4 of the grass. Josh then finishes off the last 700 square feet of the yard.

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The structure of this activity really appealed to me. In our NYC Community of Adult Math Instructors meetings, we often begin our explorations with two questions:

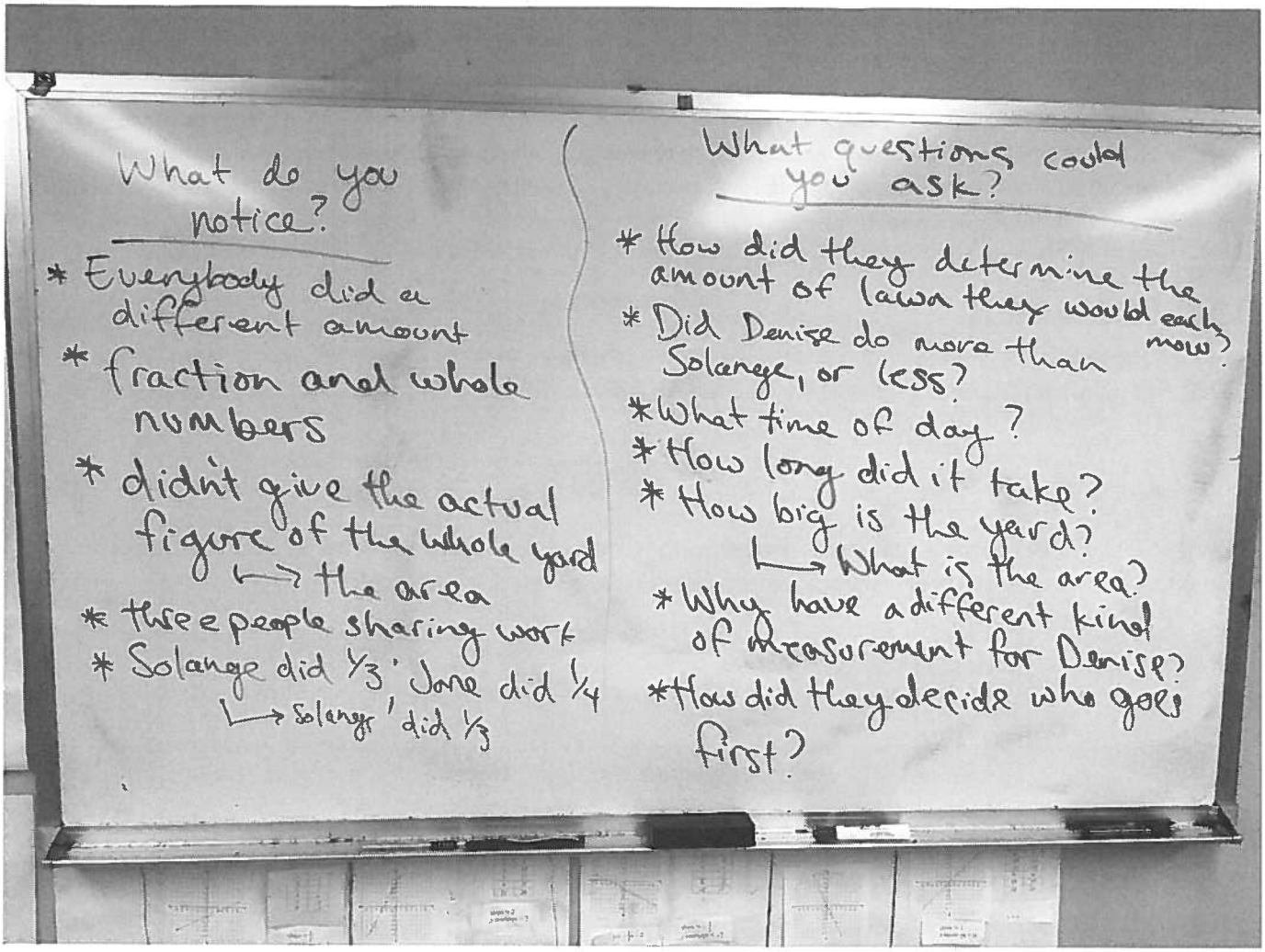
> *What do you notice?*
> *What questions do you have?*

From here, we come up with interesting ways to approach a mathematical situation, and we each tackle different questions that are interesting to us. This model has become a routine in my math class. When we look at charts and graphs, for example, we always start with these questions. This practice helps students anticipate the kind of question that they *could* be asked, and it gets them talking to one another.

