DATELINES

What's an acceptable dating range?

lesson guide

We've all seen those celebrity couples whose age difference is a little creepy. But how much of a difference is okay? The popular rule of thumb says that the youngest person you should date is seven years older than half your age.

In this lesson, students use linear relationships to examine the May-December romance and ask whether the *Half Plus Seven* rule (and its inverse, *Minus Seven Times Two*) is a good one. When dating, how young is too young? And how old is too old?

Primary Objectives

- Write and manipulate linear equations and inequalities in two variables
- Find the inverse of a linear function
- Graph a system of inequalities, determine constraints with a problem's context, and interpret the results
- Reason algebraically or numerically to determine wait-times for couples outside the bounds of *Half Plus Seven*

Content Standards (CCSS)	Mathematical Practices (CCMP)	Materials
Algebra: CED.1, CED.3, REI.3, REI.12 Functions: BF.4	MP.2, MP.4	• Student handout • LCD projector • Computer speakers

Before Beginning…

Students should be able to write the equation of and graph a linear inequality. This lesson can be used as a context to introduce writing a function's inverse, but will progress more quickly if students are already familiar with inverses.

Preview & Guiding Questions

Students begin by watching a clip from the 2003 film *Big Fish*. In it, 18-year-old Edward Bloom (played by Ewan McGregor) talks with eight-year-old Jenny, who has a crush on him, about the difference in their ages.

Jenny:	How old are you?
Edward:	18.
Jenny:	I'm 8. That means when I'm 18, you'll be 28. And when I'm 28, you'll only be 38.
Edward:	You're pretty good at arithmetic.
Jenny:	And when I'm 38, you'll be 48. That's not much difference at all.
Edward:	Sure is a lot now, though, huh?

Discuss with students whether they think ten years represents a significant age difference in a relationship. As they do, they'll likely agree that it's significant when she's young and he's older, but gets less and less significant as they age. The goal is for them to understand that it's not really the age difference that matters, but the age difference *relative* to their ages. For instance, when she's 8, ten years is longer than she's been living. But when she's 38, it represents a much smaller fraction of her lifetime.

- *On a scale of 1-10, rate how creepy you think each dating scenario is: 8 & 18; 28 & 38; and 38 & 48?*
- *Assuming students say it gets less creepy: Why is it getting less creepy? Aren't they still 10 years apart?*
- *At what point do you think it goes from being creepy to acceptable?*
- *For two people, how do you determine whether their ages are close enough so as not to be creepy?*