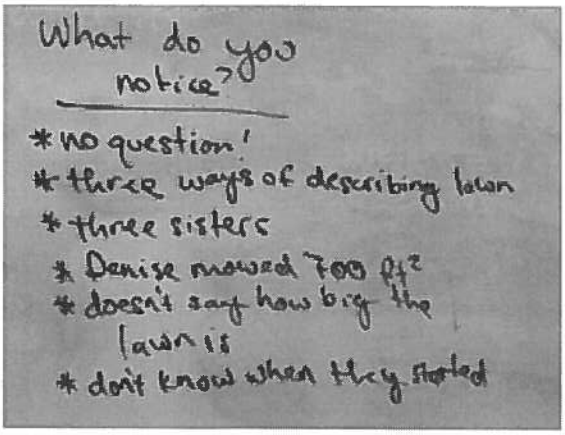
So of course I was super excited about the problem Fawn posted, and I couldn't wait to teach it. There are so many different things I like about the problem. I like that it connects spatial reasoning with proportional reasoning; I like that it's so immediately accessible but ultimately challenging to solve; and most of all I like the way it builds student interest by giving them time to talk to one another about what they notice and what math questions they might be able to ask and solve. I decided to teach this problem to two very different groups of students: one, an HSE-level class in Carroll Gardens, and two, an ABE class in Red Hook. I also changed the names of the people in the problem; I like to use the same names in my classes, as my students start to remember things like "Eric is the guy who sold the fish tanks, right?" Or, "I remember Solange from that problem about the taxi." It's fun.

Students Asking Questions

These two classes had very different levels of mathematical ability. The first class was composed primarily of HSE-level students who had been with me for a while. This problem was challenging for them, though, because we had spent so much more time on algebraic reasoning than we had on fractional and proportional reasoning. Here are some of the things they noticed and the questions that they had:



This group asked the question that I hoped they would ask: What is the area of the yard? The second class also asked this question, and they "noticed"–as I hoped they would–that this problem didn't actually ask a question!



They asked some great questions, and they were interested in figuring out who mowed the most. Some of the questions they asked were:

- How big is the lawn?
- How much is 1/3 of the lawn?
- How much is 1/4 of the lawn?

- Why use three different words for lawn?
- Who mowed the most?

Altogether, this part of the activity took about a half hour. Students took five minutes to jot down their questions, and then they talked about them in small groups for another five to ten minutes. After that, we talked about the problem as a whole class, and I noted all of their questions on the board. Both classes decided that they wanted to figure out the area of the lawn, which is exactly what I was hoping for.

How I Solved It

First I figured out what fraction of the lawn had already been mowed by Solange and Jane by adding 1/3 and 1/4.

$$1/3 + 1/4 = 4/12 + 3/12 = 7/12$$

I knew that 7/12 of the lawn had already been mowed, so the remaining 5/12 was the part left for Denise. At this point I set up a proportion to find exactly how many square feet the entire lawn was.

$$\frac{5\ \text{Denise}}{12\ \text{Total}} = \frac{700\ \text{Denise}}{?\ \text{Total}}$$

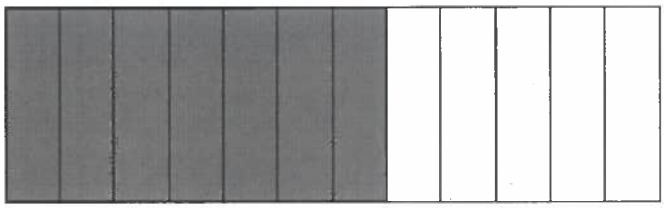
I cross-multiplied and divided to find the total area of the yard.

$$\frac{12 \times 700}{5} = 1680\ \text{square feet}$$

The yard is 1680 square feet in total.

Anticipating Student Approaches

I knew that my students were going to struggle with this problem, especially because they aren't always comfortable setting up proportions correctly. They are, however, very good at drawing pictures to help them clarify their thinking. So I assumed that several students would correctly add the fractions of the lawn that Jane and Solange mowed, and then they would be able to draw a picture of 7/12 to help them find how much Denise has left to mow. We worked with strip diagrams before, so I expected to see something like this, where the green part represents the amount that has already been mowed, the white part represents what Denise still needs to do, and the entire strip represents the lawn:



I thought that if students were able to do this, then they would be able to see that Denise's five parts are equal to 700 square feet. From here they would be able to divide by 5 to figure out the number of square feet in each "piece" of the strip

diagram.

$$700 \div 5 = 140$$

Now, to find the total area of the lawn, they could multiply $140 \times 12 = 1680$ square feet.

Supporting Productive Struggle

Once they had established that they want to find the area of the yard, many of my students struggled to figure out what to do next. Several quickly figured out that they needed to add 1/3 + 1/4 to get 7/12 but then got stuck. So to support productive struggle, I asked a lot questions about what the 7/12 meant in the context of the question.

- What is the 7/12? Who mowed 7/12 of the lawn?
- So if 7/12 has already been mowed, what fraction of the lawn still needs work?
- Will Denise need to mow more than half of the lawn, or less than half?
- Talk to me about 7/12. What does 7/12 look like to you? Could you draw a picture?

A few students tried using a proportion to solve the problem, but they forgot to label each item in the proportion, which got them into a little trouble. To help them get back on track, I asked a lot of questions about what each element in the proportion represented.

- I noticed you have 5/12 on the left side of the proportion. Can you talk to me about the 12? What does it represent? What about the 5? If you wanted to label each of those, what labels would you use?
- It looks like you have 700 on the bottom on the right side of your proportion. Can you talk to me about that? What is the 700? How would you label it?

My goal was to get students talking and to support whichever strategy they were working with.

Student Work

Glenn's Approach

Mowing the Lawn

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1. What do you notice?

Why called lawn, grass, yard, backyard.

area

$\angle W$

1/3	1/4	700 Square Feet
$\frac{4}{12}$ +	+ $\frac{3}{12} = \frac{7}{12}$	$\frac{7}{12} - \frac{7}{6}$

$700 + 1/3 + 1/4$ $\frac{4207}{6}$ $\frac{8407}{12} = 700.5833333$

2. What questions could you ask?

How big is the lawn?

Why are three different words used to describe this lawn.

Who mowed the most. What is the area.