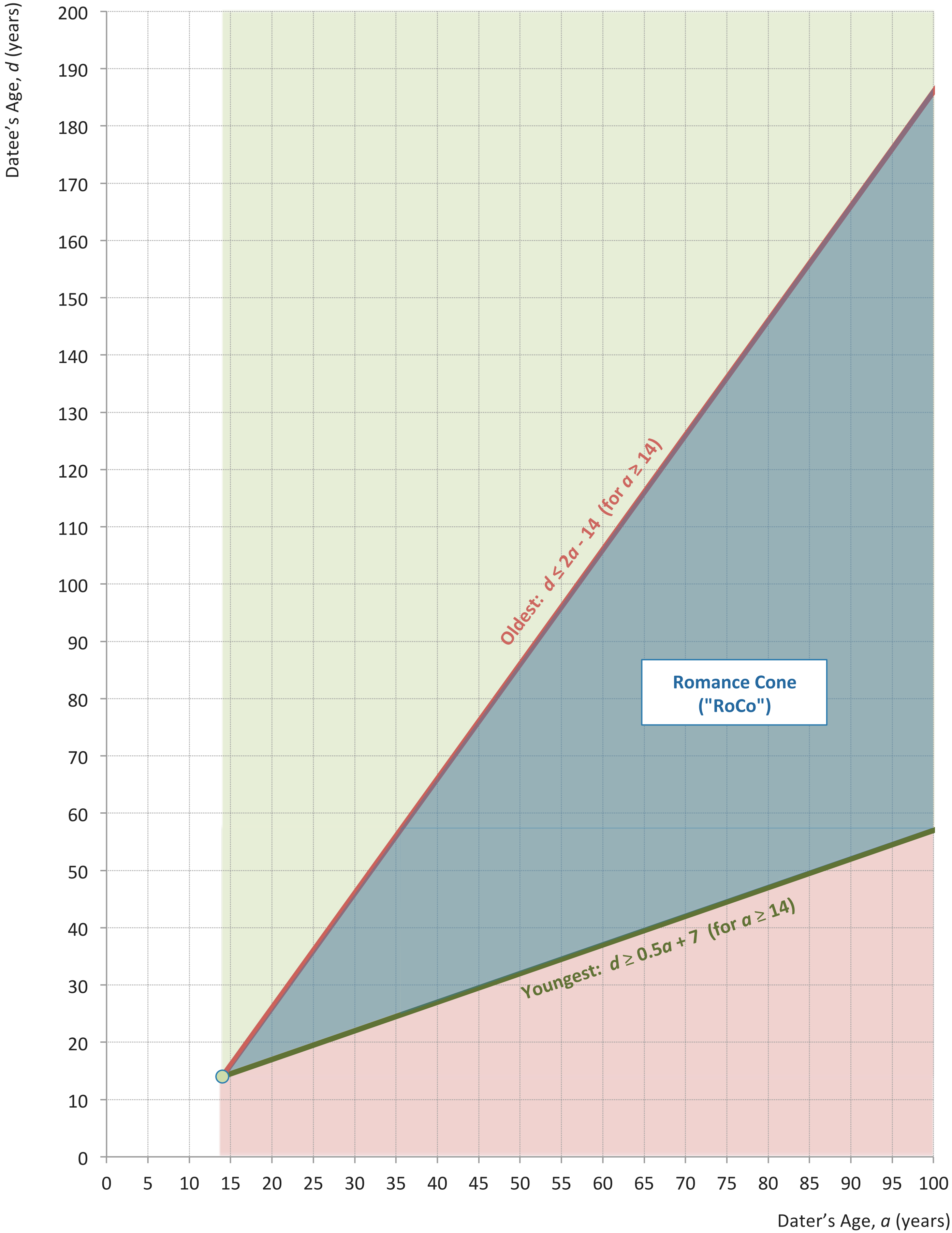
Act One

Students learn the ***Half Plus Seven*** rule and get a feel for how it works by calculating some appropriate dating ages, and discuss its limitations. After writing an inequality and graphing a region that represents the ***youngest*** person someone should date, students find the inverse of half plus seven – minus seven, times two – to determine the ***oldest*** person someone should date. At the end of Act One, students interpret the intersection of the two inequalities as the total acceptable dating region, or the "**Romance Cone**" (RoCo).

Act Two

In Act Two, students learn about four celebrity couples, and how old each person was on their wedding day. Students determine whether the couple was in the Romance Cone and, if not, how many years they'd need to wait until they are. Students will find that there's a relationship between the initial gap – the gap between how old the youngest person *needs* to be, and how old he/she actually is – and the wait time: the wait time is simply twice the initial gap. (For instance, to marry the 37-year-old Brian Austin Green, Megan Fox needed to be 25.5 years old. However, she was only 24 years old when they married, or 1.5 years too young. Thus, the couple needed to wait 1.5 years x 2 = 3 years to enter the Romance Cone.)

So once a couple is inside the Romance Cone, will they ever be outside it again? Students consider this question, and determine that once a couple enters the cone, they will never exit it again.


Datee's Age, d (years)
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Dater's Age, a (years)
Oldest: $d \leq 2a - 14$ (for $a \geq 14$)
Romance Cone ("RoCo")
Youngest: $d \geq 0.5a + 7$ (for $a \geq 14$)

Act One: The Dating Game

1 To determine the age of the youngest person you can date, some people recommend using the formula **half-plus-seven**: *start with your own age, take half of it, and add seven years*.

Use this to fill in the table below. Then, do you think half-plus-seven make sense for all ages? Explain.

Age, Older Person	80	50	*46*	30	*16*	12	10	*6*
Age, Younger Person	*47*	*32*	30	*22*	15	*13*	*12*	10

According to half-plus-seven, the youngest person a 10-year-old can date is 12 years old, which doesn't make sense. Not only that, the 12-year-old wouldn't be able to date the 10-year-old back, anyway, since the youngest person a 12-year-old can date is 13! Instead, half-plus-seven only starts to make sense at age 14. The youngest person a 14-year-old can date is another 14-year-old, so this is the age where the model should start.