700
980
1680

700 Sq. feet

1680

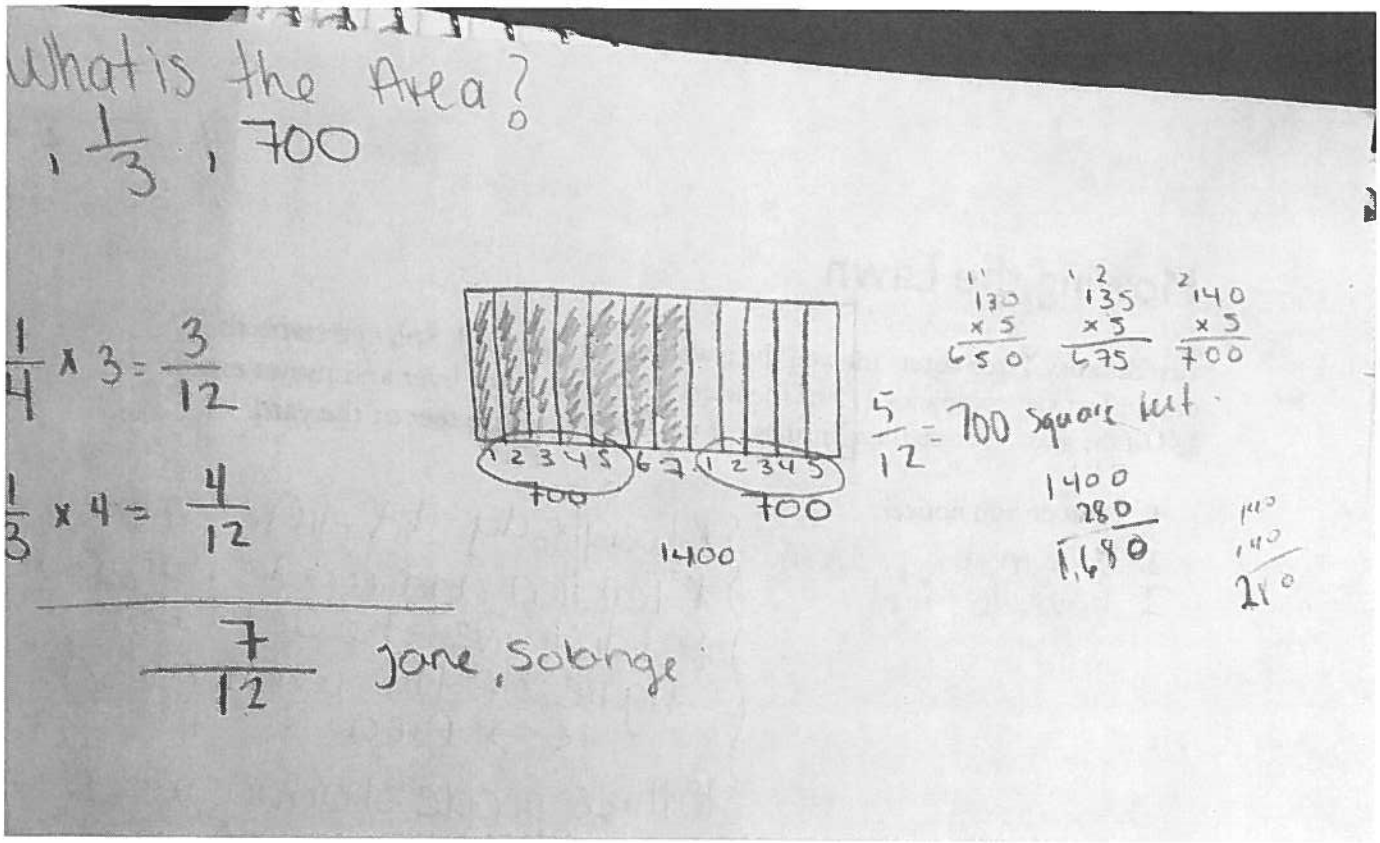
980 $\frac{5}{12}$ 140

I was interested in Glenn's approach to the problem because he started out by drawing a picture, as he has done in class before, but it didn't seem to get him very far. I thought it was interesting that in his first drawing, 1/4 is drawn larger than 1/3. Glenn had also completely forgotten how to add fractions. At first, he added 700 + 1/3 + 1/4 and got an result of 4207/6. When I asked him about this, he couldn't really explain what he did. He was sitting next to Priscilla, who I noticed had already added the fractions correctly, and so I asked Glenn to look at what she did and try to reproduce it. He was able to raise the fractions properly, though I'm not sure much of the work was his own. It did, however, give Priscilla a teaching moment, which I liked.

Glenn still needed a lot of help, so I asked him to draw the fraction. His drawing looked good, and so I asked him some questions about what the shaded and unshaded parts meant. Once he was able to tell me that the 5 unshaded pieces represented the part that Denise still needed to mow, I stepped away. He was able to do the rest on his calculator, and when I came back over, he and Priscilla were comparing answers.

Priscilla's Approach

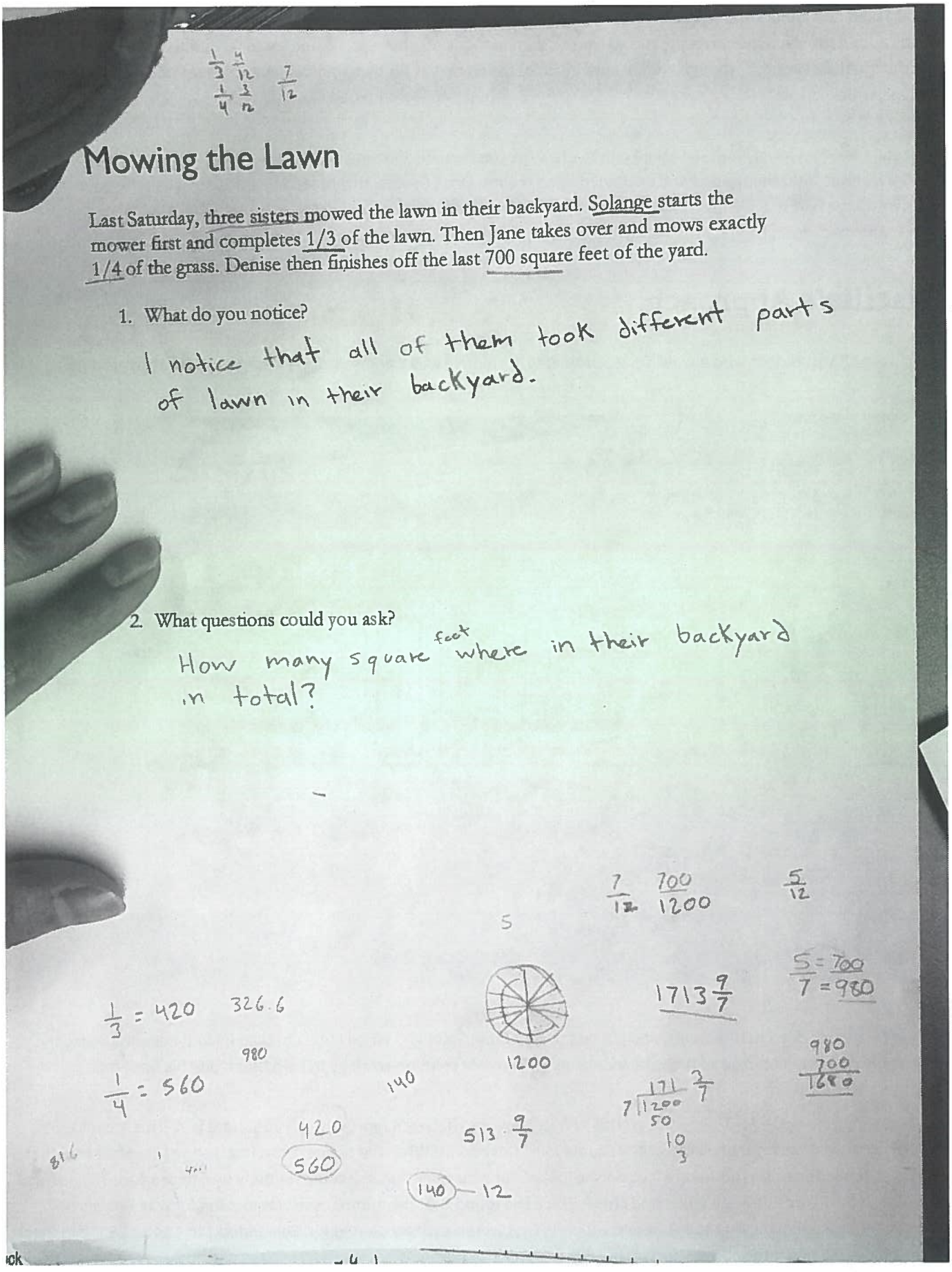
Priscilla is a sharp student who always solves problems in creative ways, and so I was pleased to see that she produced this:



Priscilla correctly added her fractions, and labeled them "Jane, Solange." When I had checked in with her and Glenn, she had already drawn a picture, and then she took a break from her solution strategy to help Glenn add his fractions.