Explanation & Guiding Questions

There are two main goals in this question. The first is for students to **calculate the younger age given the older, and calculate the older age given the younger**. Students should have a relatively easy time going from older to younger; *take half the older person's age, then add seven years*. However, they may have a bit more trouble going from younger to older, i.e. "undoing" half-plus-seven.

To help them understand how to do this, consider focusing on a specific example. In the third column we know the younger age is 30 years, and we want to determine the older age. Whatever it is, we know we'd have to 1) take half of it, then 2) add seven, to get to 30 years. Students should realize that we'd have to "land" on 23 years in that intermediate step. Since this represents half the older age, the older age must be 46 years. Thus, the inverse of half-plus-seven must be *subtract seven, then double*. Even though students aren't asked to write equations yet, this process of going forwards and backwards in the table should make it easier when they do.

The second goal is for students to realize that the **half-plus-seven rule of thumb only starts to make sense at 14 years**. This will come into play later when students consider the domain for their graph. (Note: some students might ask whether a 20-year-old can *legally* date a 17-year-old. While this adds an interesting twist to the domain, it's probably a conversation you want to avoid with a simple, "That's a good question…but not one we're going to consider in this lesson. Instead, we're just going to focus on what the *math* says.")

- *When you knew the older age (e.g. first column, 80 years), how did you find the youngest age?*
- *When you know the younger age (e.g. third column, 30 years), how did you find the oldest age?*
- *If you're not sure, what are some ages along the way that you'd have to "land on?"*
- *Why doesn't half-plus-seven make sense for the people in the last three columns?*
- *At what age does half-plus-seven start to make sense and why?*

Deeper Understanding

- *14 years is the lower bound for half-plus-seven. Should there be an upper bound, as well? (According to half-plus-seven, a 60-year-old could date a 106-year-old, so perhaps there should be an upper bound.)*

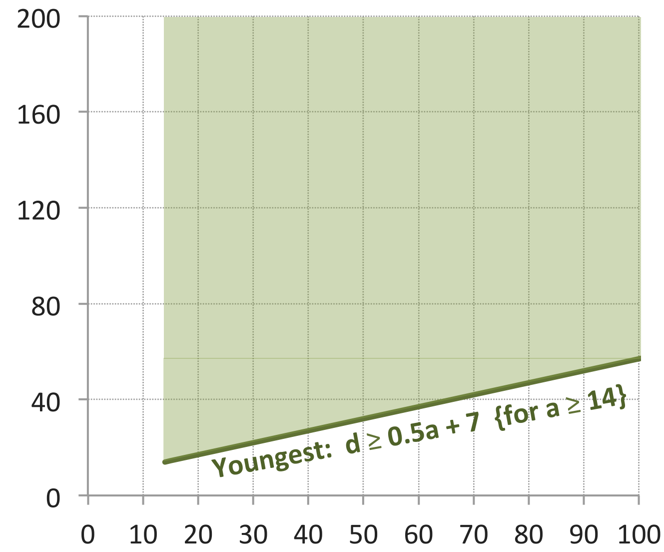
2 Let ***a*** represent your age and ***d*** that of your date. Write an equation for the age of the **youngest person** you can date, and graph it. Then, does this show *all the ages* you can date? If not, how could you modify the equation?

$$d = \frac{1}{2}a + 7 \text{ \{for } a \geq 14\text{\}}$$

This equation represents the age of the youngest person someone can date. However, he/she can still date people who are older than this. For instance, a 50-year-old can date a 32-year-old, plus people who are older than this.

On the graph, the line represents the youngest age, while the area above the line represents older ages. To make the picture more complete, we can turn the equation into an inequality:

$$d \geq \frac{1}{2}a + 7 \text{ \{for } a \geq 14\text{\}}.$$



Explanation & Guiding Questions

Students should have an easy time writing the equation of the line, d = (1/2)x + 7, which represents the precise age of the youngest datee. However, this is only the *minimum* age; you can still date people older than this. Thus, students should shade the region above the line to show other permissible ages, and rewrite their equation as an **inequality**. Also, from before, students should recognize that the model really "starts" when a = 14, and **restrict the domain** accordingly. Lastly, students should understand what the **slope** means: for every year that passes (i.e. when a increases by 1), the age of the youngest person we can date increases by half a year.

Note: students may be unsure what the axes represent. For instance, some might think the x-axis represents the age of the youngest person, and the y-axis represents the age of the oldest person. This isn't the case. Instead, the x-axis represents the age of the dater (a), while the y-axis represents the age of the datee (d).