In her drawing, she circled two separate groups of five, knowing that each represented 700 square feet. This meant that 10/12 of the strip diagram represented 1400 square feet. I loved this! What she did next was interesting too. She knew that 5 of the pieces needed to be 700 square feet, so she looked for a number that she could multiply by 5 to produce 700. Instead of dividing, she did this through guess and check. Once she found that the number she was looking for was 140, she multiplied it by 2, knowing that these were the 2 parts that weren't yet accounted for. She added 140 + 140 = 280, and then added 1400 + 280 to get the correct answer of 1680.

Edith's Approach



1/3 4/12 7/12
1/4 3/12

Mowing the Lawn

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1. What do you notice?

I notice that all of them took different parts of lawn in their backyard.

2. What questions could you ask?

How many square feet where in their backyard in total?

7/12 700 1200 5/12

5

1/3 = 420 326.6

980

1/4 = 560

140

1200

1713 9/7

5 = 700
7 = 980

980
700
1680

171 3/7
7|1200
50
10
3

513 9/7

420
560

140 — 12

816

In this photo, it's hard to see just how much work Edith had done and then erased. She knew that she could use a proportion, but at first she just couldn't make it work, no matter how hard she tried. Near the top right of her work, she creates the proportion

$$\frac{7}{12} = \frac{700}{1200}$$

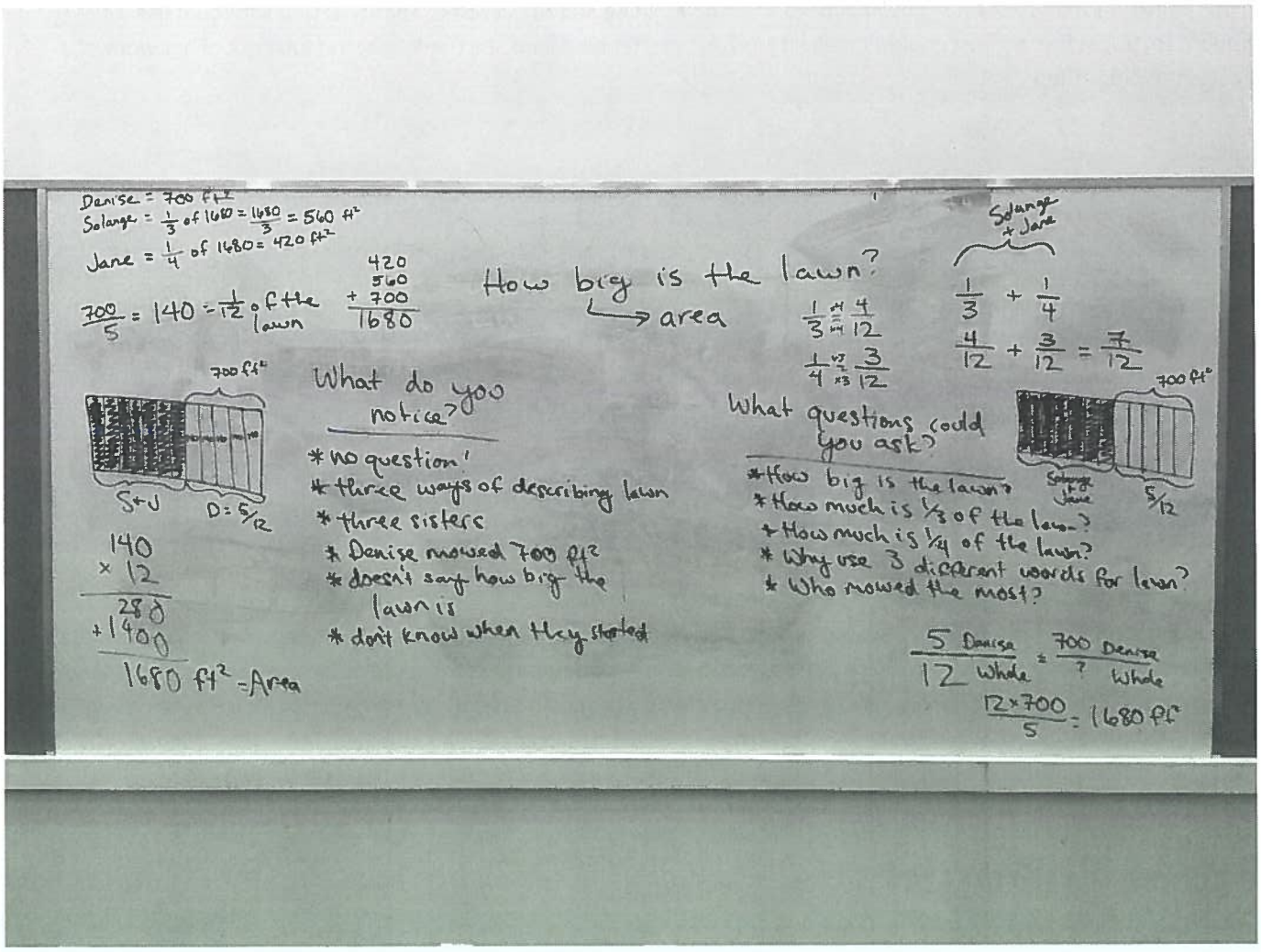
This led her to think that the total area must be 1200 square feet. I asked Edith several times to check that and see if it would work. For example, "If the total area is 1200 square feet, then how many square feet did Solange actually mow? What about Jane? Does that leave 700 square feet for Denise? Hmm....Maybe we need to rethink your proportion." So then Edith set it up a little differently. This time she wrote out the proportion:

$$\frac{5}{7} = \frac{700}{?}$$

On the left side of her proportion, she compares the 5 parts that Denise needs to mow to the 7 parts that have already been completed. She then correctly put Denise's 700 square feet in the numerator of the ratio on the right and solved. She got an answer of 980. This confused her for just a minute, but then we talked about how the 980 square feet corresponds with the 7 parts that have already been mowed, and she realized that she still needed to add the two: 700 + 980 = 1680. I really liked how Edith not only used a proportion, but also that she did it in a way different from what I was anticipating.

Final Thoughts

I really love this problem and will definitely keep it as a part of my teaching toolkit. It worked well–and was challenging for–both groups of students because the "notice/wonder" structure of the problem really helps to engage learners at all levels and give them time to think about the situation posed before they start crunching numbers. As I mentioned before, this structure has become routine in my math classes. Try it for yourself! Start problems with these two questions: "What do you notice?" and "What do you wonder?" It will allow all students an entry point into the problem. Some may ask more challenging questions and will want to struggle to answer them, while beginning students might work on other questions more appropriate to their level. I've used the notice/wonder structure with visual patterns, geometry, tables, graphs, among other things. It's been fun helping my students get into the rhythm of anticipating questions and making mental notes as soon as they start interacting with a problem. Because the students come up with the questions themselves, they also feel more invested in answering them. This is just another way for the teacher to step back, focus authority with the students, and act as a facilitator in their learning.



A final note: I can't emphasize enough how much the math Twitterverse has impacted my teaching. There is a truly robust, supportive, and creative group of math educators on Twitter, and they're always posting new ideas about lessons and good math pedagogy, as well as linking to great problems and teaching resources. For example, to find samples of activities that are structured like this one, check out the hashtag #noticewonder. To find blog posts by Fawn Nguyen, as well as other "Math Twitter Blogosphere" folks, search #MTBoS. Are there other hashtags that you follow? Leave them in the comments below. Oh, and follow me at @rezloh!

3 thoughts on "Mowing the Lawn: Let Students Ask the Questions"

August 23, 2016 at 5:11 pm