- *What does the line y = (1/2)x + 7 represent, and does it show* ***all*** *the ages that someone can date?*
- *Should we shade above the line or below the line?*
- *What are the differences between the four inequality symbols (>, <, ≤, and ≥), and which should we use?*
- *What is the domain of the graph?*
- *What do the x-axis and y-axis represent in the model?*

Deeper Understanding

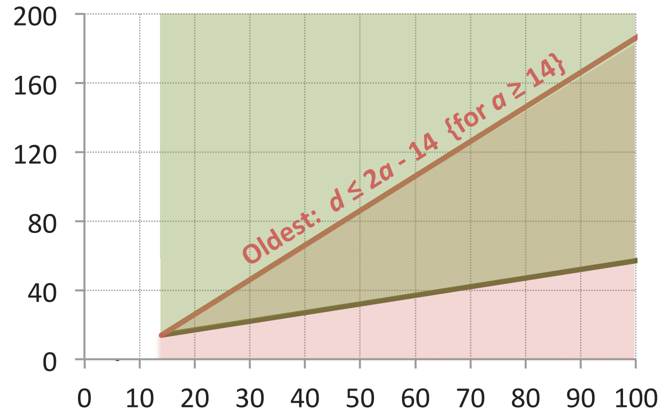
- *What is the slope of the line, and what does it tell us in this context? (The slope is 0.5 years per year. Every year that the dater ages – i.e. each year – the age of the youngest datee increases by half a year.)*

3 Now write an equation for the age of the **oldest person** you can date, and modify it to be more inclusive.

From the table earlier, we know that the oldest person a 30-year-old can date is 46 years old. To get this, we subtracted 7 years to get 23 years, then doubled this to get 46 years. If the variable a represents someone's age, we can find the oldest person he/she can date by:

$$d = (a - 7) \times 2$$
$$d = 2(a - 7)$$
$$d = 2a - 14 \quad \{\text{for } a \geq 14\}$$

Since this represents the oldest possible datee age, we can rewrite this as an inequality: $d \leq 2a - 14$.



Explanation & Guiding Questions

Students already know that the datee (d) can't be younger than half the dater's age (a), plus seven. In other words, **$d \geq (1/2)a + 7$**. Here, the dater is assumed to be older, while the datee is assumed to be younger.

But this doesn't have to be the case. The dater can be the younger one, in which case the datee will be older. In this case, and for half-plus-seven to hold, the dater can't be younger than half the *datee's* age, plus seven, either. Therefore, **$a \geq (1/2)d + 7$**. Since *a* represents the dater's age (independent), and *d* represents the datee's age (dependent), we should rewrite this inequality in terms of *a*:

$$a \geq (1/2)d + 7$$
$$a - 7 \geq (1/2)d$$
$$2(a - 7) \geq d$$
$$2a - 14 \geq d$$

Based on this, to find the age of the *oldest* person a dater can date, we simply have to subtract seven from his/her age, then double the result. (Alternatively, we can double the age, then subtract fourteen.) Of course, we've seen this before; this is exactly what students did in the table in Question 1!

Note: instead of $2a - 14$, students may choose to leave the expression as $2(a - 7)$, which is arguably more intuitive given that this is probably how we think of the process: *subtract seven, then double it*.

- *Is the datee necessarily younger than the dater?*
- *If the datee cannot be younger than twice the dater's age, plus seven, what else must be true?*
- *Which form of the expression do you find more useful in this situation: 2(a – 7) or 2a – 14?*

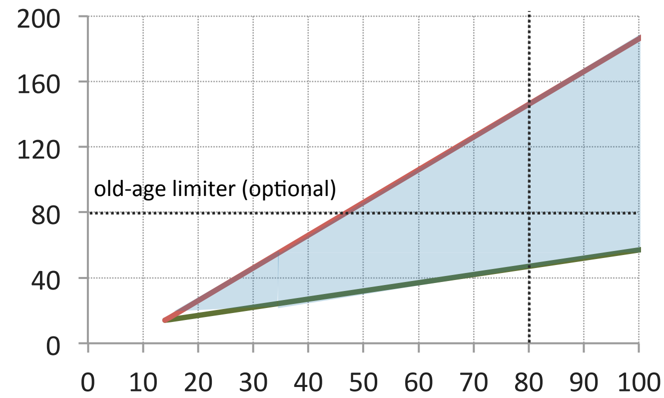
Deeper Understanding

- *What does the slope of the upper line tell us? (The slope is 2 years per year. Every year that the dater ages, the age of the oldest datee increases by two years.)*
- *There's a region on the graph where the inequalities intersect. What does this region represent? (The region between the youngest oldest lines represents the permissible datee ages.)*

4 Finally, shade in the **total acceptable dating region**. What do you notice about its shape, and what does it suggest about your dating prospects as you get older? Explain.

The total acceptable dating region is a triangular region that gets wider and wider the older someone gets. This suggests that as someone ages, the range of ages that s/he is allowed to date gets bigger.

However, this doesn't necessarily mean that the datable pool gets bigger. After all, a 60-year-old won't be able to date everyone between the ages of 37 and 106; some of these people will be married, while some will be dead.



Explanation & Guiding Questions

By this point, students should be fairly comfortable with the model. They'll quickly realize that as the dater's age increases, the cone-shaped region – the **Romance Cone**, aka "RoCo" – gets wider and wider. However, one misconception you can expect is for students to say, "As someone gets older, they can date *more and more* people." This isn't necessarily true. The graph tells us that the *range of permissible ages* within which a dater can date gets larger and larger over time, but it doesn't actually say anything about the *number* of people there are available to date. After all, some of these people are married, while others are dead.

Of course, students may realize that, at some point, the RoCo becomes unrealistic. According to the graph, a 30-year-old can date between the ages of 22 and 46 years, which seems okay. However, a 60-year-old can date between the ages of 37 and 106 years. To account for lifespan, students might choose to create a sort-of "**old-age limiter**." For instance, the average life span in the United States is around 80 years. Students might opt to include lines on the graph to further restrict the domain and range, thus morphing the Romance Cone into a Romance Kite!

- *Some people think the cone means that as you get older, you can date more people. Is this true?*
- *Why does the cone get taller and taller as the dater gets older and older?*
- *According to the graph, a 60-year-old can date between the ages of 37 and 106. Does this seem realistic?*
- *If not, how might be adjust the graph to account for lifespan?*

Deeper Understanding

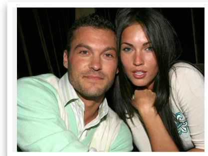
- *As someone gets older, by how much does the permissible age range change? (We know that the slope of the lower line is ½: for every year the dater ages, the age of the youngest person he/she can date increases by half a year. Meanwhile, the age of the oldest eligible person increases by two years. Putting these together, we see that the range increases by 1.5 years each year. We can demonstrate this with an example. A 40-year-old can date between 27 and 66 years, while a 41-year-old can date between 27.5 and 68 years. The permissible range increases from 39 years to 40.5 years.)*

Act Two: The Waiting Game

5 The celebrity couples below had significant age gaps on their wedding days. Was every couple inside the half-plus-seven "**Romance Cone**?" If not, how many years would they need to wait until they entered the "RoCo?"

Mariah Carey: 38
Nick Cannon: 27

Tom Cruise: 44
Katie Holmes: 28



Brian Austin Green: 37
Megan Fox: 24

Clint Eastwood: 65
Dina Ruiz: 30

She's 38, so her minimum age is 0.5(38) + 7 = 26. Nick is 27. He's already old enough, and they're safely within the RoCo.

Yrs.	Him	Min.	Her
0	44	29	28
1	45	29.5	29
2	46	30	30

Wait 2 years

Yrs.	Him	Min.	Her
0	37	25.5	24
1	38	26	25
2	39	26.5	26
3	40	27	27

Wait 3 years

Yrs.	Him	Min.	Her
0	65	39.5	30
…	…	…	…
18	83	48.5	48
19	84	49	49

Wait 19 years

Explanation & Guiding Questions

You can expect many students to use guess-and-check. If so, a table may help students organize their work. However students do it, they may notice a pattern: to determine the waiting time, we simply have to double the difference between how old the younger person *is* and how old he/she *needs to be*. For instance, Katie Holmes was 1 year too young, so she and Tom needed to wait 2 years. Megan Fox was 1.5 years too young, so she and Brian had to wait 3 years. Again, it's all due to slope; every year the minimum age increases by 0.5 years, while the younger person increases by 1 year. In other words, the gap closes by half a year each year.

When students calculate the wait time for Clint Eastwood and Dina Ruiz, they'll find that Mr. Eastwood will be 84 years old when the couple finally enters the RoCo. Depending on where students set their "old age limiter," they might decide that the couple will never actually make it.

- *Do you notice a relationship between the initial difference and the wait time?*
- *By how much is the gap closing each year?*
- *If we set the "old age limiter" at 80 years, will Clint Eastwood and Dina Ruiz make it into the RoCo?*

Deeper Understanding

Advanced students might challenge themselves to write an equation to calculate the number of years a couple will have to wait until they're in the RoCo. There are a various ways to do this. In the equations below, j represents the youngest/junior age, s represents the oldest/senior age, and t represents the wait time in years.

$$\begin{array}{rcl} j & < & 0.5s + 7 \\ j + t & = & 0.5(s + t) + 7 \\ j + t & = & 0.5s + 0.5t + 7 \\ 2j + 2t & = & s + \mathrm{t} + 14 \\ t & = & s - 2j + 14 \end{array}$$

$$\begin{array}{rcl} \text{Wait Time} & = & \text{Twice the "Starting Gap"} \\ t & = & 2 \times (\text{Minimum Age} - \text{Younger Age}) \\ t & & 2 \times [(0.5s + 7) - j)] \\ t & & 2(0.5s + 7 - j) \\ t & & s + 14 - 2j \end{array}$$

6 Finally, once someone is inside your RoCo, will (s)he ever be outside of it again? Explain.

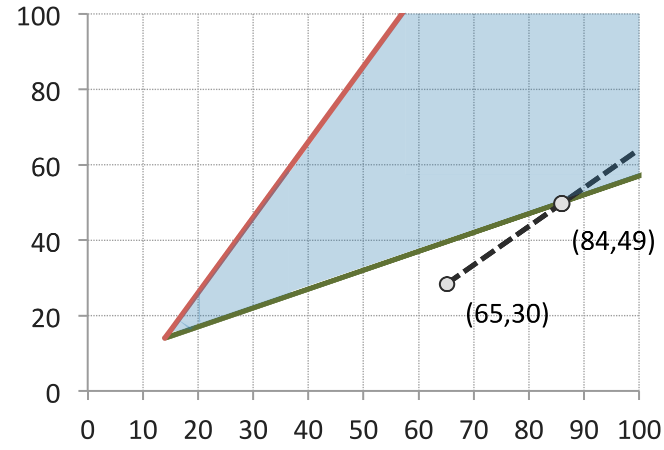
No. Once a couple is inside the RoCo, they will never be outside of it again.

The slope of the bottom line is ½: every year, the minimum datee age increases by half a year. The slope of the top line is 2: every year, the maximum datee age increases by 2 years. However, every year, each person in the couple also ages by one year, i.e. the slope of the aging process is 1. If the reason the couple didn't start inside the Romance Cone is because the younger person was too young, it's only a matter of time until he/she is old enough. Once this happens, the couple will always remain inside the cone.

Explanation & Guiding Questions

Once a couple enters the RoCo, they will never leave it. The slope of the green line is **0.5 years per year**: every year, the age of the youngest datee increases by half a year. The slope of the red line is **2 years per year**: every year, the age of the oldest datee increases by 2 years.

Of course, with every year that passes, the age of both people increases by one year, and a line representing the aging process will have a slope of **1 year per year**. Once this line enters the RoCo, it will always lie between the "youngest" line (slope = 0.5) and the "oldest" line (slope = 2). Put another way, it will always be steeper than the bottom line, but less steep than the top.



We can see this with the Clint Eastwood/Dina Ruiz example above. When they marry, he is 65, she is 30, and they are outside the cone. As they age, their ages increase by one year every year, as represented by the dashed line. The slope of this line is 1 year/year, while the slope of the "younger" line is only 0.5 years/year. Thus, Dina's age is changing at a faster rate than is the minimum age. After 19 years, the dashed line intersects the "younger" line. However, it will never intersect the "older" line (slope = 2), since the upper line is growing at an even faster rate!

- *What is the slope of the lower line, and what is the slope of the upper line?*
- *What is the slope of the aging process, and how can you show this on your graph?*
- *Once the black line enters the cone, will it ever be below the green line again? Will it ever be above the red?*